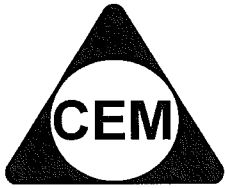


NAME :



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YEAR 12 – MATHEMATICS

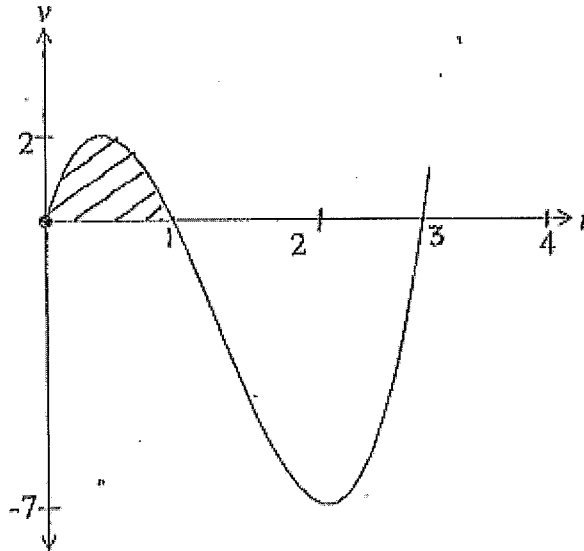
SPECIMEN PAPER 2
TOPIC : APPLICATIONS OF
CALCULUS TO PHYSICAL
WORLD

(iii) the times when the particle reverses the direction of its motion. 1

(iv) the distance travelled in the fifth second. 1

ASCHAM 2001 Q9

a)



not drawn to scale

The graph represents the velocity (in m/s) of a particle after t seconds.
The particle is moving in a straight line.

- i) What is the velocity of the particle after 0.5 seconds? (1)
- ii) What is the acceleration of the particle after 2 seconds? (1)

iii) At what time(s) does the particle change direction? (1)

iv) What does the shaded area on the graph represent? (1)

v) When is the speed of the particle maximum? ($0 \leq t \leq 3$) (1)

ASCHAM 2001 Q10

b) A particle P is moving in a straight line along the x axis. Its position at time t seconds is given by the equation $x = e^{\frac{t}{2}} - t$.

i) Find an expression for the velocity of the particle at time t . (1)

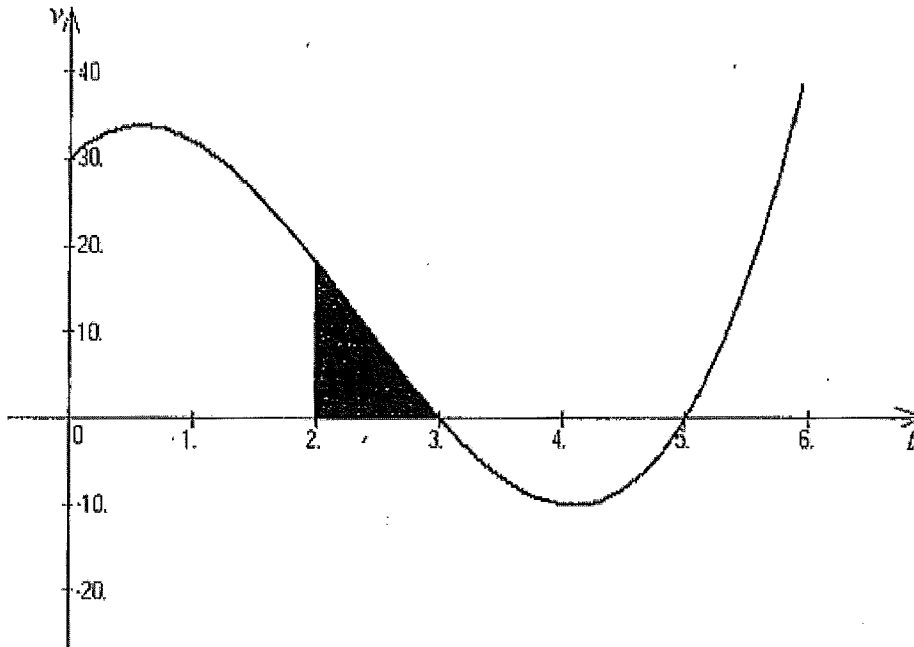
ii) In what direction is the particle moving initially? (1)

iii) When does the particle come to rest? (1)

iv) Find the acceleration of the particle at time t . (1)

v) What comment can you make about the acceleration and how does it affect the future motion of the particle. (1)

vi) Find the distance travelled by the particle in the first 3 seconds. (2)

CSSA 2001 Q6

A particle moves along a straight line for 6 seconds. The particle's velocity v at time t seconds is shown on the graph above.

- (i) When is the particle at rest? 1
- (ii) What does the shaded region represent? 1
- (iii) Is this particle further from its initial position at $t = 3$ or at $t = 5$? 1
Explain briefly.

CSSA 2001 Q9

(b) A particle is moving along the x axis. Its position x at time t is given by

$$x = 60t + 100e^{\frac{-t}{5}} \quad (t \geq 0).$$

- (i) Find the initial position of the particle. 1
- (ii) Show that the particle is always moving to the right. 2
- (iii) What happens to the acceleration eventually? 2

CSSA 2000 Q7

- (b) A particle moves in a straight line. At time t seconds, its displacement, x metres from a fixed point O on the line is given by

7

$$x = 1 - \cos \pi t$$

- (i) What is the initial displacement of the particle?
- (ii) Sketch the graph of x as a function of t .

(iii) Find an expression for the velocity of the particle at any time t .

(iv) What is the velocity of the particle at time $t = \frac{1}{6}$?

(v) At what time does the particle first reach its maximum speed?

SOLUTIONSAMP 2002 Q8

(a) $a = 6t - 18$

(i) $v = \int 6t - 18 \, dt$
 $v = 3t^2 - 18t + c \checkmark$

at $t = 0, v = 24$

$24 = c$

So $v = 3t^2 - 18t + 24 \checkmark$

(ii) $s = \int 3t^2 - 18t + 24 \, dt$
 $s = t^3 - 9t^2 + 24t + k \checkmark$

at $t = 0, s = 0 \Rightarrow k = 0$

$\therefore s = t^3 - 9t^2 + 24t \checkmark$

(iii) $v = 0$

$3t^2 - 18t + 24 = 0$

$t^2 - 6t + 8 = 0$

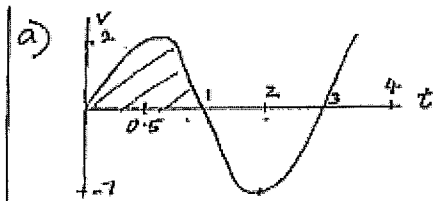
$(t - 4)(t - 2) = 0$

$t = 4 \text{ or } 2 \checkmark$

(iv) when $t = 4, s = 16 \text{ m}$

when $t = 5, s = 20 \text{ m}$

In the fifth second the distance is 4m. \checkmark

ASCHAM 2001 Q9

- i) $v = 2 \text{ m/s}$ in \rightarrow after 0.5 sec . ✓
- ii) $a = 0$ when $t = 2$ ✓
- iii) Particle changes direction when $v = 0$ i.e. $t = 1$ & $t = 3$ ✓
- iv) Shaded area represents the distance travelled by the particle in the first 1 second. ✓
- v) Speed max i.e. 7 m/s when $t = 2$ ✓

ASCHAM 2001 Q10

b $x = e^{\frac{t}{2}} - t$

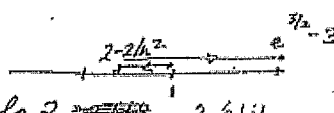
i) $v = \frac{1}{2} e^{\frac{t}{2}} - 1$ ✓

ii) $t=0 \quad v = \frac{1}{2} \times 1 - 1$
 $= -\frac{1}{2} < 0$
 \therefore particle moving to left ✓

iii) $v=0$
 $\frac{1}{2} e^{\frac{t}{2}} = 1$
 $e^{\frac{t}{2}} = 2$
 $\ln 2 = \frac{t}{2}$
 $t = 2 \ln 2$
 $= 1.39 \text{ (2dp)}$
 \therefore particle comes to rest ✓
 after $2 \ln 2$ or 1.39 sec .

iv) $a = \frac{1}{4} e^{\frac{t}{2}}$ ✓

v) $a > 0$ for all t
 \therefore after $2 \ln 2$ sec when
 particle is moving to right $v > 0$ ✓
 particle continues moving to
 right & speeds up. ($a > 0, v > 0$)

vi) $t=0 \quad x=1$  $2[1 - (2-2\ln 2)] + e^{3/2} - 3 - 1$

$t=2 \ln 2 \quad x = 2 - 2 \ln 2$ ≈ 0.511

$t=3 \quad x = e^{3/2} - 3$

Particle's distance = $1 - (2-2\ln 2) + e^{3/2} - 3 - (2-2\ln 2)$ $\frac{dx}{dt}$
Use definite
integrals

$= 1 - 2 + 2\ln 2 + e^{3/2} - 3 - 2 + 2\ln 2$

$= -6 + e^{3/2} + 4\ln 2$ ✓✓ (7)

$= 1.254 \text{ units (3dp)}$

CSSA 2001 Q6

- (c) (i) $t = 3$ or $t = 5$
- (ii) The shaded region represents the distance travelled during the third second.
- (iii) The particle changes direction at $t = 3$ (after it has come to rest) and begins to move back towards its initial position. Hence, the particle is further from its initial position at $t = 3$.

CSSA 2001 Q9

(b) $x = 60t + 100e^{-\frac{t}{5}}$

When $t = 0, x = 100e^0$

(i) $= 100$

Initially, the particle is 100 units to the right of the origin

(ii)

$$\frac{dx}{dt} = 60 - \frac{1}{5} \cdot 100e^{-\frac{t}{5}}$$

$$= 60 - 20e^{-\frac{t}{5}}$$

$$> 0 \text{ (for all } t \geq 0)$$

hence particle is always moving to the right

(iii)

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\frac{1}{5} \cdot -20e^{-\frac{t}{5}}$$

$$= 4e^{-\frac{t}{5}}$$

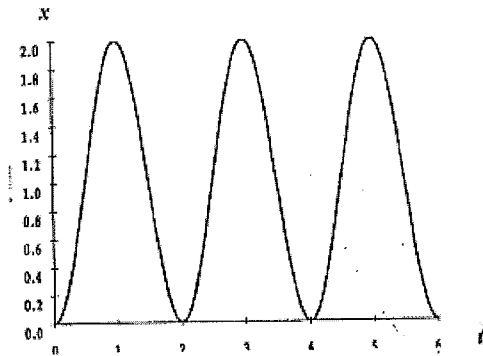
$$\text{As } t \rightarrow \infty \frac{d^2x}{dt^2} \rightarrow 0$$

CSSA 2000 Q7

- (b)
 (i) When $t = 0$, $x = 1 - \cos 0 = 0$
 \therefore The particle is initially at $x = 0$.

(iii) $v = \pi \sin \pi t$.

(ii)



(iv) When $t = \frac{1}{6}$, $v = \pi \sin \frac{\pi}{6} = \frac{\pi}{2}$ m/s

(v) Maximum speed when acceleration is zero.

$$a = \pi^2 \cos \pi t = 0$$

$$\therefore \cos \pi t = 0$$

$$\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$\therefore t = \frac{1}{2}$ second when the particle first reaches its maximum speed.