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MATHEMATICS
SPECIMEN PAPER 1

ARITHMETIC SERIES & SEQUENCES

1. The 12^{th} term of an arithmetic progression is 32.5, and the 20^{th} term is 52.5.

   Find (a) the first term [2]

   (b) the common difference [2]

   (c) the sum of the first 18 terms. [2]

2. Given the arithmetic progression $-5, -2, 1, 4 \ldots\ldots,$
   how many terms are needed so that the sum of the series is 918. [4]

3. The 10^{th} term of an arithmetic progression is -2, whilst the sum of the first thirty terms is 105.

   (a) Find the common difference, d [5]

   (b) Find the sum of the first sixty terms [2]

4. For the series 22 + 18 + 14 + 10 + ...

(a) Find the 30th term. [2]

(b) Find the sum of the first fifty terms. [2]

5. Given that:

(a) An arithmetic series has first term a and common difference d. Prove that the sum of the first n terms, S_n, is given by

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$ [3]

(b) The first term of an arithmetic progression is – 12 and the common difference is 2. How many terms of the *AP* are required before the sum is positive? [3]

6. The first term of an arithmetic progression is 1, and the sum of the first 20 terms is 1,540

(a) Find the common difference [3]

(b) Find the 30^{th} term of the series [2]

Solutions:

1. The n^{th} term of an arithmetic progression is

$$T_n = a + (n-1)d$$

$$\therefore \quad a + 11d = 32.5 \quad - \quad \{1\}$$

$$a + 19d = 52.5 \quad - \quad \{2\}$$

Now solve the equations {1} and {2} simultaneously

$$\{2\} - \{1\} \quad 8d = 20$$

$$d = 2.5$$

Substitute in {1}

$$a + 11 \times 2.5 = 32.5$$

$$\therefore \Rightarrow \quad a = 5$$

(a) First term, $a = 5$

(b) Common difference $d = 2.5$

(c) The sum of n terms $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\therefore$ The sum of 18 terms $S_{18} = \frac{18}{2}\left[2 \times 5 + (18-1) \times 2.5\right]$

$$\Rightarrow \quad S_{18} = 472.5$$

2. $a = -5 \qquad d = -2 - (-5)$

$$d = 3$$

The sum of n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\therefore \quad \frac{n}{2}\left(2 \times (-5) + (n-1) \times 3\right) = 918$$

$$\frac{n}{2}\left(-10 + 3n - 3\right) = 918$$

$$\frac{n}{2}\left(3n - 13\right) = 918$$

$$\times 2 \quad n(3n - 13) = 1836$$

$$3n^2 - 13n - 1836 = 0$$

$\Rightarrow \quad (3n + 68)(n - 27) = 0$

$\Rightarrow \quad 3n + 68 = 0 \quad$ or $\quad n - 27 = 0$

$n = -\dfrac{68}{3} \quad$ or $\quad n = 27$

As n must be a positive integer
27 terms are required so that the sum is 918

3.

(a) The n^{th} term of an arithmetic series is

$$T_n = a + (n - 1)d$$

$\therefore \quad a + 9d = -2 \quad - \quad \{1\}$

The Sum of n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\frac{30}{2}(2a + 29d) = 105$$

$\div 15 \quad 2a + 29d = 7 \quad - \quad \{2\}$

$\{1\} \times 2 \quad 2a + 18d = -4 \quad - \quad \{3\}$

$\{2\} - \{3\} \quad 11d = 11$

$d = 1$

(b) Firstly, find the first term
Substitute $d = 1$ in $\{1\}$

$a + 9 \times 1 = -2$

$\Rightarrow \quad a = -11$

The sum of the first 60 terms

$$S_{60} = \frac{60}{2}\left[2\times(-11) + 59\times 1\right]$$

$$S_{60} = 1110$$

4.

(a) $22 + 18 + 14 + 10 + \ldots.$
is an arithmetic series because

$18 - 22 = -4$

$14 - 18 = -4$

$10 - 14 = -4$

This means that the common difference $d = -4$, and $a = 22$
The n^{th} term of the arithmetic series is

$$T_n = a + (n - 1)d$$

the 30^{th} term =

$$\begin{aligned} T_{30} &= 22 + (30 - 1) \times (-4) \\ &= 22 - 29 \times 4 \\ &= 22 - 116 \\ T_{30} &= -94 \end{aligned}$$

(b) The sum of the first n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

The sum of the first 50 terms

$$\begin{aligned} S_{50} &= \frac{50}{2}\left[2\times 22+(50-1)\times(-4)\right] \\ &= 25(44 - 49 \times 4) \\ S_{50} &= -3,800 \end{aligned}$$

5.

(a) $S_n = a + (a + d) + (a + 2d) + \ldots. + (a + (n - 1)d)$

Now reverse the order and add

$S_n = a + (n - 1)d + (a + (n - 2)d) + (a + (n - 3)d) + \ldots + a$

$2S_n = (2a + (n - 1)d) + (2a + (n - 1)d) + \ldots.. + (2a + (n - 1)d)$

By reversing, each pair added has the same sum.

$\therefore \quad 2S_n = n(2a + (n - 1)d)$

$\div 2 \quad S_n = \frac{n}{2}\left(2a+(n-1)d\right)$

(b) $a = -12, d = 2$

$\therefore$ For $S_n \geq 0$

$$\begin{aligned} \frac{n}{2}\left(2\times(-12)+(n-1)2\right) &\geq 0 \\ \frac{n}{2}(2n-26) &\geq 0 \\ \frac{2n}{n}(n-13) &\geq 0 \end{aligned}$$

$$n(n-13) \geq 0$$

$\therefore$ As n is positive

$$n-13 \geq 0$$
$$n \geq 13$$

$\therefore$ At least 13 terms must be added in order that the sum is positive.

6.

(a) The sum of the first n terms of an arithmetic series is

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$a = 1$

$$\therefore \quad \frac{20}{2}\left(2\times1+(20-1)d\right) = 1{,}540$$
$$10(2 + 19d) = 1{,}540$$
$$\div 10 \qquad 2 + 19d = 154$$
$$19d = 152$$
$$\div 19 \qquad d = 8$$

(b) The n^{th} term of an arithmetic series is

$$T_n = a + (n-1)d$$
$$\therefore 30^{\text{th}} \text{ term } = \quad T_{30} = 1 + (30-1)8$$
$$T_{30} = 233$$
