NAME :

Centre of Excellence in Mathematics
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YEAR 12 – MATHEMATICS

SPECIMEN PAPER 2

TOPIC : BASIC & CALCULUS WITH TRIG. FUNCTIONS

AMP 2002Q3

(b) Differentiate with respect to x.

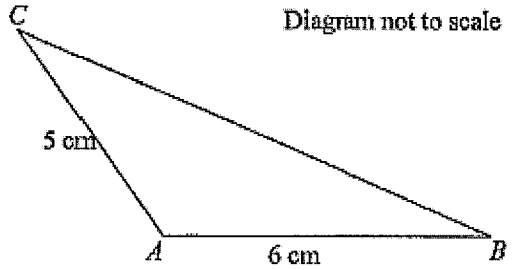
(iii) $\dfrac{\cos 3x}{x}$

AMP 2002 Q4

(c) Evaluate $\int_0^{\frac{\pi}{3}} \sin 3x \, dx$. 2

AMP 2002 Q6

(b) In the triangle ABC, $b = 5$ cm and $c = 6$ cm.



The area of the triangle is 7.5 sq. cm.

(i) Find the size of the obtuse angle BAC. **2**

(ii) Find the length of CB correct to two decimal places. **2**

AMP 2002 Q10

(d) If $y = A\cos 4x + B\sin 4x$, show that $\frac{d^2y}{dx^2} + 16y = 0$. 3

CSSA 2001 Q1

(d) Find the value of $\cos\frac{\pi}{8}$, correct to 3 decimal places. 2

(e) Solve $\tan\theta = -\frac{1}{\sqrt{3}}$ for $0^\circ \leq \theta \leq 360^\circ$. 2

CSSA 2001 Q3

(a) Find:

(i) $\int \sec^2 4x\,dx$ **1**

(d) Differentiate the following

(i) $x^3 \sin x$ **2**

CSSA 2001 Q7

(a) (i) Show that $x = \frac{2\pi}{3}$ is a solution of $\cos x = \cos 2x$. 1

(ii) On the same set of axes, sketch the graphs of $y = \cos x$ and $y = \cos 2x$ for $0 \leq x \leq \pi$, showing the x coordinate of all points of intersection. 2

(iii) Find the exact **area** of the region bounded by the curves $y = \cos x$ and $y = \cos 2x$ over the interval $0 \leq x \leq \frac{2\pi}{3}$. 3

CSSA 2000 Q3

(a) Differentiate the following expressions with respect to x:

(ii) $3x \cos 2x$ 2

(b) Evaluate the following definite integrals:

(i) $\int_0^{\pi/4} \sin 3x \, dx$ 2

CSSA 2000 Q8

(c) (i) On the same number plane, sketch the curves $y = \sin x$ and $y = \tan \frac{x}{2}$ in the domain $0 \leq x \leq 2\pi$. **3**

(ii) Hence find the number of real solutions to the equation $\sin x = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$.

SOLUTIONS

AMP 2002 Q3

(iii) $\frac{d}{dx}\left(\frac{\cos 3x}{x}\right) = \frac{x(-\sin 3x).3 - \cos 3x.1}{x^2}$ ✓

$= \frac{-3x\sin 3x - \cos 3x}{x^2}$ ✓

AMP 2002 Q4

(c) $\int_0^{\frac{\pi}{3}} \sin 3x \, dx$

$= \left[-\frac{\cos 3x}{3}\right]_0^{\frac{\pi}{3}}$ ✓

$= \left(-\frac{\cos \pi}{3}\right) - \left(-\frac{\cos 0}{3}\right)$

$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ ✓

AMP 2002 Q6

(i) $7.5 = \frac{1}{2} \times 5 \times 6 \times \sin\theta$ ✓

$0.5 = \sin\theta$

$150^0 = \theta$ (θ is obtuse) ✓

(ii) $CB^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos 150^0$ ✓

$CB^2 = 112.9615$

$CB = 10.63$ cm ✓

AMP 2002 Q10

(d) $y = A\cos 4x + B\sin 4x$

$\frac{dy}{dx} = -4A\sin 4x + 4B\cos 4x$ ✓

$\frac{d^2y}{dx^2} = -16A\cos 4x - 16B\sin 4x$ ✓

$\frac{d^2y}{dx^2} = -16(A\cos 4x + B\sin 4x)$

$\frac{d^2y}{dx^2} = -16y$ ✓

CSSA 2001 Q1

(d) $\cos\frac{\pi}{8} = 0.9238795...$

$= 0.924$ correct to 3 d.pl.

(e) $\theta = 180^\circ - 30^\circ$ or $360^\circ - 30^\circ$

$= 150^\circ$ or 330°

CSSA 2001 Q3

(a) (i) $\int \sec^2 4x\,dx = \frac{1}{4}\tan dx = \frac{1}{4}\tan 4x + c$

(d) (i) $x^3\cos x + 3x^2\sin x$

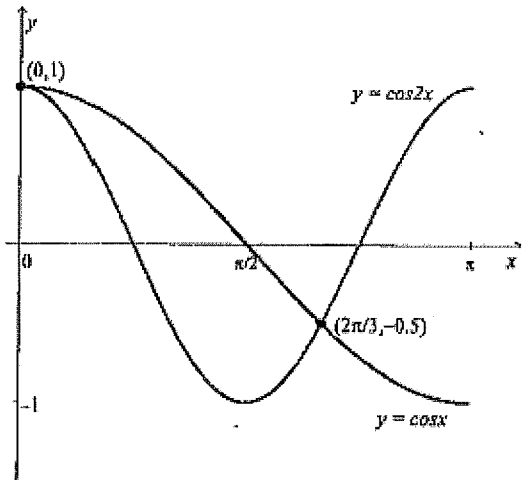
CSSA 2001 Q7

(a) (i) $\cos\frac{2\pi}{3} = -\frac{1}{2}$

$\cos 2\left(\frac{2\pi}{3}\right) = \cos\frac{4\pi}{3}$

$= -\frac{1}{2}$

(ii)



(iii) Area $\int_0^{\frac{2\pi}{3}} (\cos x - \cos 2x)\,dx$

$$= \left[\sin x - \frac{1}{2}\sin 2x\right]_0^{\frac{2\pi}{3}}$$

$$= \sin\frac{2\pi}{3} - \frac{1}{2}\sin\frac{4\pi}{3} - (0-0)$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{4}\text{ units}^2$$

CSSA 2000 Q3

(ii) $\frac{d}{dx}(3x \cos 2x)$

$= 3x \times -2 \sin 2x + 3 \cos 2x$

$= 3 \cos 2x - 6x \sin 2x$

(i) $\int_0^{\pi/4} \sin 3x \, dx = \left[-\frac{1}{3} \cos 3x \right]_0^{\pi/4}$

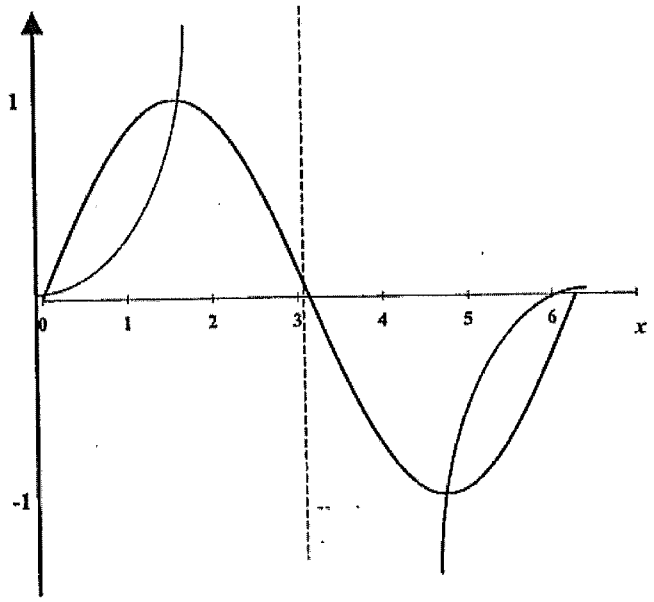
$= -\frac{1}{3} \left[\cos \frac{3\pi}{4} - \cos 0 \right]$

$= -\frac{1}{3} \left[-\frac{1}{\sqrt{2}} - 1 \right] = \frac{1}{3} \left(\frac{1+\sqrt{2}}{\sqrt{2}} \right)$

CSSA 2000 Q8

(c)

(i)



(ii) $\sin x = \tan \frac{x}{2}$ has 4 solutions in the domain $0 \le x \le 2\pi$.