NAME :

Centre of Excellence in Mathematics
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YEAR 12 – ADVANCED MATHS

REVIEW TOPIC (SP1)

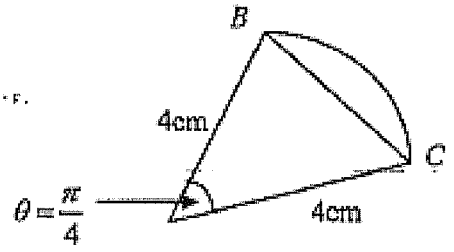
CIRCULAR FUNCTIONS I &II

CEM – Yr 12 – 2U Circular Functions 1 & 2 – Review Paper 2

1. A circular disc has area $A = 16\pi \text{ cm}^2$. A sector, with area $2\pi \text{ cm}^2$, is cut out of the disc.

   (i) Show that the sector has radius = 4 cm and angle $\theta = \frac{\pi}{4}$ radians.

   (ii) Show that the exact area of the minor segment cut off by the chord BC is $(2\pi - 4\sqrt{2}) \text{ cm}^2$



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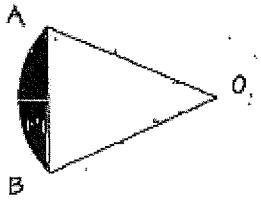
2. In the figure below OA and OB are radii of a circle with centre O. They both measure 10cm in length. The arc AB subtends an angle of $\frac{\pi}{3}$ radians at O. AB is a chord of the circle.



i) Calculate the exact area of the sector AOB

ii) Calculate the exact area of the triangle AOB and hence find the area of the shaded segment of the circle.

3. iv.

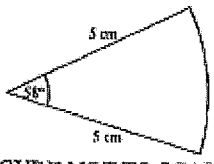


FIGURE NOT TO SCALE.

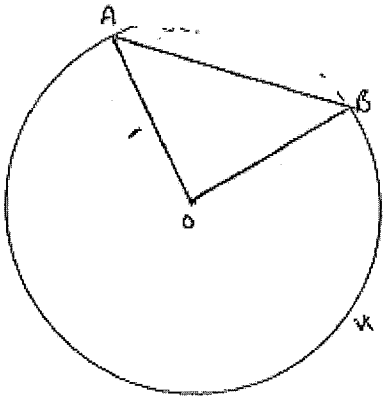
The diagram shows the sector of a circle.
Find the area of this sector. Give your answer to the nearest square centimetre.

4. A major sector of a circle, radius 1 unit, has an arc length of 4 units.

(i) Find the angle of the major sector, at the centre of the circle, in radians.

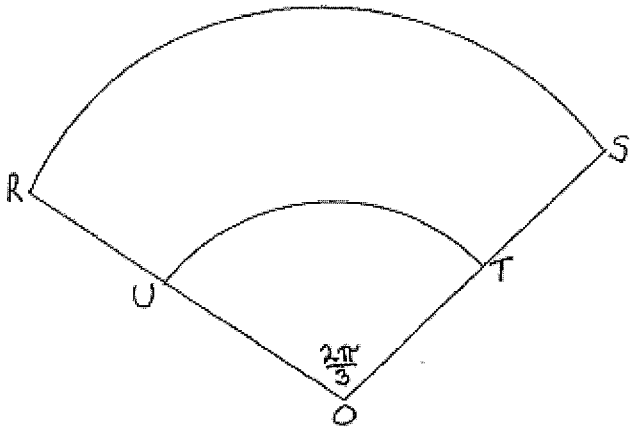
(ii) Find the area of the major sector.

(iii) Calculate the area of the triangle OAB cut off by the chord joining the endpoints of the two radii.

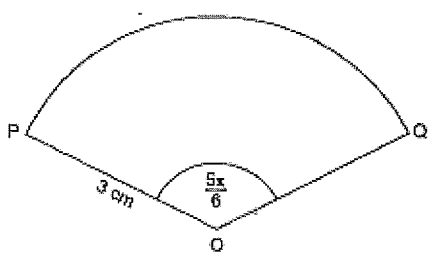


5.) A car windscreen wiper traces out the area *RSTU* where *RS* and *UT* are arcs of circles centre O, radius 40cm and 20 cm respectively, as shown in the figure.

Calculate the exact perimeter of RSTU.



6. In the diagram below, PQ is the arc of a circle with centre O.
The radius $OP = 3$cm and the angle POQ is $\frac{5\pi}{6}$ radians.



Find the exact length of the arc PQ.

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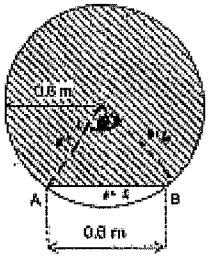
7. A table top is in the shape of a circle with a small segment removed as shown. The circle has centre O and radius 0.6 metres. The length of the straight edge AB is also 0.6 metres.



i) Explain why $\angle AOB = \frac{\pi}{3}$ radians

ii) Find the area of the table top to 2 decimal places

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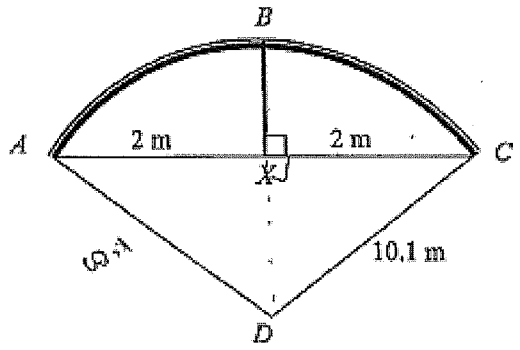
8. A bridge's steel arch ABC is part of a circle of radius 10.1 metres. BX bisects the chord AC which is 4 metres long.

NOT TO SCALE



(i) Find the size of angle ADC correct to the nearest degree.

(ii) Find the length of steel needed to make the arch ABC.

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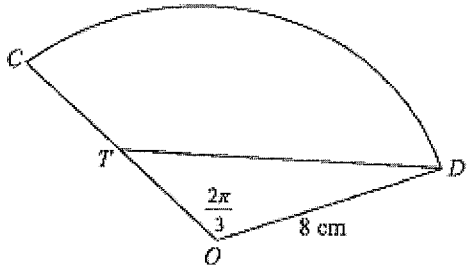
9. In the diagram, CD is an arc of a circle with radius 8 cm and centre O.
T is the midpoint of OC. Angle COD is $\frac{2\pi}{3}$.



NOT TO SCALE

Find the perimeter of CTD in exact form.

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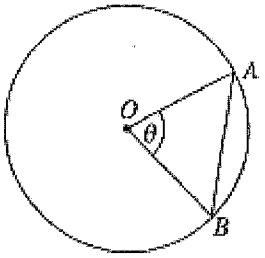
10. The diagram shows a circle with centre O and radius 2 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$.



NOT TO SCALE

(i) There are two possible values of θ for which the area of $\triangle AOB$ is $\sqrt{3}$ square centimetres. One value is $\frac{\pi}{3}$. **2**

Find the other value.

(ii) Suppose that $\theta = \frac{\pi}{3}$.

(1) Find the area of the sector AOB. **1**

(2) Find the exact length of the perimeter of the minor segment bounded by the chord AB and the arc AB. **2**

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Answers

1. $\pi r^2 = 16\pi \quad \therefore r = 4$

$\frac{1}{2}(4)^2\theta = 2\pi \quad \therefore \theta = \frac{\pi}{4}$

$$\text{Area} = \frac{1}{2}(4)^2\left[\frac{\pi}{4} - \sin\frac{\pi}{4}\right]$$
$$= 2\pi - 8\sin\frac{\pi}{4}$$
$$= 2\pi - 8\times\frac{1}{\sqrt{2}} = 2\pi - 4\sqrt{2}$$

2. i- $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 10^2 \times \frac{\pi}{3}$ ✓

$= \frac{50\pi}{3}\ cm^2$ ✓

ii- $A_\Delta = \frac{1}{2}\times 10^2 \times \sin\frac{\pi}{3}$

$= 50\times\frac{\sqrt{3}}{2}$

$= 25\sqrt{3}\ cm^2$

* A_s = area of sector − area of triangle

$= \frac{50\pi}{3} - 25\sqrt{3}$

$\doteq 9.06\ cm^2$ ✓

3. (iv) $A = \frac{58}{360}\times\pi\times 5^2$

$= 12.65$

$\doteq 13$

4. (i) $l = r\theta \qquad 4 = 1\times\theta$

$\theta = 4$ radians

(ii) Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 1^2\times 4$

$= 2\ units^2$

(iii)

$A_\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}r^2\sin(2\pi - \theta)$

$= \frac{1}{2}\times 1^2\times\sin(2\pi - 4)$

$\doteq 0.3784\ units^2$

(to 4 d.p.)

5. $R = 40cm \quad r = 20cm$

$\theta = \frac{2\pi}{3}$

Arc $RS = 40\times\frac{2\pi}{3}$

Arc $UT = 20\times\frac{2\pi}{3}$

$QU = ST = 40 - 20 = 20$

Perimeter $= \frac{2\pi}{3}(40+20) + 2\times 20$

$= \frac{120\pi}{3} + 40$

$= 40\pi + 40$

$= 40(\pi + 1)$

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6. $l = r\theta = 3 \times \frac{5\pi}{6} = \frac{5\pi}{2}$ cm

7. i) Triangle is equilateral, all sides 0.6m — 1 mark

ii) Area of segment =

$= \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2}(0.6)^2 \left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$

$= 0.0326$ — 1 mark (area of minor segment)

Hence area of table = circle – segment =

$= \pi(0.6)^2 - 0.0326...$

$= 1.098...$

$= 1.10$ sq units (2dp) — 1 mark (set up with subtraction from whole circle area)

1 mark (final answer to 2dp)

[minus ½ mark for incorrect rounding]

8. (b) (i) (2 marks)

$\cos\angle ADC = \frac{10.1^2 + 10.1^2 - 4^2}{2 \times 10.1 \times 10.1}$

$= 0.92$

$\angle ADC = 23°$

(b) (ii) (2 marks)

$23° = \frac{\pi}{180} \times 23$ radians

$= 0.4$ radians

$l = r\theta$

$= 10.1 \times 0.4$

$= 4.04$ metres

9. Length DT

$OT = 4$ cm (midpoint of OC)

$DT^2 = 4^2 + 8^2 - 2 \times 4 \times 8 \cos\left(\frac{2\pi}{3}\right)$

$DT^2 = 112$

$DT = 4\sqrt{7}$

Perimeter $= DT +$ arc $CD + CT$

$= \left(4\sqrt{7} + \frac{16\pi}{3} + 4\right)$ cm

Arc length CD

$l = r\theta$

$CD = 8 \times \frac{2\pi}{3}$

$= \frac{16\pi}{3}$

10. (i) Area of $\Delta AOB = \frac{1}{2} ab \sin O$

$\sqrt{3} = \frac{1}{2} \times 2 \times 2 \times \sin\theta$

$\sqrt{3} = 2\sin\theta$

$\therefore \sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}, \pi - \frac{\pi}{3}$

$= \frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore$ Other value of $\theta = \frac{2\pi}{3}$.

(ii) (1) Area of sector $AOB = \frac{1}{2} r^2\theta$

$= \frac{1}{2} \times 2^2 \times \frac{\pi}{3}$

$= \frac{2\pi}{3}$ cm^2.

(2) $\theta = \frac{\pi}{3} = 60°$

$\therefore \angle A = \angle B = 60°$

$\therefore AB = 2$ cm

since ΔAOB is equilateral

Arc AB: $l = r\theta$

$= 2 \times \frac{\pi}{3} = \frac{2\pi}{3}$ cm

$\therefore$ Perimeter $= \left(\frac{2\pi}{3} + 2\right)$ cm.