

Mixed exercises:

(1) $O(0,0), P(1,0), Q(2,1)$ are the vertices of a triangle. Find:

(a) The equation of the line PQ in general form.

$$x - y - 1 = 0$$

(b) The equation of the line through O

(i) parallel to PQ .

$$x - y = 0$$

(ii) perpendicular to PQ .

$$x + y = 0$$

- (2) Find the equation of the line through the point $(5, 2)$ which is perpendicular to the line $5x + 2y = 20$.

- (3) The line PQ is perpendicular to the line $x - y - 1 = 0$, meets the line in the point P and intersects the y -axis in $Q(0, 5)$. Find the coordinates of P . (Hint: draw a sketch).

$$2x - 5y = 0$$
$$x + y - 5 = 0, P(3, 2)$$

- (4) Find the coordinates of the foot of the perpendicular from the point $(-5, 4)$ to the line $3x + y - 21 = 0$.

$$\left(4\frac{3}{5}, 7\frac{1}{5}\right)$$

- (5) Show that the triangle whose sides satisfy $2x - y = 0$, $x + 2y = 5$, $x - 3y = 20$ is a right triangle.

- (6) Find the equation of the line through the intersection of the lines $x + y - 3 = 0$ and $3x + 4y - 1 = 0$ and perpendicular to the line $3x - 4y + 15 = 0$.

$$k = -\frac{1}{7}; 4x + 3y - 20 = 0$$

(7) Find the equations of the lines given by $(2x - y + 4) + k(3x + 2y - 9) = 0$ when k takes the values $-1, 0, 1$. Verify that these three lines are concurrent.

(8) Find the (i) midpoint and (ii) the distance between these pairs of points:

(a) $A(4,3)$ and $B(7,5)$

$$\boxed{\text{(i) } \left(\frac{11}{2}, 4\right) \text{ (ii) } \sqrt{13}}$$

(b) $P(-2,-3)$ and $Q(-5,-4)$

$$\boxed{\text{(i) } \left(-\frac{7}{2}, -\frac{7}{2}\right) \text{ (ii) } \sqrt{10}}$$

(9) Find the perpendicular distance from the point $(6, -5)$ to the line $4x - 3y - 9 = 0$.

6

(10) Determine whether the given points $(3, 2)$ and $(7, 3)$ lie on the same or opposite sides of the given line $2x - 5y + 3 = 0$.

opposite sides

(11) Show that the line $4x + 3y - 8 = 0$ is a tangent to the circle with centre $(-2, -3)$ and radius 5 units.

(12) Find the distance between these parallel lines:

$$5x - 12y + 21 = 0 \text{ and } 5x - 12y - 5 = 0.$$

(13) $ABCD$ is a rhombus where $A(0, -1)$, $B(3, -2)$ and $C(4, 1)$ are three vertices. Find the coordinates of the fourth vertex.

$(1, 2)$

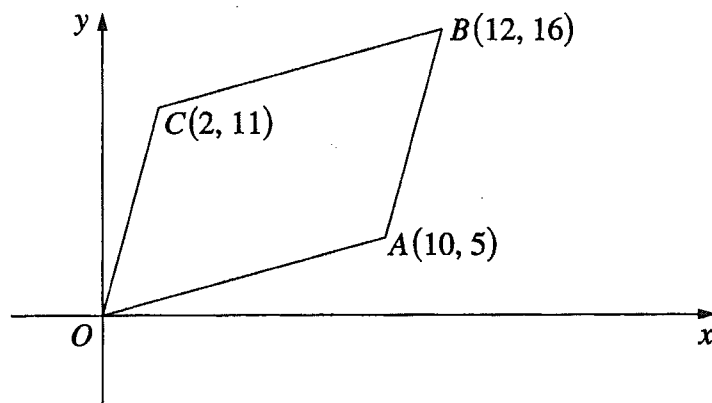
PAST HSC QUESTIONS:**HSC 07**

(1)

- (f) Find the equation of the line that passes through the point $(-1, 3)$ and is perpendicular to $2x + y + 4 = 0$. 2

$$x - 2y + 7 = 0$$

(a)

NOT
TO
SCALE

(3)

In the diagram, A , B and C are the points $(10, 5)$, $(12, 16)$ and $(2, 11)$ respectively.

Copy or trace this diagram into your writing booklet.

(i) Find the distance AC .

1

(ii) Find the midpoint of AC .

 $\boxed{10}$
1

(iii) Show that $OB \perp AC$.

 $\boxed{(6,8)}$
2

(iv) Find the midpoint of OB and hence explain why $OABC$ is a rhombus.

2

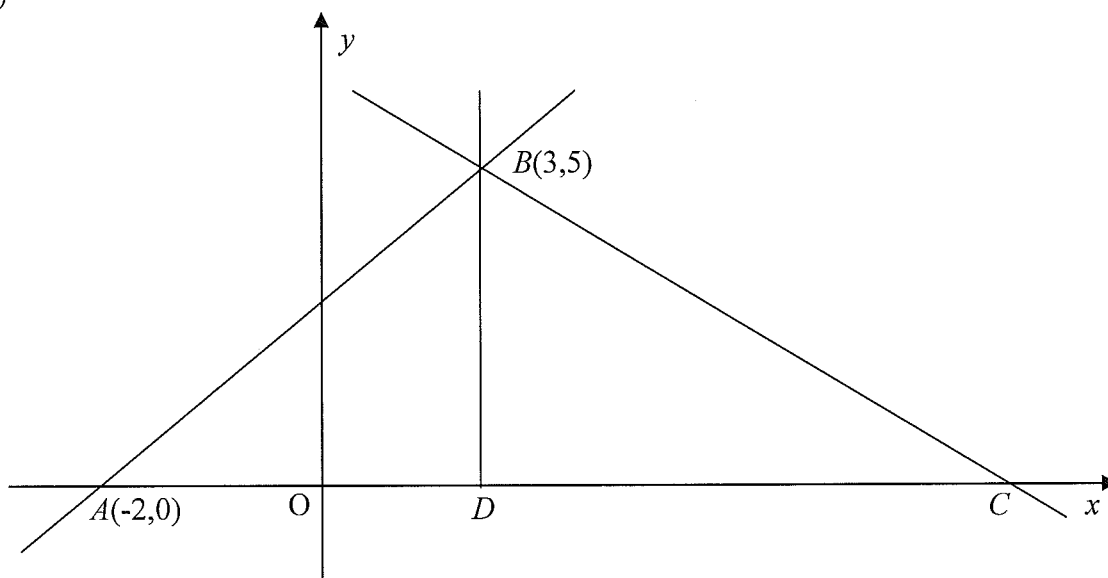
(v) Hence, or otherwise, find the area of $OABC$.

1

100 units ²

HSC '99

(2) (b)



The diagram shows the points $A(-2,0)$, $B(3,5)$ and the point C which lies on the x axis. The point D also lies on the x axis such that BD is perpendicular to AC .

(i) Show that the gradient of AB is 1.

(ii) Find the equation of the line AB .

$$y = x + 2$$

(iii) What is the size of $\angle BAC$?

45°

(iv) The length of BC is 13units. Find the length of DC .

12 units

(v) Calculate the area of $\triangle ABC$.

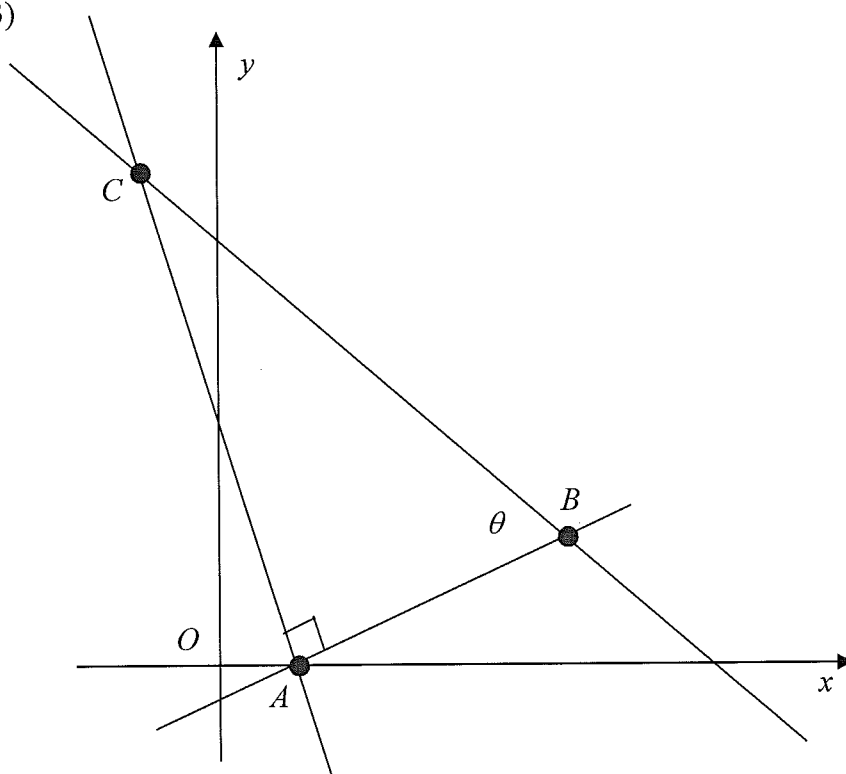
42.5 sq. units

(vi) Calculate the size of $\angle ABC$ to the nearest degree.

112°

HSC '98

(3)



The diagram shows points $A(1,0)$, $B(4,1)$ and $C(-1,6)$ in the Cartesian plane. Angle ABC is θ .

Copy or trace this diagram into your Writing Booklet.

(a) Show that A and C lie on the line $3x + y = 3$.

2

(b) Show that the gradient of AB is $\frac{1}{3}$.

1

(c) Show that the length of AB is $\sqrt{10}$ units.

1

(d) Show that AB and AC are perpendicular.

1

(e) Find $\tan \theta$.

2

(f) Find the equation of the circle with centre A that passes through B .

2

$$x^2 + y^2 - 2x - 9 = 0$$

(g) The point D is not shown on the diagram. The point D lies on the line $3x + y = 3$ between A and C , and $AD = AB$. Find the coordinates of D .

2

$$(0, 3)$$

(h) On your diagram, shade the region satisfying the inequality $3x + y \leq 3$.

1

HSC '97

(3) (b) Let A and B be the points $(0, 1)$ and $(2, 3)$ respectively.

6

(i) Find the coordinates of the midpoint of AB .

 $(1, 2)$

(ii) Find the slope of the line AB .

1

(iii) Find the equation of the perpendicular bisector of AB .

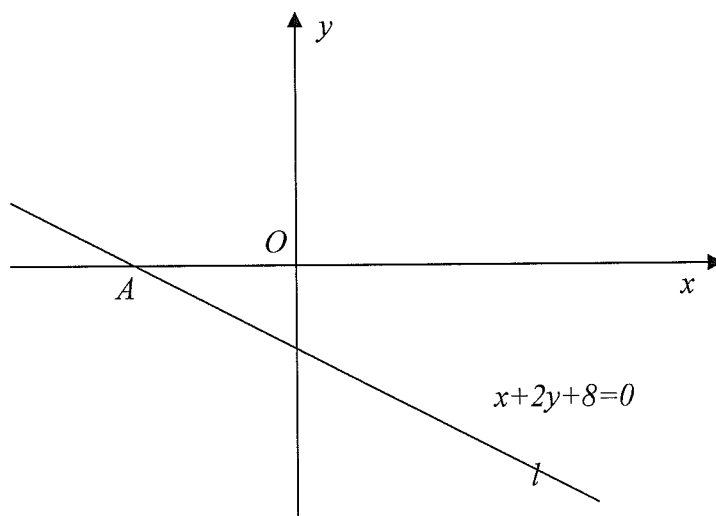
 $x + y - 3 = 0$

(iv) The point P lies on the line $y = 2x - 9$ and is equidistant from A and B . Find the coordinates of P .

 $P(4, -1)$

HSC '96

(2)



The line l is shown in the diagram. It has equation $x + 2y + 8 = 0$ and cuts the x axis at A .

The line k has equation $y = -\frac{1}{2}x + 6$, and is not shown on the diagram.

Copy and trace the diagram.

(a) Find the coordinates of A .

1

$(-8, 0)$

(b) Explain why k is parallel to l .

1

(c) Draw the graph of k on your diagram, indicating where it cuts the axes.

2

(d) Shade the region $x + 2y + 8 \leq 0$ on your diagram.

1

(e) Write down a pair of inequalities which define the region between k and l .

2

$\frac{1}{2}x + y - 6 < 0 \text{ and } x + 2y + 8 > 0$
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(f) Show that $P(8, 2)$ lies on k .

1

(g) Find the perpendicular distance from P to l .

2

 $4\sqrt{5}$

(h) $Q(4, -6)$ lies on l . Show that Q is the point on l which is closest to P .

2

Show that the perp. distance is $4\sqrt{5}$