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YEAR 12 – MATHEMATICS

SPECIMEN PAPER 2

TOPIC : CURVE SKETCHING USING CALCULUS

AMP 2002 Q9

(b) The gradient function of a curve is given by $\frac{dy}{dx} = 6x^2 - 12x - 18$. The curve passes through the point $(-2, -3)$.

(i) Find the equation of the curve. **2**

(ii) Find the coordinates of the stationary points and determine their nature. **2**

(iii) Find the coordinates of the point of inflexion. **2**

(iv) Sketch the curve showing its main features in the domain $-2 \leq x \leq 5$. **2**

ASCHAM 2001 Q6

b) For the curve $y = x(x-3)^2$

i) Find any stationary points and describe their nature. (4)

ii) Find the point of inflexion. (2)

iii) Draw the graph of $y = x(x-3)^2$ for $-1 \leq x \leq 5$, showing significant points (2)

iv) What is the maximum value of $x(x-3)^2$ for $-1 \leq x \leq 5$? (1)

v) For what values of x is $y = x(x-3)^2$ concave down for $-1 \leq x \leq 5$? (1)

ASCHAM 2000 Q6

Consider the curve given by $y = 3x - x^3$.

(a) Show that the curve represents an odd function. [1]

(b) Find all stationary points and their nature. [3]

(c) Find any points of inflexion. [2]

(d) Sketch the curve for $-2 \leq x \leq 2.5$ [2]

(e) State the absolute minimum and maximum values in $-2 \leq x \leq 2.5$ [1]

(f) Calculate the area bounded by the curve $y = 3x - x^3$ and the x-axis. [3]

CSSA 2001 Q5

(a) Consider the curve given by $y = 2x^3 - 3x^2 - 12x$.

(i) Find $\frac{dy}{dx}$. **1**

(ii) Find the coordinates of the two stationary points. **3**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve for $-2 \leq x \leq 3$. **2**

SOLUTIONS

AMP 2002 Q9

(b)(i) $\frac{dy}{dx} = 6x^2 - 12x - 18$

$y = 2x^3 - 6x^2 - 18x + c$ ✓

Substituting $x = -2, y = -3 \Rightarrow c = 1$

$\therefore\ y = 2x^3 - 6x^2 - 18x + 1$ ✓

(ii) Stationary points when $\frac{dy}{dx} = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or -1

$f''(x) = 12x - 12$

At $x = 3, y = 2(3^3) - 6(3^2) - 18(3) + 1 = -53$

$f''(3) = 12 \times 3 - 12 > 0$

$\therefore\ (3, -53)$ is a relative minimum turning point ✓

At $x = -1, y = 11;\ f''(-1) = -24 < 0$,

$\therefore\ (-1, 11)$ is a relative maximum turning point ✓

(iii) For points of inflexion, let $\frac{d^2y}{dx^2} = 0$,

$12x - 12 = 0$

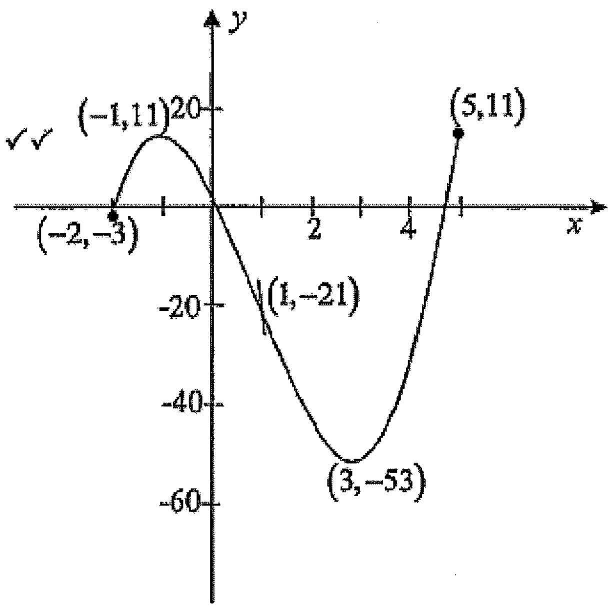
$x = 1$ ✓

Test at $x = 2, f''(2) > 0$

Test at $x = 0, f''(0) < 0$

Change of sign suggests a point of inflexion exists at $(1, -21)$ ✓

(iv)



ASCHAM 2001 Q6

b) $y = x(x-3)^2$

i) $y' = x\,2(x-3) + (x-3)^2$

$= (x-3)(2x + x - 3)$

$= (x-3)(3x-3)$ ✓

$= 3(x-3)(x-1)$.

Stationary points at $(3,0)$ ✓

and $(1,4)$ ✓

$y'' = 6x - 12$

At $(3,0)$ $y'' = 6 > 0$

$\therefore (3,0)$ a min t/pt ½

At $(1,4)$ $y'' = -6 < 0$

$\therefore (1,4)$ a max t/pt ½ (4)

ii) Poss Pt of inflexion at (2, 2) ✓

Check change in concavity

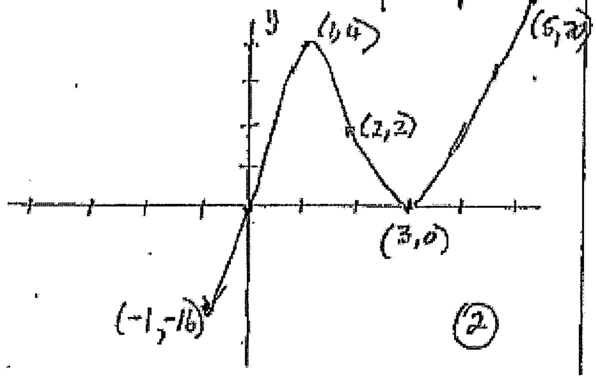
x	$2-\varepsilon$ $(1\frac{1}{2})$	2	$2+\varepsilon$ $(2\frac{1}{2})$
y''	−3	0	3
	−		+

✓ ②

∴ change in concavity

∴ (2, 2) a point of inflexion

iii)



②

iv) Max value is 20 ①

v) Concave down for $-1 < x < 2$ ①

ASCHAM 2000 Q6

A) $y = 3x - x^3$

Let $y = f(x)$

$f(x) = 3x - x^3$

$f(-x) = -3x - (-x)^3 = -3x + x^3$

$-f(x) = -(3x - x^3) = -3x + x^3$

Since $f(-x) = -f(x)$, $f(x)$ is an odd function.

B) $f'(x) = 3 - 3x^2$

At stat. pts, $f'(x) = 0$

$3 - 3x^2 = 0$

$1 - x^2 = 0$; $x = 1$ or $x = -1$

$y = 2$ $\quad$ $y = -2$

Determine nature ~

$f''(x) = -6x$

At $(1, 2)$, $f''(x) = -6 < 0$ $\therefore$ $(1, 2)$ is a max. t.pt

At $(-1, -2)$, $f''(x) = 6 > 0$ $\therefore$ $(-1, -2)$ MIN t.pt

C) At possible pts of inflexion, $f''(x) = 0$

$-6x = 0$

$x = 0$; $y = 0$

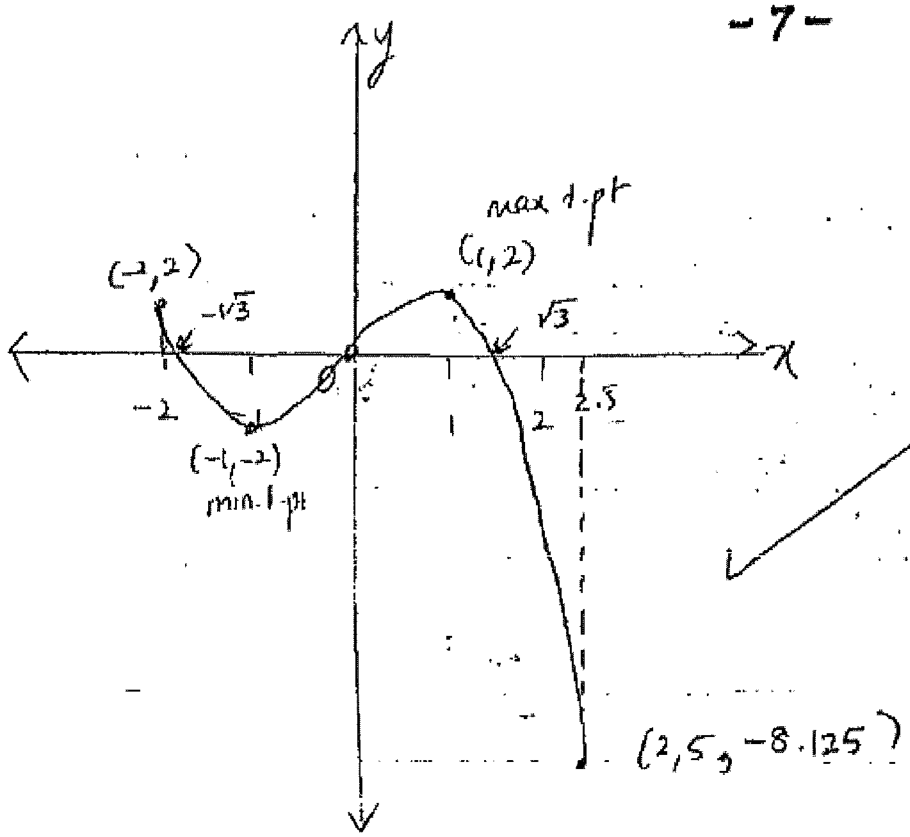
Check for change in concavity.

x	0^-	0	0^+
y''	>0	0	<0

$\therefore$ since there is a change in concavity, $(0, 0)$ is a pt of inflexion.

– 7 –

D)



E) Absolute minimum value = −8.125

" max " = 2

F) ~~Area~~ $y = x(3 - x^2)$

It cuts the x axis at $x = 0$, $x = \sqrt{3}$ and $x = -\sqrt{3}$

$$\text{Area} = \int_{-2}^{-\sqrt{3}} 3x - x^3 dx + \left| \int_{-\sqrt{3}}^{0} 3x - x^3 dx \right| + \int_{0}^{\sqrt{3}} 3x - x^3 dx + \left| \int_{\sqrt{3}}^{2.5} 3x - x^3 dx \right|$$

$$= \left(\frac{3x^2}{2} - \frac{x^4}{4}\right)_{-2}^{-\sqrt{3}} + \left| \left(\frac{3x^2}{2} - \frac{x^4}{4}\right)_{-\sqrt{3}}^{0} \right| + \left(\frac{3x^2}{2} - \frac{x^4}{4}\right)_{0}^{\sqrt{3}} + \left| \left(\frac{3x^2}{2} - \frac{x^4}{4}\right)_{\sqrt{3}}^{2.5} \right|$$

$$= \left(\frac{9}{2} - \frac{9}{4} - 6 + 4\right) + \left| \frac{9}{4} - \frac{9}{2} \right| + \left(\frac{9}{2} - \frac{9}{4}\right) + \left| \frac{3 \times 2.5^2}{2} - \frac{39.0625}{4} - \frac{9}{2} + \frac{9}{4} \right|$$

$= 0.25 + 2.25 + 2.25 + 2.640625$

$= \underline{7.390625}\ u^2$

CSSA 2001 Q5

(a) (i) $\frac{dy}{dx} = 6x^2 - 6x - 12$

(ii)

$6x^2 - 6x - 12 = 0$ stationary points occur where $\frac{dy}{dx} = 0$

$\Rightarrow x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1 \qquad x = 2$

$y = 7 \qquad y = -20$

The stationary points are
$(-1,7), (2,-20)$

(iii) $\frac{d^2y}{dx^2} = 12x - 6$

At $(-1,7)\ \frac{d^2y}{dx^2} = -12 - 6 < 0$

$\Rightarrow (-1,7)$ is a maximum t.pt.

At $(2,-20)\ \frac{d^2y}{dx^2} = 24 - 6 > 0$

$\Rightarrow (2,-20)$ is a minimum t.pt.

(iv) When $x = 0,\ y = 0$

$x = -2,\ y = -4$

$x = 3,\ y = -9$