NAME :

Centre of Excellence in Mathematics
S201 / 414 GARDENERS RD. ROSEBERY 2018
www.cemtuition.com.au

MOBILE 0412 880 475

PHONE 96663331

YEAR 12 – ADVANCED MATHS

REVIEW TOPIC (SP1) CURVE SKETCHING WITH CALCULUS

HSC 02

(6)

(b) The gradient function of a curve is given by

$$f'(x) = 3(x+1)(x-3)$$

and the curve $y = f(x)$ passes through the point (0, 12).

(i) Find the equation of the curve $y = f(x)$. **2**

(This question requires knowledge of "***primitive functions***").

$y = x^3 - 3x^2 - 9x + 12$

(ii) Sketch the curve $y = f(x)$, clearly labelling turning points and the y intercept. **3**

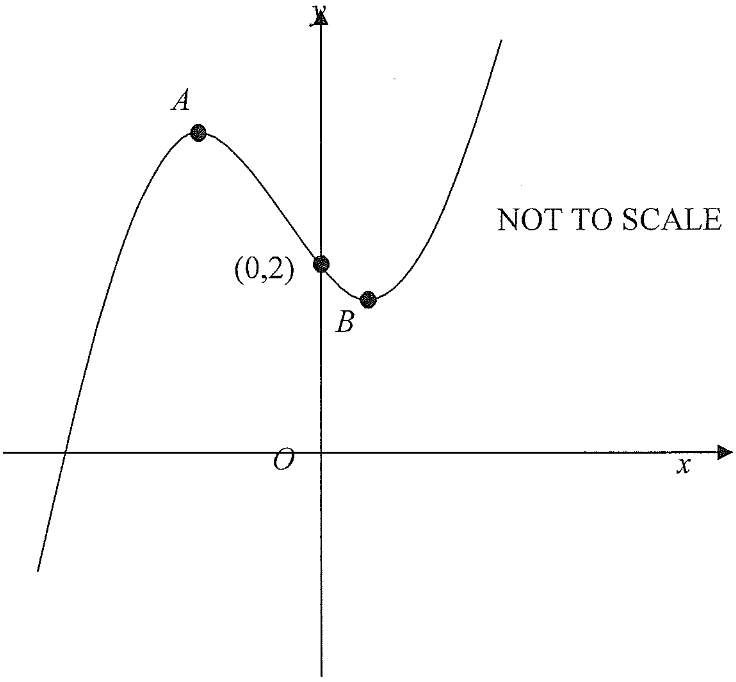
$(-1,17)$ max, $(3,-15)$ min; $(1,1)$ p.o.i.

(iii) For what values of x is the curve concave up? **1**

$x > 1$

HSC 01

(4) (c) The graph of $y = x^3 + x^2 - x + 2$ is sketched below. The points A and B are the turning points. **2**



(i) Find the coordinates of A and B. **3**

$A(-1,3)$; $B\left(\frac{1}{3}, 1\frac{22}{27}\right)$

(ii) For what values of x is the curve concave up? Give reasons for your answer. **2**

$x > -\frac{1}{3}$; p.o.i. at $x = -\frac{1}{3}$

(iii) For what values of k has the equation $x^3 + x^2 - x + 2 = k$ three real solutions? **1**

$1\frac{22}{27} < k < 3$

HSC 2000 (Practical applications of calculus)

(6)

(b) The number N of students logged onto a website at any time over a five-hour period is approximated by the formula **9**

$$N = 175 + 18t^2 - t^4, \quad 0 \le t \le 5.$$

(i) What was the initial number of students logged onto the website?

175

(ii) How many students were logged onto the website at the end of the five hours?

0

(iii) What was the maximum number of students logged onto the website?

256

(iv) When were the students logging onto the website most rapidly?

$\sqrt{3}$ hrs

(v) Sketch the curve $N = 175 + 18t^2 - t^4$ for $0 \le t \le 5$.

HSC 99

(5) (a) Consider the curve given by $y = x^3 - 6x^2 + 9x + 1$. **8**

(i) Find $\frac{dy}{dx}$

$\boxed{3x^2 - 12x + 9}$

(ii) Find the coordinates of the two stationary points.

$\boxed{(1, 5), (3, 1)}$

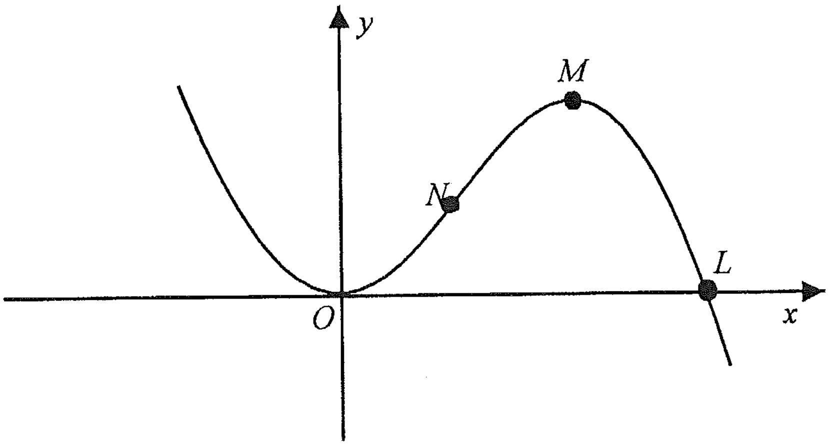
(iii) Determine the nature of the stationary points.

$\boxed{(1, 5) \text{ rel. max}, (3, 1) \text{ rel.min}}$

(iv) Sketch the curve for $x \geq 0$.

HSC '97

(2) (a) **6**



The diagram shows a sketch of the curve $y = 6x^2 - x^3$. The curve cuts the x axis at L, and has a local maximum at M and a point of inflection at N.

(i) Find the coordinates of L.

$(6, 0)$

(ii) Find the coordinates of M.

$(4, 32)$

(iii) Find the coordinates of N.

$(2, 16)$

HSC 96

(5) (d) For all x in the domain $0 \le x \le 4$, a function $g(x)$ satisfies $g'(x) < 0$ and $g''(x) < 0$. **2**

Sketch a possible graph of $y = g(x)$ in this domain.

(6) (a) Lee takes some medicine. The amount, M, of medicine present in Lee's bloodstream t hours later is given by **7**

$$M = 4t^2 - t^3, \quad \text{for} \quad 0 \le t \le 3$$

Sketch the above function, showing any stationary points.

min(0, 0); max $\left(\frac{8}{3}, 9.4\dot{8}\dot{1}\right)$

(ii) At what time is the amount of medicine in Lee's bloodstream a maximum?

2 hrs 40 mins

(iii) When is the amount present increasing most rapidly ?

1 hrs 20 mins

HSC 95

(5) (b) Consider the curve given by $y = 7 + 4x^3 - 3x^4$.

(i) Find the co-ordinates of the two stationary points.

$(0,7),(1,8)$

(ii) Find all values of x for which $\frac{d^2y}{dx^2} = 0$.

$x = 0$ or $\frac{2}{3}$

(iii) Determine the nature of the stationary points

$(0, 7)$ pt. of inflection, $(1, 8)$ local max.

(iv) Sketch the curve for the domain $-1 \le x \le 2$.

HSC '94

(6) (c) Consider the curve given by $y = x^3 - 6x + 4$.

(i) Find the coordinates of the stationary points and determine their nature.

$\left(\sqrt{2}, -1.7\right)$ local min, $\left(-\sqrt{2}, 9.7\right)$ local max.

(ii) Find the coordinates of any point of inflexion.

$(0, 4)$

(iii) Sketch the curve for the domain $-3 \le x \le 3$.

(iv) What is the maximum value of $x^3 - 6x + 4$ in the domain $-3 \le x \le 3$.

$y_{max} = 13$ when $x = 3$

HSC '93

(6) (a) Consider the curve given by $y = \frac{1}{4}x^4 - x^3$.

(i) Find any turning points and determine their nature.

$(0, 0)$ h.p.o.i, $\left(3, -6\frac{3}{4}\right)$ rel. min. t.p.

(ii) Find any points of inflexion.

$(0, 0), (2, -4)$

(iii) Sketch the curve for $-1.5 \leq x \leq 4.5$, indicating where the curve crosses the x–axis.

(iv) For what values of x is the curve concave down ?

$0 < x < 2$

HSC '91

(6) (a) Consider the curve given by $y = 3x^2 - x^3$.

(i) Find the stationary points and determine their nature.

(0, 0) local min, (2, 4) local max.

(ii) Sketch the curve, indicating where it crosses the x–axis.

x-intercepts at 0 and 3.

(iv) Find the equation of the tangent to the curve at the point $R(-1, 4)$.

$9x + y + 5 = 0$

HSC '90

(5) (a) Consider the curve given by $y = 1 + 3x - x^3$, for $-2 \le x \le 3$.

(i) Find the stationary points and determine their nature.

$(-1,-1)$ local min, $(1,3)$ local max.

(ii) Find the point of inflexion.

$(0,1)$

(iii) Sketch the curve for $-2 \leq x \leq 3$.

$(-2, 3)$ and $(3, -17)$ are endpoints

(iv) What is the minimum value of y for $-2 \leq x \leq 3$?

$y_{\min} = -17$ at $x = 3$.