NAME :

Centre of Excellence in Mathematics
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YEAR 12 – ADVANCED MATHS

REVIEW TOPIC (SP2):

Derivative & Integral of Trigonometric Functions

CEM – Yr 12 – 2U Derivatives and Integrals of Trig functions – Review (SP2)

1. Differentiate $\frac{\cos x}{x}$

2. Differentiate the following function: $x^3 \cos x$

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3. Differentiate $\dfrac{\sin(ax+1)}{\cos(ax+1)}$

4. Evaluate this indefinite integral: $\int \dfrac{6}{\operatorname{cosec} 2x}\,dx$

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5. Evaluate this indefinite integral: $\int \sec^2 6x \, dx$

6. Differentiate the following:
   (i) $y = 3\sin 2x$
   (ii) $y = \sin^2 x$

7. Evaluate the definite integral

$$\int_0^{\frac{\pi}{2}} \sec^2 \tfrac{x}{2}\, dx.$$

8. Find $\frac{dy}{dx}$ for each function:

(i) $y = e^{\cos x}$

(ii) $y = \ln(\sin x)$

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9. Differentiate each of the following:

i) $\frac{\sin x}{x}$

ii) $\cos^4 3x$

10. Find:

i) $\int (2\sin x - \sec^2 2x)\,dx$

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Answers

1. $\frac{d\left(\frac{\cos x}{x}\right)}{dx}$

$= \frac{(-\sin x) \times x - 1 \times \cos x}{x^2}$

$= -\frac{x\sin x + \cos x}{x^2}$

$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{u'v - uv'}{v^2}$

$u = \cos x \quad u' = -\sin x$

$v = x \quad v' = 1$

2 $y = x^3 \cos x$

let $\frac{dy}{dx} = 3x^2 \cos x - x^3 \sin x$

$= x^2(3\cos x - x\sin x)$

3. $y = \frac{\sin(ax+1)}{\cos(ax+1)}$

$= \tan(ax+1)$

$y' = \sec^2(ax+1) \times a$

$= a\sec^2(ax+1)$

4. $I = \int \frac{6}{\operatorname{cosec} 2x}\,dx$

$= \int 6\sin 2x\,dx$

$= 6x - \frac{1}{2}\cos 2x + C$

$= -3\cos 2x + C$

5. $I = \int \sec^2 6x\,dx$

$= \frac{1}{6}\tan 6x + C$

6. (i) $y = 3\sin 2x$

$y' = 6\cos 2x$ ✓

(ii) $y = \sin^2 x$

$y' = 2\sin x \cos x$ ✓

7. $\int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}}{2}\,dx$

$= \left[2\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$ ✓

$= 2\left(\tan\frac{\pi}{4} - \tan 0\right)$

$= 2$ ✓

8. (i) $y = e^{\cos x}$

$\frac{dy}{dx} = -\sin x \cdot e^{\cos x}$ ✓

(ii) $y = \ln(\sin x)$

$\frac{dy}{dx} = \frac{\cos x}{\sin x}$ ✓

$= \cot x$

9. i) $\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \boxed{\frac{x\cos x - \sin x}{x^2}}$

ii) $\frac{d}{dx}(\cos^4 3x) = 4\cos^3 3x \cdot -\sin 3x \cdot 3$

$= \boxed{-12\cos^3 3x \cdot \sin 3x}$

10. $\int (2\sin x - \sec^2 2x)\,dx$

$= -2\cos x - \frac{\tan 2x}{2}$ ✓ $+ C$ ✓