NAME :

Centre of Excellence in Mathematics
S201 / 414 GARDENERS RD. ROSEBERY 2018
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YEAR 12 – ADVANCED MATHS

REVIEW TOPIC (SP1)
DISPLACEMENT, VELOCITY & ACCELERATION

HSC '99 **Marks**

(7)(b) A particle P is moving along the x–axis. Its position at time t seconds is given by 7

$$x = 2\sin t - t, \quad t \geq 0.$$

(i) Find an expression for the velocity of the particle.

$\boxed{v = 2\cos t - 1}$

(ii) In what direction is the particle moving at $t = 0$?

To the right

(iii) Determine when the particle first comes to rest.

$\boxed{\frac{\pi}{3}\text{ s}}$

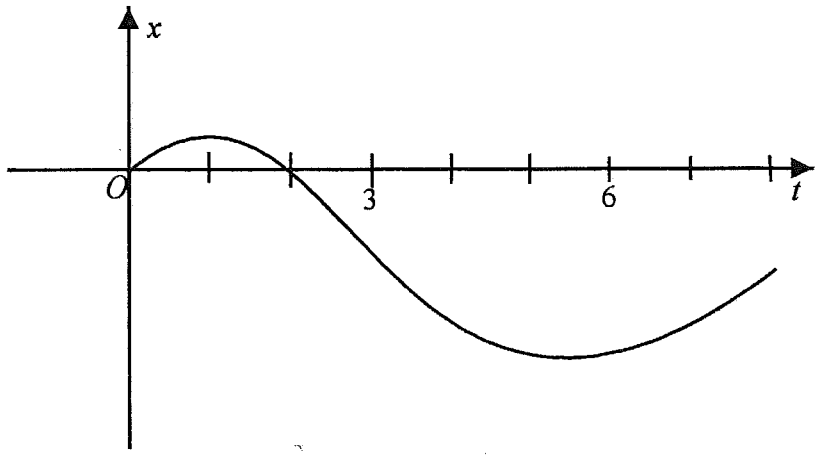
(iv) When is the acceleration negative for $0 \leq t \leq 2\pi$?

$\boxed{0 < t < \pi}$

(v) Calculate the total distance travelled by the particle in the first π seconds.

$\boxed{\frac{\pi}{3} + 2\sqrt{3}}$

(6) (a) A particle P moves along a straight line for 8 seconds, starting at the fixed point S at time $t = 0$. At time t seconds, P is $x(t)$ metres to the right of S. The graph of $x(t)$ is shown in the diagram. **5**



(i) At approximately what times is the velocity of the particle equal to 0 ?

$t = 1,\ 5.5$ s

(ii) At approximately what time is the acceleration of the particle equal to 0 ?

$t = 3$ s

(iii) At approximately what time is the distance from S greatest ?

$t = 5.5$ s

(iv) At approximately what time is the particle moving with the greatest velocity ?

$t = 8$ s

HSC '97 **Marks**

(8) (b) A particle is moving along the x–axis. Its position at time t is given by $x = t + \sin t$. **8**

(i) At what times during the period $0 < t < 3\pi$ is the particle stationary ?

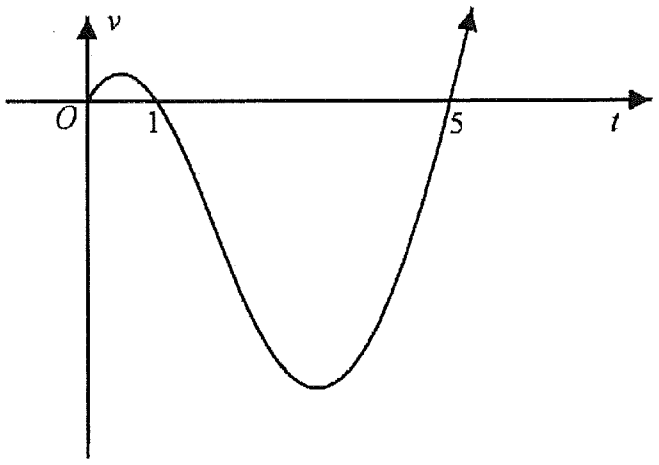
$t = \pi$

(ii) At what times during the period $0 < t < 3\pi$ is the acceleration equal to 0 ?

$t = \pi$ or 2π

(iii) Carefully sketch the graph of $x = t + \sin t$ for $0 < t < 3\pi$.
Clearly label any stationary points and any points of inflection.

(9) (b) 7



A pen moves along the x-axis, ruling a line. The diagram shows the graph of the velocity of the tip of the pen as a function of time.

The velocity, in centimetres per second, is given by the equation

$$v = 4t^3 - 24t^2 + 20t,$$

where t is the time in seconds. When $t = 0$, the tip of the pen is at $x = 3$. That is, the tip is initially 3 centimetres to the right of the origin.

(i) Find an expression for x, the position of the tip of the pen, as a function of time.

$\boxed{x = t^4 - 8t^3 + 10t^2 + 3}$

(ii) What feature will the graph of x as a function of t have at $t = 1$?

$\boxed{\text{Max. at } t = 1}$

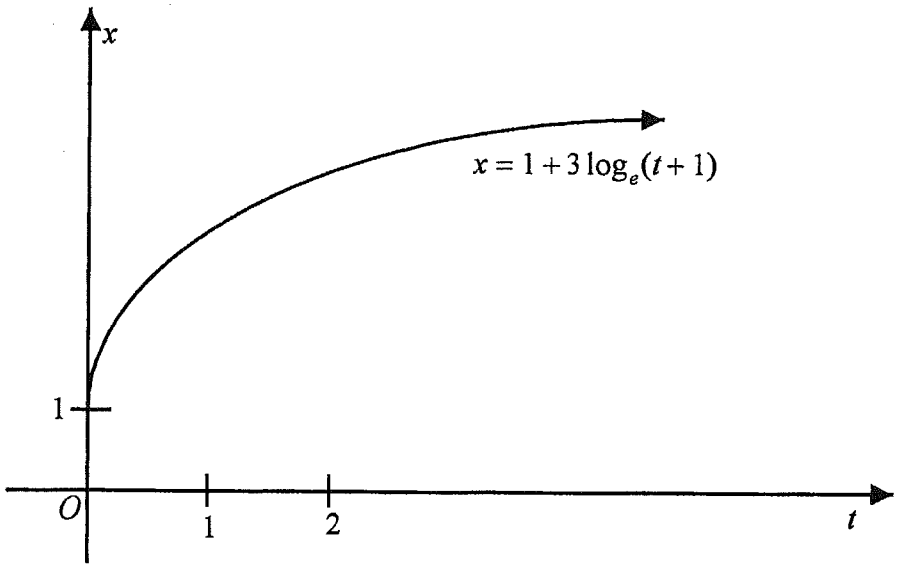
(iii) The pen uses 0.05 milligrams of ink per centimetre travelled.
How much ink is used between $t = 0$ and $t = 2$?

$x = 14$ cm; Ink used = 0.7 mg

HSC '96

(9) Two particles P and Q start moving along the x–axis at time $t = 0$ and never meet. Particle P is initially at $x = 4$ and its velocity v at time t is given by $v = 2t + 4$.

The position of particle Q is given by $x = 1 + 3\log_e(t+1)$. The diagram shows the graph of $x = 1 + 3\log_e(t+1)$.



(a) Find an expression for the position of P at time t.

$\boxed{x = (t+2)^2}$

(b) Copy the diagram into your Writing Booklet and, on the same axes, draw the graph of the function found in part (a).

(c) P and Q are joined by an elastic string and M is the midpoint of the string.
Show that the position of M at time t is given by

$$x = \tfrac{1}{2}\left[t^2 + 4t + 3\log_e(t+1) + 5\right].$$

$\boxed{x = \frac{1}{2}[t^2 + 4t + 3\ln(t+1) + 5]}$

(d) Find the time at which the acceleration of M is zero.

$\boxed{t = -1 \pm \sqrt{\frac{3}{2}}}$

(e) Find the minimum distance between P and Q.

$x_{min} = 3$ units.

HSC '95

(10) (b) Two particles A and B start moving on the x-axis at time $t = 0$. The position of particle A at time t is given by

$$x = -6 + 2t - \frac{1}{2}t^2$$

and the position of particle B at time t is given by

$$x = 4\sin t$$

(i) Find expressions for the velocities of the two particles.

$v_A = 2 - t, v_B = 4\cos t$

HSC '94

(5) (c) A particle moves along a straight line so that its distance x metres from a fixed point O is given by

$$x = 6 - 2t + 8\ln(t+3)$$

where the time t is measured in seconds.

(i) What is the position of the particle when $t = 0$?

14.79 m (to 2 dp)

(ii) Find expressions for the velocity and acceleration of the particle at time t.

$v = -2 + \frac{8}{t+3}; a = -\frac{8}{(t+3)^2}$

(iii) Find the time t when the velocity of the particle is zero.

$t = 1$ when $v = 0$

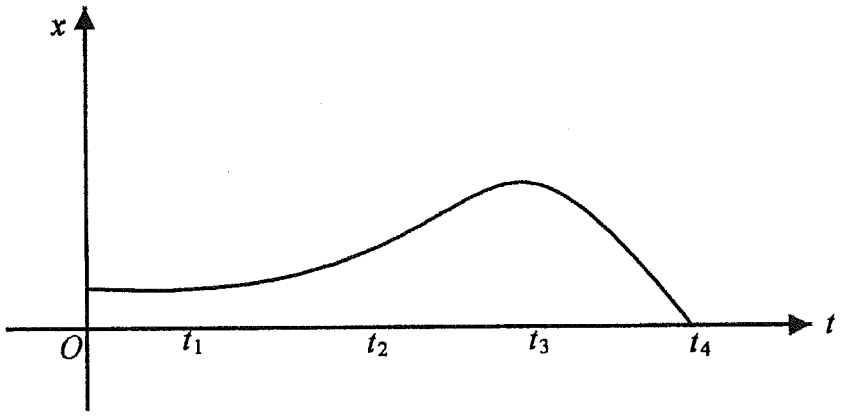
HSC '93

(9)(a) 'The car moved away from where it had stopped, its speed increasing at a constant rate, and after exactly 10 seconds it was travelling at 25 m/s. It continued at a constant speed for a further 20 seconds. Then the brakes were applied causing it to slow down at a constant rate, so that 5 seconds later it was travelling at 5 m/s.'

(i) Let the car's speed be v m/s. Graph v as a function of time t, measured in seconds.

(ii) Let the distance travelled by the car be s metres from where it had stopped. On a separate diagram, graph s as a function of t.

HSC '92



A particle moves in a straight line and the above graph shows the distance x of the particle from a fixed point at time t.

(i) Is the particle moving faster at time t_1 or at time t_2 ? Why ?

(ii) What is the velocity at time $t = 0$? Why ?

(iii) Sketch the graph of the velocity v as a function of t.