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CENTRE OF EXCELLENCE IN MATHS TUITION

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MATHEMATICS SPECIMEN PAPER 1

EXPONENTIALS & LOGARITHMS REVIEW

1. Using the substitution $u = 7^x$ in the equation

$$7^x = 5 + 24 \times 7^{-x}$$

find the value of x correct to 4 decimal places [4]

2. Solve the equation

$$2^{2y} - 2^{y+3} - 2^{y+2} + 32 = 0$$

by forming a quadratic equation in x where $x = 2^y$. [5]

3.

(a) Solve the equation

$$2 \ln x = \ln 4 + \ln(2x + 5)$$ [2]

(b) Solve the equation

$$e^{2x} - 3e^{x} = 54,$$

giving your answers correct to 3 decimal places. [3]

4. Solve the equation

$$2^y + \frac{16}{2^y} = 17$$ [4]

5.

(a) Assuming that $x = e^{\ln x}$ where $x > 0$, show that

$$\ln x^m = m \ln x$$ [3]

(b) If $4^{(x+1)} = 7 \times 8^{(x-2)}$

find x to 3 decimal places [6]

Extension 1: Q6 - 7

6.

(a) Factorise the expression $3x^3 - 4x^2 - 5x + 2$ [4]

(b) Hence, using a suitable substitution, solve the equation

$$3e^{3y} - 4e^{2y} - 5e^{y} + 2 = 0$$

giving your values of y correct to 4 decimal places [3]

7.

(a) Using the factor theorem, factorise the expression

$$2u^3 - 3u^2 - 8u + 12$$

[4]

(b) By using the substitution $u = e^x$, solve the equation

$$2e^x - 8e^{-x} + 12e^{-2x} = 3$$

giving your answers correct to 3 decimal places. [3]

SOLUTIONS:

1.

$$7^x = 5 + 24 \times 7^{-x}$$

$\Rightarrow$ $$7^x = 5 + 24 \times \frac{1}{7^x}$$

Let $u = 7^x$

$\Rightarrow$ $$u = 5 + 24 \times \frac{1}{u}$$

$\times u \Rightarrow$ $$u^2 = 5u + 24$$

$\Rightarrow$ $$u^2 - 5u - 24 = 0$$

$$(u - 8)(u + 3) = 0$$

$\Rightarrow$ $$u - 8 = 0 \quad \text{or} \quad u + 3 = 0$$

$\Rightarrow$ $$u = 8 \quad \text{or} \quad u = -3$$

$\Rightarrow$ $$7^x = 8 \quad \text{or} \quad 7^x = -3$$

Take logarithms of both sides

$$\ln 7^x = \ln 8 \qquad \text{no solution}$$

$$x \ln 7 = \ln 8$$

$$x = \frac{\ln 8}{\ln 7}$$

$x = 1.0686$ to 4 decimal places

2.

$$2^{2y} - 2^{y+3} - 2^{y+2} + 32 = 0$$

$$(2^y)^2 - 2^y \times 2^3 - 2^y \times 2^2 + 32 = 0$$

$$(2^y)^2 - 8 \times 2^y - 4 \times 2^y + 32 = 0$$

$$(2^y)^2 - 12 \times 2^y + 32 = 0$$

Let $x = 2^y$

$$x^2 - 12x + 32 = 0$$

$$(x - 8)(x - 4) = 0$$

$x - 8 = 0$ or $x - 4 = 0$

$x = 8$ or $x = 4$

$\Rightarrow$ $2^y = 8$ or $2^y = 4$

$2^y = 2^3$ or $2^y = 2^2$

$\Rightarrow$ $y = 3$ or $y = 2$

3.

(a) $$2 \ln x = \ln 4 + \ln(2x + 5)$$

$\Rightarrow$ $$\ln x^2 = \ln\big(4(2x + 5)\big)$$

$\Rightarrow$ $$x^2 = 4(2x + 5)$$

$\Rightarrow$ $$x^2 = 8x + 20$$

$$x^2 - 8x - 20 = 0$$

$$(x - 10)(x + 2) = 0$$

$\Rightarrow$ $$x - 10 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 10 \quad \text{or} \quad x = -2$$

As $x > 0$ then $x = 10$

(b)

$$e^{2x} - 3e^{x} = 54$$
$$(e^{x})^2 - 3(e^{x}) = 54$$

Let $u = e^{x}$

$\Rightarrow \quad u^2 - 3u = 54$

$u^2 - 3u - 54 = 0$

$(u - 9)(u + 6) = 0$

$\Rightarrow \quad u - 9 = 0$ or $u + 6 = 0$

$u = 9$ or $u = -6$

$\Rightarrow \quad e^{x} = 9$ or $e^{x} = -6$

$x = \ln 9$ No solution

$x = 2.197$ to 3 decimal places

4.

$$2^{y} + \frac{16}{2^{y}} = 17$$

$\times 2^{y} \quad (2^{y})^2 + 16 = 17 \times 2^{y}$

$(2^{y})^2 - 17 \times 2^{y} + 16 = 0$

Let $x = 2^{y}$

$x^2 - 17x + 16 = 0$

$(x - 1)(x - 16) = 0$

	$x - 1 = 0$	or	$x - 16 = 0$
	$x = 1$	or	$x = 16$
	$2^{y} = 1$	or	$2^{y} = 16$
	$2^{y} = 2^{0}$	or	$2^{y} = 2^{4}$
$\Rightarrow$	$y = 0$	or	$y = 4$

5.

(a)

$x = e^{\ln x}$ -{*}

$\Rightarrow \quad x^{m} = (e^{\ln x})^{m}$

$\Rightarrow \quad x^{m} = e^{(m \ln x)}$

But, using {*} $\quad x^{m} = e^{\ln x^{m}}$

$\Rightarrow \quad \ln x^{m} = m \ln x$

(b)

$4^{(x+1)} = 7 \times 8^{(x-2)}$

Take logarithms of both sides

$\ln 4^{(x+1)} = \ln (7 \times 8^{(x-2)})$

$\Rightarrow \quad (x + 1) \ln 4 = \ln 7 + \ln 8^{(x-2)}$

$\Rightarrow \quad (x + 1) \ln 4 = \ln 7 + (x - 2) \ln 8$

$\Rightarrow \quad x \ln 4 + \ln 4 = \ln 7 + x \ln 8 - 2 \ln 8$

Re-arranging

$2 \ln 8 + \ln 4 - \ln 7 = x \ln 8 - x \ln 4$

$\ln 8^2 + \ln 4 - \ln 7 = x(\ln 8 - \ln 4)$

$$\Rightarrow \quad \ln\left(\frac{64 \times 4}{7}\right) = x \ln\left(\frac{8}{4}\right)$$

$$\Rightarrow \quad \ln\left(\frac{256}{7}\right) = x\ln 2$$

$$\Rightarrow \quad x = \frac{\ln\left(\frac{256}{7}\right)}{\ln 2}$$

$$x = 5.193 \text{ to 3 decimal places}$$

6.

(a) Let $f(x) = 3x^3 - 4x^2 - 5x + 2$

Using the factor theorem

$x = 1 \Rightarrow \quad f(1) = 3(1)^3 - 4(1)^2 - 5(1) + 2 = -4$

$\Rightarrow (x - 1)$ is not a factor

$x = -1 \Rightarrow \quad f(-1) = 3(-1)^3 - 4(-1)^2 - 5(-1) + 2 = 0$

$\Rightarrow \quad (x + 1)$ is a factor of $f(x)$

To find the other factor divide $(x + 1)$ into $f(x)$

$$\begin{array}{r} 3x^2 - 7x + 2 \\ (x+1)\overline{)3x^3 - 4x^2 - 5x + 2} \\ \underline{3x^3 + 3x^2} \qquad\qquad \\ -7x^2 - 5x \qquad \\ \underline{-7x^2 - 7x} \qquad \\ 2x + 2 \\ \underline{2x + 2} \\ \underline{-\ \ -} \end{array}$$

$\Rightarrow \quad f(x) = (x + 1)(3x^2 - 7x + 2)$

$= (x + 1)(3x - 1)(x - 2)$

$\Rightarrow \quad 3x^3 - 4x^2 - 5x + 2 = (x + 1)(3x - 1)(x - 2)$

(b) $3e^{3y} - 4e^{2y} - 5e^y + 2 = 0$

$\Rightarrow 3(e^y)^3 - 4(e^y)^2 - 5(e^y) + 2 = 0$

Let $x = e^y$

$3x^3 - 4x^2 - 5x + 2 = 0$

$\Rightarrow \quad (x + 1)(3x - 1)(x - 2) = 0$

$\Rightarrow \quad (x + 1) = 0$ or $(3x - 1) = 0$ or $(x - 2) = 0$

$x = -1$ or $x = \frac{1}{3}$ or $x = 2$

$\Rightarrow \quad e^y = -1 \qquad e^y = \frac{1}{3} \qquad e^y = 2$

No solution $\quad y = \ln\left(\frac{1}{3}\right)$ or $y = \ln 2$

$y = -1.0986$ or $y = 0.6931$ to 4 decimal places

7.

(a) Let $\quad f(u) = 2u^3 - 3u^2 - 8u + 12$

$u = 1 \Rightarrow \quad f(1) = 2(1)^3 - 3(1)^2 - 8(1) + 12 = 3$

$\Rightarrow (u-1)$ is not a factor

$u = -1 \Rightarrow \quad f(-1) = 2(-1)^3 - 3(-1)^2 - 8(-1) + 12 = 15$

$\Rightarrow (u+1)$ is not a factor

$u = 2 \quad f(2) = 2(2)^3 - 3(2)^2 - 8(2) + 12 = 0$

$\Rightarrow \quad (u-2)$ is a factor

To find the other factor, divide $(u-2)$ into $f(u)$

$$\begin{array}{r} 2u^2 + u - 6 \\ (u-2)\overline{)2u^3 - 3u^2 - 8u + 12} \\ \underline{2u^3 - 4u^2} \\ u^2 - 8u \\ \underline{u^2 - 2u} \\ -6u + 12 \\ \underline{-6u + 12} \\ \underline{- \quad -} \end{array}$$

$\Rightarrow \quad f(u) = (u-2)(2u^2 + u - 6)$

$f(u) = (u-2)(2u-3)(u+2)$

(b) $\quad 2e^x - 8e^{-x} + 12e^{-2x} = 3$

$\Rightarrow \quad 2e^x - \frac{8}{e^x} + \frac{12}{e^{2x}} = 3$

$\Rightarrow \quad 2e^x - \frac{8}{e^x} + \frac{12}{(e^x)^2} = 3$

Let $u = e^x$

$\Rightarrow \quad 2u - \frac{8}{u} + \frac{12}{u^2} = 3$

$\times u^2 \quad 2u^3 - 8u + 12 = 3u^2$

$\Rightarrow \quad 2u^3 - 3u^2 - 8u + 12 = 0$

$\Rightarrow \quad (u+2)(2u-3)(u-2) = 0$

$\Rightarrow \quad (u+2)=0$ or $(2u-3)=0$ or $(u-2)=0$

$u=-2$ or $u=\frac{3}{2}$ or $u=2$

$e^x=-2 \qquad e^x=\frac{3}{2} \qquad e^x=2$

No solution $\qquad x=\ln\left(\frac{3}{2}\right)$ or $x = \ln 2$

$x=0.405,\ 0.693$ to 3 decimal places