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YEAR 12 – MATHEMATICS

SPECIMEN PAPER 2
TOPIC :FINANCE MATHS

ASCHAM 2001 Q10

a) Jacqui borrowed $400 000 from the bank on 1 January 1998. She was not charged interest for the first 6 months. Thereafter she was charged interest of 15% per annum compounded monthly.

She agreed to repay the loan by equal annual instalments of M dollars on 31 December of each year. The loan was to be repaid by the end of 2001.

i) How much did she owe on 31 December 1998 after making her first payment? (1)

ii) How much did she owe on 31 December 1999 after making her second payment? (1)

iii) Show that $M = \dfrac{400000(1.0125)^{42}(1.0125^{12}-1)}{1.0125^{48}-1}$ (2)

iv) Hence find her annual instalment. (1)

ASCHAM 2000 Q8

(c) Jane borrows $5 000 from her father to pay for her Olympic tickets. They agree that Jane should pay interest of 1.5% every month and that she should pay her father back an instalment every two months.

(i) Letting $\$A_n$ be the amount owing after n months and $T be the value of each two-monthly instalment derive an expression, involving T, for the amount owing after 12 months.

(ii) Hence find the value of T to the nearest dollar if she pays back the loan after 2 years. [6]

CSSA 2001 Q9

(a) Mia would like to save \$60 000 for a deposit on her first home.
She decides to invest her net monthly salary of \$3000 in a bank account that pays interest at a rate of 6% per annum compounded monthly.
Mia intends to withdraw $\$E$ at the end of each month from this account for living expenses, immediately after the interest has been paid.

(i) Show that the amount of money in the account following the second withdrawal of $\$E$ is given by 2

$$\$3000(1\cdot 005^2 + 1\cdot 005) - \$E(1\cdot 005 + 1).$$

(ii) Calculate the value of E if Mia is to reach her goal after 4 years. 3

CSSA 2000 Q8

(a) When Jack left school, he borrowed \$15 000 to buy his first car. The interest rate on the loan was 18% p.a. and Jack planned to pay back the loan in 60 equal monthly instalments of \$M. **6**

(i) Show that immediately after making his first monthly instalment, Jack owed

$$\$[15\,000 \times 1{\cdot}015 - M]$$

(ii) Show that immediately after making his third monthly instalment, Jack owed

$$\$[15\,000 \times 1{\cdot}015^{3} - M(1 + 1{\cdot}015 + 1{\cdot}015^{2})]$$

(iii) Calculate the value of M.

CSSA 2002 Q10

(b) Mr and Mrs Matthews decide to borrow \$250 000 to buy a house. Interest is calculated monthly on the balance still owing, at a rate of 6.06% per annum. The loan is to be repaid at the end of 15 years with equal monthly repayments of \$$M$.

Let \$$A_n$ be the amount owing after the nth repayment.

(i) Derive an expression for A_{60} . **1**

(ii) Find the value of M. **2**

(iii) Hence, calculate the amount still owing after 5 years of payment at this rate. **2**

(iv) At the end of five years, the interest rate is increased to 7.2% per annum and Mr and Mrs Matthews change their payments to \$1800 per month. How many more months are needed to pay off the remainder of the loan? **2**

SYDNEY GRAMMAR 2000 Q8

(a) Kerry deposits \$1500 into a superannuation fund on January 1st 2001. He makes further deposits of \$1500 on the first of each month up to and including December 1st 2010. The fund pays compound interest at a monthly rate of 0·75%. In each of the following questions give your answer to the nearest dollar.

1 (i) How much is in the fund on January 31st 2001?

1 (ii) How much is the first \$1500 deposit worth on December 31st 2010?

[3] (iii) Form a geometric series and hence determine the total amount in the fund on December 31st 2010.

[2] (iv) If each deposit was increased to $1600, what difference does it make to the total amount in the fund on December 31st 2010?

SOLUTIONS

ASCHAM 2001 Q10

a) Let A_n be amount owing after n years

15% pa $= 1.25\%$ p month $= 0.0125$

i) $A_1 = 400000(1.0125)^6 - M$ ✓

ii) $A_2 = A_1(1.0125)^{12} - M$

$= [400000(1.0125)^6 - M](1.0125)^{12} - M$

$= 400000(1.0125)^{18} - M(1.0125)^{12} - M$

$= 400000(1.0125)^{18} - M(1.0125^{12} + 1)$ ✓

iii) $A_4 = 400000(1.0125)^{42} - M(1.0125^{36} + 1.0125^{24} + 1.0125 + 1)$ ✓

But $A_4 = 0$

$\therefore 400000(1.0125)^{42} = M\left[1 \cdot \frac{1.0125^{12\times 4} - 1}{1.0125^{12} - 1}\right]$ ✓

$M = \frac{400000(1.0125)^{42} \cdot (1.0125^{12} \ldots}{1.0125^{48} - 1}$

iv) $M = 132882.52$

$\therefore$ annual instalment is $\$132882.52$. ✓ (5)

ASCHAM 2000 Q8

c) i.) $A_1 = \$5,000(1.015)$ A_n = amount owing after n months.

$A_2 = \$5,000(1.015)^2 - T$

$A_3 = \$5,000(1.015)^3 - T(1.015)$

$A_4 = \$5,000(1.015)^4 - T(1.015)^2 - T$ ✓

$A_4 = \$5000(1.015)^4 - T(1+1.015^2)$

$A_n = \$5000(1.015)^n - T(1+1.015^2+ \ldots 1.015^{n-2})$

$A_{12} = \$5000(1.015)^{12} - T(1+1.015^2+ \ldots 1.015^{10})$ ✓

ii.) Note $A_{24} = 0$.

$A_{24} = \$5000(1.015)^{24} - T(1+1.015^2+ \ldots 1.015^{22})$

$0 = \$5000(1.015)^{24} - T\left(\frac{(1.015^2)^{12}-1}{1.015^2-1}\right)$ ✓

$T\left(\frac{1.015^{24}-1}{1.015^2-1}\right) = \$5000(1.015)^{24}$; $T = \$503$ (to nearest dollar) ✓

CSSA 2001 Q9

(a) (i) Following the first withdrawal of \$E, Mia has \$3000 (1.005) - \$E
Following the second withdrawal of \$E, she has

$[\$3000(1.005) - \$E)]1.005 + \$3000(1.005) - \$E$

$= \$3000(1.005)^2 - \$E(1.005) + \$3000(1.005) - \$E$

$= \$3000(1.005^2 + 1.005) - \$E(1.005+1)$

(ii) After 4 years or 48 months, Mia has

$\$3000(1.005^{48} + 1.0005^{47} + \ldots + 1.005) - \$E(1.005^{47} + \ldots + 1.005 + 1)$

But she has saved \$60000 after 4 years

Solve $\$3000(1.005 + 1.005^2 + \ldots + 1.005^{48}) - \$E(1 + 1.005 + \ldots + 1.005^{47}) = \$60000$

$$\Rightarrow E = \frac{3000 \times 1.005\frac{(1.005^{48}-1)}{0.005} - 60000}{1\frac{(1.005^{48}-1)}{0.005}}$$

$= 1905.898\ldots$

CSSA 2000 Q8

(a)

(i) Interest rate 18% pa = 1.5% per month = 0.015 per month.
Interest charged at the end of the first month $(0.015 × 15 000)
∴ Total amount owing after making the first instalment is $(15 000 + 0.015 × 15 000 – M)
= $[15 000(1.015) – M]

(ii) After making the second instalment the amount owing is:
$[15 000(1.015) – M](1.015) – M
= $[15 000(1.015)^2 – 1.015M – M]

Immediately after making the third instalment the amount owing is:
$[15 000(1.015)^2 – 1.015M – M](1.015) – M]
= $[15 000(1.015)^3 – (1.015)^{2}M – 1.015M – M]
=$[15 000(1.015)^3 – M(1 + 1.015 + 1.015^2)]

(iii) ∴ Immediately after making the 60th instalment the amount owing is 0.
∴ 15 000(1.015)^{60} – M(1 + 1.015 + 1.015^2 + ... + 1.015^{59}) = 0

$$\therefore M\left[\frac{1(1.015^{60} - 1)}{1.015 - 1}\right] = 15\,000(1.015)^{60}$$

$$M = \frac{15\,000(1.015)^{60} \times 0.015}{1.015^{60} - 1}$$

M = $380.90

CSSA 2002 Q10

(b) \$260000 6.06% p.a $= 0.505\%$ p.m

$n = 15 \times 12$

$n = 180$

(i) $A_1 = 250000 \times 1.00505 - M$

$A_2 = 250000 \times 1.00505^2 - M \times 1.00505 - M$

$A_3 = 250000 \times 1.00505^3 - M(1.00505^2 + 1.00505 + 1)$ ✓

$\vdots$

$A_{60} = 250000 \times 1.00505^{60} - M(1 + 1.00505 + 1.00505^2 + \dots 1.00505^{59})$

(ii) $A_{180} = 250000 \times 1.00505^{180} - M\left(\frac{1.00505^{180} - 1}{1.00505 - 1}\right)$

$= 0$

$M = 260000 \times 1.00505^{180} \times \frac{0.00505}{1.00505^{180} - 1}$ ✓

$= \$2117.75$ ✓

(iii) 5 years Amount owing is A_{60}

$A_{60} = 250000 \times 1.00505^{60} - 2117.75\left(\frac{1.00505^{60} - 1}{0.00505}\right)$ ✓

$= \$190\ 236.76$ ✓

(iv) 7.2% p.a $= 0.6\%$ p.m

$190\ 236.76 \times 1.006^n = 1800 \times \frac{(1.006^n - 1)}{0.006}$ ✓

$= 300\ 000 \times (1.006^n - 1)$

$1.006^n = \frac{300000}{300000 - 190236.76}$

$n = \frac{\ln\left(\frac{300000}{30000 - 190236.76}\right)}{\ln 1.006}$

$n = 168.01\ldots$

ie Approx 169 months. ✓

SYDNEY GRAMMAR 2000 Q8

a) (i) $A_1 = 1500(1\cdot0075)$ ✓
$= \$1511\cdot25$

(ii) $A = 1500(1\cdot0075)^{120}$ ✓
$= \$3677\cdot04$

(iii) Total Amount $= 1500\left[1\cdot0075^{120} + 1\cdot0075^{119} + 1\cdot0075^{118} + \ldots + 1\cdot0075\right]$ ✓

$= 1500\left[1\cdot0075\,\dfrac{(1\cdot0075^{120} - 1)}{(1\cdot0075 - 1)}\right]$ ✓

$= 1500 \times 194\cdot9656342$ ✓
$= \$292448$

(iv) New Total $= 1600 \times 194\cdot9656342$ ✓
$= \$311945$

$\therefore$ Difference $= \$19497$ ✓