Review of 2 Unit Financial Maths – Specimen Paper 1

1. Simple Interest and AP

Simple interest is when we get interest based on our principal [the money initially put in]. Interest = $P\,R\,n$, P = principal R = interest rate in decimals [1% = 0.01] n = units of time

$\boxed{A_n = P + P\,R\,n}$, A_n = the total amount in the bank after n units of time

Obviously, A_n forms an AP with the first term $P + PR$ and common difference PR

Note: for simple interest and all other financial problems, $\boxed{n \text{ must be in the same unit as the interest rate}}$; you can always convert the rate, e.g. 12% pa is 1% per month; but there is another thing governing the unit of n, namely the compounding period.

1.1 Compound Interest and GP

Compound interest is we get interest based on the total money we have in the bank at the end of the compounding period. If your money is compounded monthly, n must be expressed in month. If this period is not stated, it's the same as in the given interest rate.

$\boxed{A_n = P(1+R)^n}$, which is a GP with first term $P(1+R)$ and ratio $1+R$

1.2 Depreciation and GP

Depreciation is similar to compound interest. $A_n = P(1-R)^n$, P is the initial worth of the item and A_n is how much the item is worth after n units of time.

Exercises:

(1) Mary invests $2 000 in a bank account that pays 4% compound interest p.a., paid quarterly. How much money does Mary have in her account after 5 years?

$\boxed{\$2440.38}$

(2) The Bank of Saturn offers interest at 5.8% p.a. paid monthly, while the Bank of Neptune pays interest at 6%, paid twice a year. If I have $5 000 to invest, which bank should I use, and how much extra money will I have after 7 years?

Neptune; $66.27 extra

(3) Mark invests $1 000 in a bank account that pays interest compounded quarterly. After four years he has $1219.89 in the bank. What is the interest rate p.a.?

5% p.a.

2. Investing Money by Regular Instalments (such as in Superannuation)

Each individual instalment earns compound interest for a different length of time. Find what each instalment grows to as it accrues compound interest. These final amounts turn out to form a GP, which we can then sum. $A_n = that\ sum$ and we use this result to answer what are asked. You need to show the process.

Because of the large numbers involved, we usually use the pronumeral M to represent the instalment.

Exercises:

(1) You are investing $10 000 on 1st July each year, beginning in 2000. The money earns compound interest at 8% pa.

(a) How much will the fund amount to by 30th June 2020 (to the nearest dollar).

$494 229

(b) Find the year in which the fund first exceeds $700 000 on 30th June

In 2024

(c) What annual instalment would have produced $1 000 000 by 2020. (Answer in dollars and cents),

$20 233.53

(2) (a) Grandma wants to invest a certain amount of money for her grandchild so that she will have $5 000 in ten years time. If the bank account pays 5% p.a. interest, paid annually, how much will she need to invest now?

$3069.57

(b) Instead of investing just one amount now, Grandma decides to bank $500 at the beginning of each year for ten years. If the money earns 5% p.a., how much money will she have after ten years?

$6603.39

(3) Mark starts work and invests \$200 in a superannuation fund at the beginning of each year until he retires 30 years later. If the money earns 9% p.a., how much will Mark have when he retires?

\$29715.04

3. Paying Off a Loan by Regular Instalments

Compound interest (or in this context it's more appropriately called reducible interest since the dollar amount of interest reduces over time) is charged on the balance still owing at the time of payment (of the instalment). That is, if the interest is 1% then $A_n = A_{n-1}(1+0.01) - M$, A_n is the amount still owing and M is the instalment. It can be shown that this is equivalent to the principal getting interest and the individual instalments also getting interest. Again we develop a formula for A_n, and you need to show the process.

Exercises:

(1) You took a huge loan of \$200 000 on 1 January 2002. Interest is charged at 12% pa, compounded monthly. You decided to pay off the load in monthly instalments of \$2200 [actually, if it's compounded monthly then you automatically need to pay monthly, and vice versa].

(a) Show that the amount owing at the end of n months is

$$A_n = P \times 1.01^n - 100M\left(1.01^n - 1\right)$$

(b) find how long it takes to repay (i) the full loan

20 years

(ii) half the loan

15 years

(c) By justifying your answer, explain why would instalments of $1900 per month never repay the loan.

(2) Maria applies for a loan of $15 000 to be paid back in monthly instalments over 5 years. The interest on the loan is 16% p.a.

(a) How much does Maria owe after one month?

$\boxed{\$15200 - M}$

(b) What is the amount of each monthly instalment?

$\boxed{\$364.77}$

(c) How much money will Maria pay on the loan altogether?

$\boxed{\$21886.25}$

(3) Farmer Brown buys a harvester for \$250 000 and pays it off over 3 years in equal yearly instalments. If the interest rate on the harvester is 14.5% p.a., find the amount of each yearly instalment.

\$108587.44

(4) Furniture Galore has a special deal where you can buy a lounge on hire purchase and pay it off over two years. No repayments are needed for the first three months. Hire purchase interest is 18% p.a. If George buys a \$3 000 lounge, find
(a) the amount he owes after three months

\$3137.04

(b) the amount of each monthly repayment.

\$167.70

Lily

Review of 2 Unit Financial Maths – Specimen Paper 1

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Note: for simple interest and all other financial problems, n must be in the same unit as the interest rate; you can always convert the rate, e.g. 12% pa is 1% per month; but there is another thing governing the unit of n, namely the compounding period.

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$A_n = P(1+R)^n$, which is a GP with first term $P(1+R)$ and ratio $1+R$

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Depreciation is similar to compound interest. $A_n = P(1-R)^n$, P is the initial worth of the item and A_n is how much the item is worth after n units of time.

Exercises:

(1) Mary invests $2 000 in a bank account that pays 4% compound interest p.a., paid quarterly. How much money does Mary have in her account after 5 years?

~~$A_5 = 2000 \times 1.04^5$~~

$4\% = 1\%$

$R = 1.01$

5×4 (quarterly) ✓

$=$

$\therefore A_{20} = 2000 \times 1.01^{20}$

$= 2440.38$ ✓

$A_1 = 2000$

$2440.38

(2) The Bank of Saturn offers interest at 5.8% p.a. paid monthly, while the Bank of Neptune pays interest at 6%, paid twice a year. If I have $5 000 to invest, which bank should I use, and how much extra money will I have after 7 years?

$\frac{5.8}{12} = 0.48\dot{3}$

Saturn: $(1.00483)^{84}$

$A_{84} = 5000 \times$ ~~$(0.483)^{84}$~~

$= 7496.68$ ✓

Neptune:

$A_{14} = 5000 \times (1.03)^{14}$

$= 7562.95$ ✓

$\therefore$ Neptune gives $66.27 more ✓

Neptune; $66.27 extra

(3) Mark invests $1 000 in a bank account that pays interest compounded quarterly. After four years he has $1219.89 in the bank. What is the interest rate?

$A_{16} = 1000 \times R^n$

$1219.89 = 1000 \times R^n$ ✓

$1.21989 = R^n$ ✓

~~$\ln 1.21989 = \ln R^n$~~

$1.21989 = R^{16}$ ✓

$R = 1.0125$

$\therefore 0.0125 = \frac{r}{4}$ ✓

$r = 0.05 \quad \therefore r = 5\%$

5%

2. *Investing Money by Regular Instalments (such as in Superannuation)*

Each individual instalment earns compound interest for a different length of time. Find what each instalment grows to as it accrues compound interest. These final amounts turn out to form a GP, which we can then sum. A_n = *that sum* and we use this result to answer what are asked. You need to show the process.
Because of the large numbers involved, we usually use the pronumeral M to represent the instalment.

Exercises:
(1) You are investing $10 000 on 1st July each year, beginning in 2000. The money earns compound interest at 8% pa.

(a) How much will the fund amount to by 30th June 2020 (to the nearest dollar).

$R = 1.08$

$A_1 = MR$

$A_2 = MR^2 + MR$

$A_3 = MR(1 + R + R^2)$

⋮

$A_{20} = MR(1 + R + \dots + R^{19})$

$\hookrightarrow S_{20} = \frac{1(R^{20}-1)}{R-1} = 45.76$

$A_{20} = \$494\ 229.21$

$= \$494\ 229.21$

$494229

(b) Find the year in which the fund first exceeds $700 000 on 30th June

$700{,}000 = MR(1 + R + \dots + R^n)$

$\therefore 64.815 = (1 + R + \dots + R^n)$

$\hookrightarrow S_n = \frac{1(R^n - 1)}{R-1}$

$64.815 \times 0.08 = R^n - 1$

$R^n = 6.18519$

$\ln 6.18519 = \ln R^n$

$n = \frac{\ln 6.18519}{\ln 1.08}$

$= 23.67$

$\doteq 24$

$\therefore$ in year 2024

In 2024

(c) What annual instalment would have produced $1 000 000 by 2020. (Answer in dollars and cents),

~~1000000 = 10,000 × R × S20~~

~~92.59~~

$1000{,}000 = MRS_{20}$

~~M = 74147.6~~

$M = \frac{1000000}{49.4229} = \$20{,}233.53$

$20233.53

$5000 = MK$

$M = 3069.57$

(2) (a) Grandma wants to invest a certain amount of money for her grandchild so that she will have $5 000 in ten years time. If the bank account pays 5% p.a. interest, paid annually, how much will she need to invest now?

$R = 1.05$

$A_1 = MR$

$A_2 = MR^2 + MR$

$A_3 = MR(1 + R + R^2)$

⋮

$A_{10} = MR(1 + R + \dots + R^9)$

$\hookrightarrow S_{10} = \frac{1(R^{10}-1)}{R-1}$

$\therefore 5000 = M \times 1.05 \times 12.57789$

$M \doteq 378.59$

$3069.57

(b) Instead of investing just one amount now, Grandma decides to bank $500 at the beginning of each year for ten years. If the money earns 5% p.a., how much money will she have after ten years?

$A_1 = MR$

$A_2 = MR^2 + MR$

$A_3 = MR(1 + R + R^2)$

⋮

$A_{10} = MR(1 + R + \dots + R^9)$

$\hookrightarrow S_{10} = \frac{1(1.05^{10}-1)}{1.05-1} = 12.57789\dots$

$= 500 \times 1.05 \times 12.57789\dots$

$= \$6603.39$

$6603.39

(3) Mark starts work and invests $200 in a superannuation fund at the beginning of each year until he retires 30 years later. If the money earns 9% p.a., how much will Mark have when he retires?

$M = 200$

$R = 1.09$

$A_1 = MR$

$A_2 = MR^2 + MR$

$A_3 = MR(1+R+R^2)$ ✓

$A_{30} = MR(1+R+\dots+R^{29})$ → $S_{30} = \frac{1(R^{30}-1)}{R-1} = 136.3075$

$\therefore = 200 \times 1.09 \times 136.3075$

$= \$29\,715.04$

$29715.04

3. *Paying Off a Loan by Regular Instalments*

Compound interest (or in this context it's more appropriately called reducible interest since the dollar amount of interest reduces over time) is charged on the balance still owing at the time of payment (of the instalment). That is, if the interest is 1% then $A_n = A_{n-1}(1+0.01) - M$, A_n is the amount still owing and M is the instalment. It can be shown that this is equivalent to the principal getting interest and the individual instalments also getting interest. Again we develop a formula for A_n, and you need to show the process.

Exercises:

(1) You took a huge loan of $200 000 on 1 January 2002. Interest is charged at 12% pa, compounded monthly. You decided to pay off the load in monthly instalments of $2200 [actually, if it's compounded monthly then you automatically need to pay monthly, and vice versa].

(a) Show that the amount owing at the end of n months is

$\frac{12\%}{12} = 1$, $R = 1.01$

$A_n = P \times 1.01^n - 100M(1.01^n - 1)$

$A_1 = PR - M$

$A_2 = (PR - M)R - M$

$= PR^2 - RM - M$

$A_3 = (PR^2 - RM - M)R - M$

$= PR^3 - R^2M - RM - M$

$= PR^3 - M(1+R+R^2)$

$A_n = PR^n - M(1+R+R^2+\dots+R^{n-1})$ → $S_n = \frac{1(R^n-1)}{R-1} = \frac{1.01^n - 1}{0.01} = 100(1.01^n - 1)$

$\therefore A_n = P \times (1.01)^n - 100M(1.01^n - 1)$ ✓

(b) find how long it takes to repay (i) the full loan

$P \times (1.01)^n = 100M(1.01^n - 1)$

$M = 2200$

$200{,}000 \times (1.01)^n = 100 \times 2200 (1.01^n - 1)$

$\frac{10}{11}(1.01)^n = 1.01^n - 1$

$\frac{10}{11} = \frac{1.01^n - 1}{1.01^n}$ $\frac{10}{11} = 1 - \frac{1}{1.01^n}$

$\frac{1}{1.01^n} = \frac{1}{11}$ $11 = 1.01^n$

$\ln 11 = n \ln 1.01$

$n = \frac{\ln 11}{\ln 1.01} = 240$ (months) $= 20$ years.

20 years

(ii) half the loan

$\frac{P}{2} = P(1.01)^n - 100M(1.01^n - 1)$

$\frac{P}{2} = P(1.01)^n - 220000(1.01^n - 1)$

$100{,}000 = 200{,}000(1.01)^n - 220\,000(1.01^n - 1)$

$\frac{1}{2} = (1.01)^n - \frac{11}{10}(1.01^n - 1)$ ✓

$5 = 10(1.01^n) - 11(1.01^n) + 11$

$1.01^n = 6$

$n \ln 1.01 = \ln 6$

$n = \frac{\ln 6}{\ln 1.01}$ ✓

$= 180$ (months)

15 years $= 15$ yrs

(c) By justifying your answer, explain why would instalments of $1900 per month never repay the loan.

$0 = 200{,}000(1.01)^n - 190\,000(1.01^n - 1)$

$190{,}000(1.01^n - 1) = 200{,}000(1.01)^n$

$1.01^n - 1 = \frac{20}{19}(1.01)^n$

$\frac{1.01^n - 1}{1.01^n} = \frac{20}{19}$

$1 - \frac{1}{1.01^n} = \frac{20}{19}$ ✓

$-0.0526 = \frac{1}{1.01^n}$

$1.01^n = \frac{-1}{0.0526}$

→ interest accumulates at more than $2000 ✓ with $1900 installments, interest will be growing too fast for loan to be paid back. ✓

can't take log of a neg number

(2) Maria applies for a loan of $15 000 to be paid back in monthly instalments over 5 years. The interest on the loan is 16% p.a.

(a) How much does Maria owe after one month?

$\frac{16}{12} = 1.\dot{3}$

$A_1 = PR - M$

$= 15000 \times 1.01\dot{3} - M$

$= 15200 - M$

$\therefore$ Maria owes $15200

$15200

(b) What is the amount of each monthly instalment?

$A_{60} = PR^{60} - M(1 + R + \ldots + R^{59})$

$15000 \times (1.01\dot{3})^{60} = M\left(\frac{1(R^{60}-1)}{0.01\dot{3}}\right)$

$\frac{33\,207.10323}{91.0355} = M$

$M = \$364.77$

$364.77

(c) How much money will Maria pay on the loan altogether?

total $= 60 \times 364.77$

$= \$21886.2$

$21886.25

(3) Farmer Brown buys a harvester for $250 000 and pays it off over 3 years in equal yearly instalments. If the interest rate on the harvester is 14.5% p.a., find the amount of each yearly instalment.

$A_1 = PR - M$

$A_2 = PR^2 - RM - M$

$A_3 = PR^3 - M(1 + R + R^2)$

$\therefore 0 = 250{,}000 \times (1.145)^3 - M(1 + 1.145 + 1.145^2)$

$M = \frac{250\,000 \times (1.145)^3}{3.456025}$

$= \$108587.44$

$108587.44

(4) Furniture Galore has a special deal where you can buy a lounge on hire purchase and pay it off over two years. No repayments are needed for the first three months. Hire purchase interest is 18% p.a. If George buys a $3 000 lounge, find

(a) the amount he owes after three months

$\frac{18}{12} = 1.5\%$

$R = 1.015$

$A_1 = PR$

$A_2 = PR^2$

$A_3 = PR^3$

NO M

$= 3000 \times 1.015^3$

$= \$3137.04$

$3137.04

(b) the amount of each monthly repayment.

$A_1 = PR$

$A_2 = PR^2$

$A_3 = PR^3$

$A_4 = PR^4 - M$

$A_5 = PR^5 - RM - M$

$= PR^5 - M(R+1)$

$\vdots$

$A_{24} = PR^{24} - M(1 + R + \ldots + R^{20})$

$PR^{24} = M(1 + R + \ldots + R^{20})$

$\hookrightarrow S_{21} = \frac{1(R^{21}-1)}{0.015} =$

$\frac{4288.508}{24.47} = M$

$M = \$175.25$

$175.25