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MATHEMATICS SPECIMEN PAPER 1

GEOMETRIC SERIES & SEQUENCES

1. In a geometric progression, the fifth term is 27, and the seventh term is 243.

(a) Find two possible values for the common ratio. [4]

(b) Find the value of the eighth term. [3]

2. If the first term of a geometric progression is 5 and the common ratio is 2, find the number of terms that gives a sum equal to 5115. [3]

3. In a geometric series the sum of the first two terms is 45, and the third term is 12.

(a) Find the two possible values for the common ratio. Also find corresponding values for the first term. [4]

(b) Find the sum of the first five terms of the series in each case. [2]

4. The sum of the first three terms of a geometric series is 27.1, and the sum to infinity is 100.

(a) Write down two equations involving a and r, the first term and common ratio respectively. [2]

(b) Hence find the values of a and r. [2]

(c) Write down an expression for the sum of n terms of this series. [1]

(d) How many terms would be required for the sum to be greater than 95? [3]

Solutions:

1.

(a) The nth term of a geometric progression is

$$T_n = ar^{n-1}$$

$$T_5 = ar^4 = 27 \qquad -\{1\}$$

$$T_7 = ar^6 = 243 \qquad -\{2\}$$

$$\frac{\{2\}}{\{1\}} \qquad \frac{ar^6}{ar^4} = \frac{243}{27}$$

$$\Rightarrow \qquad r^2 = 9$$

$$\Rightarrow \text{ Common ratio } \qquad r = \pm 3$$

(b) Substitute $r = \pm 3$ in $\{1\}$

$$a \times 3^4 = 27$$

$$a \times 81 = 27$$

$$a = \frac{27}{81}$$

$$\text{First term } a = \frac{1}{3}$$

$$\therefore \qquad \text{Eighth term } T_8 = \frac{1}{3}3^7$$

$$= 729$$

$$\text{or} \qquad T_8 = \frac{1}{3}(-3)^7$$

$$= -729$$

$$\Rightarrow \qquad \text{Eighth term} = \pm 729$$

2. The sum of n terms of a geometric equation is

$$S_n = \frac{a(r^n - 1)}{(r-1)}$$

$$\therefore \qquad \frac{5(2^n - 1)}{2-1} = 5115$$

$$\div 5 \qquad 2^n - 1 = 1023$$

$$2^n = 1024$$

$$2^n = 2^{10}$$

$$\Rightarrow \qquad n = 10$$

$\therefore$ 10 terms give a sum equal to 5115

3.

(a)

$$a + ar = 45 \qquad -\{1\}$$
$$ar^2 = 12 \qquad -\{2\}$$

From {2} $\quad a = \frac{12}{r^2}$

Substitute in {1}

$$\frac{12}{r^2} + \frac{12}{r^2} \times r = 45$$
$$\frac{12}{r^2} + \frac{12}{r} = 45$$

$\times r^2 \quad 12 + 12r = 45r^2$

$\Rightarrow \quad 45r^2 - 12r - 12 = 0$

$\div 3 \quad 15r^2 - 4r - 4 = 0$

$$(5r + 2)(3r - 2) = 0$$
$$5r + 2 = 0 \text{ or } 3r - 2 = 0$$
$$r = -\frac{2}{5} \text{ or } r = \frac{2}{3}$$

When $r = -\frac{2}{5}$, substituting in {2}

$$a\left(-\frac{2}{5}\right)^2 = 12$$
$$\frac{4a}{25} = 12$$

$\Rightarrow \quad a = 75$

When $r = \frac{2}{3}$ substitute in {2}

$$a\left(\frac{2}{3}\right)^2 = 12$$
$$\frac{4a}{9} = 12$$
$$a = 27$$

(b) The sum of the first n terms of a geometric series is

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum of the first 5 terms when $a = 75, r = -\frac{2}{5}$ is

$$S_5 = 75\frac{\left(1-\left(-\frac{2}{5}\right)^5\right)}{1-\left(-\frac{2}{5}\right)}$$

$$= 75\frac{\left(1+\frac{32}{3125}\right)}{\frac{7}{5}}$$

$$= \frac{9471}{125}\times\frac{5}{7}$$

$$S_5 = 54\frac{3}{25}$$

Sum of the first 5 terms when $a = 27, r = \frac{2}{3}$, is

$$S_5 = 27\frac{\left(1-\left(\frac{2}{3}\right)^5\right)}{\left(1-\frac{2}{3}\right)}$$

$$= 27\frac{\left(1-\frac{32}{243}\right)}{\frac{1}{3}}$$

$$S_5 = 70\frac{1}{3}$$

4.

(a)

$$a + ar + ar^2 = 27.1 \qquad -\{1\}$$

$$\frac{a}{(1-r)} = 100 \qquad -\{2\}$$

(b) From {2} $\quad a = 100(1-r)$

Substitute in {1}

$$100(1-r) + 100r(1-r) + 100r^2(1-r) = 27.1$$

$$\Rightarrow \quad 100 - 100r^3 = 27.1$$

$$100(1-r^3) = 27.1$$

$$(1-r^3) = 0.271$$

$$r^3 = 1 - 0.271$$

$$r^3 = 0.729$$

$$r = \sqrt[3]{0.729}$$

$$r = 0.9$$

$$\therefore \quad a = 100(1 - 0.9)$$
$$a = 10$$

(c) Sum of n terms of a geometric series is

$$S_n = a\frac{(1-r^n)}{(1-r)}$$
$$= 10\frac{(1-0.9^n)}{(1-0.9)}$$
$$S_n = 100(1-0.9^n)$$

(d) If

$$S_n > 95$$
$$100(1 - 0.9^n) > 95$$
$$\div 100 \quad 1 - 0.9^n > 0.95$$
$$\Rightarrow \quad 0.9^n < 0.05$$

Take ln of both sides

$$\ln 0.9^n < \ln 0.05$$
$$n \ln 0.9 < \ln 0.05$$
$$-0.1054n < -2.9957$$
$$\div -0.1054 \quad n > \frac{-2.9957}{-0.1054}$$

(Remember to reverse the inequality)

$$n > 28.4$$

$\Rightarrow$ 29 terms would be required for the sum to be greater than 95.