NAME :

Centre of Excellence in Mathematics
S201 / 414 GARDENERS RD. ROSEBERY 2018
www.cemtuition.com.au

MOBILE 0412 880 475

PHONE 9666 3331

YEAR 12 – MATHEMATICS

SPECIMEN PAPER 2
TOPIC : GROWTH & DECAY

AMP 2002 Q10

(c) The diameter of a tree (D cm), t years after the start of a particular growth period is given by:

$$D = 80e^{kt}.$$

(i) Show that $\frac{dD}{dt} = kD$ where k is a constant. 2

(ii) If $k = 0.018$, how long will it take for the diameter of the tree to measure 90cm (to the nearest whole number)? 2

ASCHAM 2001 Q8

b) The amount M grams of a chemical is given by

$M = M_0 e^{-kt}$ where M_0 and k are positive constants and time t is measured in years.

i) Show that M satisfies the equation $\frac{dM}{dt} = -kM$ (1)

ii) Find k (in exact form) if 200 grams of the chemical decomposes to 150 grams at the end of 2 years. (2)

iii) Find the amount of the chemical which has decomposed by the end of 10 years (to the nearest gram.) (2)

CSSA 2001 Q6

(b) The number N of bacteria in a colony is growing at a rate that is proportional to the current number. The number at time t hours is given by

$N = N_0 e^{kt}$ where N_0 and k are positive constants.

(i) If the size of the colony doubles every half hour, find the value of k. **2**

(ii) If the colony now contains 600 million bacteria, how long ago did the colony contain 3 million bacteria? **2**

(iii) Show that the numbers of bacteria present at consecutive integer hours form a geometric sequence. **2**

CSSA 2000 Q7

(a) An experimental vaccine was injected into a cat. The amount, M millilitres, of vaccine present in the bloodstream of the cat, t hours later was given by $M = e^{-2t} + 3$. **5**

(i) How much vaccine was initially injected into the cat?

(ii) At what rate was the amount of vaccine decreasing at the end of 3 hours?

(iii) Show that there will always be more than 3 millilitres of vaccine present in the cat's bloodstream.

(iv) Sketch the curve of $M = e^{-2t} + 3$ to show how the amount of vaccine present in the cat's bloodstream changes over time.

CSSA 2002 Q5

(c) The population P of a town is growing at a rate proportional to the town's current population. The population at time t years is given by $P = A\,e^{kt}$, where A and k are constants.

The population 20 years ago was 100 000 people and today the population of the town is 150 000 people.

(i) Find the value of A. **1**

(ii) Find the value of k. **1**

(iii) Find the population that will be present 20 years from now. **2**

SYDNEY GRAMMAR 2000 Q9

(a) The value $\$V$ of a car is given by the formula $V = Ce^{-kt}$, where C and k are constants and t is the time measured in years. Michael bought a car on June 30th 2001 which cost \$65 000 and which was worth \$55 000 after one year.

[2] (i) Evaluate the constants C and k.

[1] (ii) Find the value of the car after 5 years. Give your answer correct to the nearest dollar.

2 (iii) In which year will the value of the car fall below half its cost price for the first time?

SOLUTIONS

AMP 2002 Q10

(c) (i) $D = 80e^{kt}$

$\frac{dD}{dt} = 80e^{kt} \times k$ (but $D = 80e^{kt}$) ✓

$\therefore \frac{dD}{dt} = kD$ ✓

(ii) $90 = 80e^{0.018t}$

$\frac{90}{80} = e^{0.018t}$

$\ln\left(\frac{9}{8}\right) = 0.018t$ ✓

$6.54 = t$

7 years $= t$ ✓

ASCHAM 2001 Q8

b) $M = M_0 e^{-kt}$

i) $\frac{dM}{dt} = M_0 \cdot -k e^{-kt}$

$= -k M_0 e^{-kt}$ (1)

$= -kM.$

ii) $M = 200 e^{-kt}$

$M = 150, t = 2$: $150 = 200 e^{-k \times 2}$

$\frac{150}{200} = e^{-2k}$

$\frac{3}{4} = e^{-2k}$

$k = \frac{\ln 3/4}{-2}$ (2)

iii) $t=10$: $M = 200\, e^{-\frac{\ln\frac{3}{4}}{-2} \times 10}$

$= 200\, e^{\ln 3/4}$

$\doteqdot 47.4609 \ldots$ (2)

∴ amount of chemical left after 10 years is 47 g (to n gram)

∴ Amt decomposed is 153 g (to n gram)

CSSA 2001 Q6

(b) (i)

$N = 2N_o$ when $t = 0.5$

Solve $2N_o = N_o e^{0.5k}$

$\Rightarrow e^{0.5k} = 2$

$\Rightarrow 0.5k = \ln 2$

$\Rightarrow k = \dfrac{\ln 2}{0.5} = 1.38629\ldots$

(ii)

$600 = 3e^{1.386\ldots t}$

$\ln 200 = 1.386\ldots t$

$\Rightarrow t = \dfrac{\ln 200}{1.386\ldots}$

$= 3.8219\ldots \text{h}$

(iii)

when $t = 0,\ N = N_o$

$t = 1,\ N = N_o e^k$

$t = 2,\ N = N_o e^{2k}$

$t = 3,\ N = N_o e^{3k}$

$t = 4,\ N = N_o e^{4k}$

$$\frac{N_o e^{2k}}{N_o e^k} = \frac{N_o e^{3k}}{N_o e^{2k}} = \frac{N_o e^{4k}}{N_o e^{3k}} = \frac{N_o e^k}{N_o} = e^k$$

CSSA 2000 Q7

(a)

(i) $M = e^{-2(0)} + 3$
$= 4$ mL was initially injected into the cat.

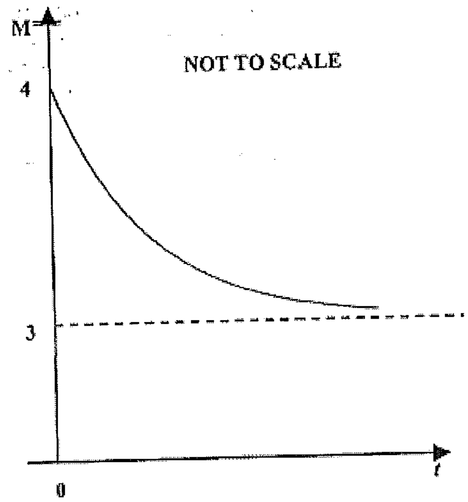
(ii) $\frac{dM}{dt} = -2e^{-2t}$

When $t = 3$, $\frac{dM}{dt} = -2e^{-2(3)} = -2e^{-6}$

$\therefore$ The amt of vaccine is decreasing at the rate of $2e^{-6}$ mL/h which is approximately 0.005 mL/h.

(iii) As $t \to \infty$, $e^{-2t} \to 0$, $\therefore$ M $\to 3$.
$\therefore$ There will always be more than 3 mL of vaccine present in the cat's bloodstream.

(iv)



CSSA 2002 Q5

(c) $P = Ae^{kt}$

(i) $A = 100\,000$ $(t=0 \Rightarrow e^{kt}=1)$ ✓

(ii) $150000 = 100\,000\,e^{20k}$

$e^{20k} = 1.5$

$20k = \ln 1.5$

$k = \frac{1}{20}\ln 1.5$ ✓

(iii) $t = 40$

$P = 100000\,e^{40 \times \frac{1}{20}\ln 1.5}$ ✓

$= 100000\,e^{2\ln 1.5}$

$= 225000$

∴ Pop^n in 20 years time will be 225000 ✓

SYDNEY GRAMMAR 2000 Q9

a) (i) $V = Ce^{-kt}$

When $t = 0$, $V = 65000$

$\therefore C = 65000$ ✓

When $t = 1$, $V = 55000$

$\therefore 55000 = 65000\,e^{-k}$

$e^{-k} = \frac{11}{13}$

$-k = \ln\left(\frac{11}{13}\right)$

$\therefore k = -\ln\left(\frac{11}{13}\right)$

$\doteqdot 0{\cdot}16705\ldots$ ✓ (either)

(ii) When $t = 5$,

$V = 65000e^{-5k}$

$= \$28194$ ✓

(iii) We need t such that

$V < \frac{65000}{2}$

ie/ $65000\,e^{-kt} < \frac{65000}{2}$

$e^{-kt} < \frac{1}{2}$

$-kt < \ln\frac{1}{2}$

$t > 4{\cdot}149\ldots$ ✓ (any)

$\therefore$ Car falls below half its cost price in 2005 ✓