C.E.M. TUITION

Name : ______________________________

Review Topic : Integration

(HSC - PAPER 2)

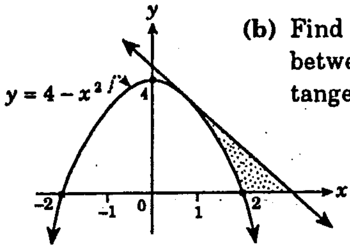
Year 12 - 2 Unit

7. (a) **(i)** Sketch the graphs of the curves $x^2+y^2=1$ and $y^2=1-x$ for $-1\le x\le 1$.

(ii) Find all points of intersection of the two graphs.

(b) The area in the first quadrant contained between the two curves is rotated about the x axis. Find the volume of revolution so formed.

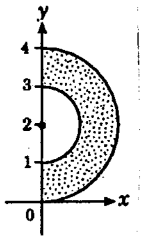
8. (a) Find the equation of the tangent to $y = 4 - x^2$ at the point where $x = 1$.

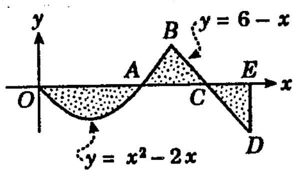


(b) Find the shaded area above the x axis between the curve $y = 4 - x^2$ and the tangent to the curve at $x = 1$.

9. The shaded region is the area between the curves $x^2+(y-2)^2=1$ and $x^2+(y-2)^2=4$.

Find the volume generated when the shaded region is rotated about the y axis.



10. 

The shaded region *OABCDE* is bounded by the lines $x = 0$ and $x = 7$, the curve $y = x^2 - 2x$, the line $y = 6 - x$ and the x axis.

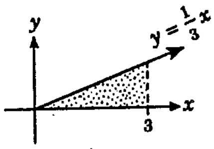
(a) Find the coordinates of the points *A*, *C*, *E*.

(b) Show that the coordinates of *B* are (3, 3).

(c) Calculate the area of the shaded region *OABCDE*.

11. **(a)** Find $\int (3x-10)^{12}\, dx$.

(b)

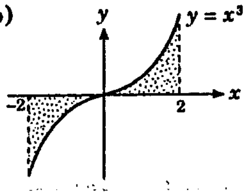


The shaded region is bounded by $y = \frac{1}{3}x$, the x axis and the line $x = 3$.

Find the volume of the conical shape formed when this region is rotated about the x axis.

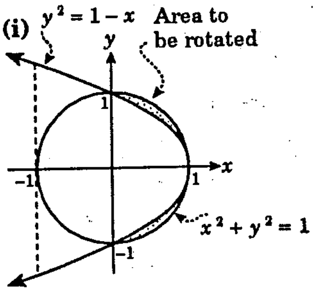
12. **(a)** Evaluate $\int_{-2}^{2} (x^2 - x^4)\, dx$.

(b)



Find the area bounded by the curve $y = x^3$, the x axis and the lines $x = -2$ and $x = 2$.

7. (a) (i)



$$\boxed{\begin{array}{l} x^2+y^2=1 \\ y^2=1-x^2 \\ \hline y^2=1-x \end{array}}$$

(ii) Curves intersect at (0, 1), (0, −1) and touch at (1, 0). [By observation]

(b) Volume required is difference between volumes obtained by rotating sections of curves $x^2+y^2=1$ and $y^2=1-x$ in first quadrant.

$$V=\pi\int_0^1(1-x^2)\,dx-\pi\int_0^1(1-x)\,dx$$
$$=\pi\left(\left[x-\frac{1}{3}x^3\right]_0^1-\left[x-\frac{1}{2}x^2\right]_0^1\right)$$
$$=\pi\left(\left[1-\frac{1}{3}\right]-0-\left[\left(1-\frac{1}{2}\right)-0\right]\right)$$
$$=\pi\left(\frac{2}{3}-\frac{1}{2}\right)$$
$=\frac{\pi}{6}$. Volume is $\frac{\pi}{6}$. units³.

8. (a) $y=4-x^2$ [$y=3$ when $x=1$]

$\therefore\ \frac{dy}{dx}=-2x$

$=-2$ at $x=1$.

Gradient of tangent = −2.

Equation of tangent is of form $y-3=-2(x-1)$

$=-2x+2,$

i.e., $2x+y=5$.

Eqn. is $2x+y=5$.

(b) Line $2x+y=5$ intersects x axis at $x=\frac{5}{2}$ (when $y=0$).

Required area

= (area of Δ base $1\frac{1}{2}$, height 3) $-\int_1^2(4-x^2)\,dx$

$$A=\frac{1}{2}\times\frac{3}{2}\times3-\left[4x-\frac{1}{3}x^3\right]_1^2$$
$$=\frac{9}{4}-\left[\left(8-\frac{8}{3}\right)-\left(4-\frac{1}{3}\right)\right]$$
$$=\frac{9}{4}-\left[4-\frac{7}{3}\right]$$
$$=\frac{9}{4}-\frac{5}{3}$$
$$=\frac{7}{12}.$$

Required area is $\frac{7}{12}$ units².

9. $x^2=1-(y-2)^2$ and $x^2=4-(y-2)^2$

$$V=\pi\int_0^4 4-(y-2)^2\,dy-\pi\int_0^4 1-(y-2)^2\,dy$$
$$=\pi\int_0^4 3-(y-2)^2+(y-2)^2\,dy$$
$$=\pi\int_0^4 3\,dy$$
$$=\pi[3y]_0^4$$
$$=\pi[12]$$
$$=12\pi.$$

Volume is 12π units³.

10. (a) For A, $y=x^2-2x$,

when $y=0$, $x(x-2)-0$,

i.e., $x=0$ or $x=2$.

A is (2, 0).

For C, $y=6-x$

when $y=0$, $x=6$.

C is (6, 0) E is (7, 0)

$A(2, 0)$, $E(7, 0)$, $C(6, 0)$

(b) $y=x^2-2x$ —①

$y=6-x$ —②

① in ②: $x^2-2x=6-x$

$\therefore\ x^2-x-6=0$

$(x-3)(x+2)=0$

i.e., $x=3$ or -2.

When $x=3$ subs. in ②:

$y=6-3=3$

B is (3, 3).

(c) Area between O and A is given by

$$\left|\int_0^2 x^2-2x\,dx\right|$$
$$=\left|\left[\frac{1}{3}x^3-x^2\right]_0^2\right|$$
$$=\left|\left(\frac{8}{3}-4\right)-(0)\right|$$
$$=\frac{4}{3}\text{ units}^2.$$

Area between AB

$$=\int_2^3(x^2-2x)\,dx$$
$$=\left[\frac{1}{3}x^3-x^2\right]_2^3$$
$$=(9-9)-\left(\frac{8}{3}-4\right)$$
$$=\frac{4}{3}\text{ units}^2.$$

Area between BC

= Area of Δ

$=\frac{1}{2}\times3\times3$ $\boxed{A=\frac{1}{2}bh}$

$=4\cdot5$ units².

Area CDE = Area of Δ $\boxed{D=(7,-1)}$

$=\frac{1}{2}\times1\times1$

$=0\cdot5$ unit².

Total $A=2\times\frac{4}{3}+4\cdot5+0\cdot5$

$=7\frac{2}{3}$ units².

11. (a) $\int(3x-10)^{12}\,dx$

$$=\frac{1}{13}(3x-10)^{13}\times\frac{1}{3}+c$$

$\boxed{\text{cw}\int x^{12}\,dx}$

$$=\frac{1}{39}(3x-10)^{13}.$$

(b) $y = \frac{1}{3}x \quad \therefore \; y^2 = \frac{1}{9}x^2$

$$V = \pi \int_0^3 \left(\frac{1}{9}x^2\right) dx$$
$$= \frac{1}{9}\pi \int_0^3 x^2 \, dx$$
$$= \frac{1}{9}\pi \left[\frac{1}{3}x^3\right]_0^3$$
$$= \frac{1}{9}\pi \left[\frac{1}{3}.27\right]$$
$$= \pi.$$

Volume is π units^3.

12. (a) $\int_{-2}^{2} (x^2 - x^4) \, dx$

$$= 2\int_0^2 (x^2 - x^4) \, dx$$

[as $f(x) = x^2 - x^4$ is an even function]

$f(x) = x^2 - x^4$
$f(-x) = (-x)^2 - (-x)^4$
$= x^2 - x^4$

$$= 2\left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_0^2$$
$$= 2\left[\frac{8}{3} - \frac{32}{5}\right]$$
$$= 2\left[-3\frac{11}{15}\right]$$
$$= -7\frac{7}{15}.$$

(b) Function is an odd function.

$$\text{Area} = 2\int_0^2 x^3 \, dx$$
$$= 2\left[\frac{1}{4}x^4\right]_0^2$$
$$= 2\left[\frac{1}{4}.16\right]$$
$$= 8.$$

Area is 8 units^2.