C.E.M.TUITION

Student Name : ________________________________

Review Topic : Linear Functions

(Preliminary Course - Paper 1)

Year 11 - 2 Unit

Question 1

Without using square paper, plot on the Cartesian plane the three points A, B, C, whose coordinates are $(-5,3)$, $(1,-5)$, $(2,2)$, respectively.

(a) Calculate the length AB.

(b) Find the equation of the line AB.

(c) The line through C, perpendicular to AB, meets AB at N. Find the coordinates of N.

(d) Hence, or otherwise, find the area of ΔABC.

(a) 10 units (b) $4x+3y+11=0$ (c) $(-2,-1)$ (d) 25 units^2

Question 2 :

The point $Q(-2, 1)$ lies on the line k whose equation is $9x - 2y + 20 = 0$.
The point $R(4, -2)$ lies on the line l whose equation is $3x + y - 10 = 0$.

(a) Show that k and l intersects at a point P on the y–axis.

(b) Find the equation of the line m which joins Q and R.

(c) Show, by shading on a sketch (not on graph paper), the region defined by the three inequalities

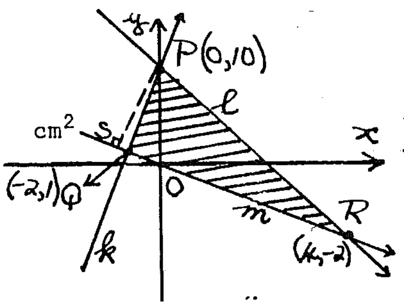
$$9x - 2y + 20 \geq 0, \quad 3x + y - 10 \leq 0, \quad x + 2y \geq 0.$$

(d) Find, as a surd, the perpendicular distance from P to m.

(e) Hence, or otherwise, find the exact value of the area of the triangle bounded by the three lines k, l and m.

(a) intersect at (0, 10) (b) $x + 2y = 0$ (d) $4\sqrt{5}$ (e) 30 units^2

(c)



Question 3 :

(a) $A(1,8)$, $B(3,7)$ and $C(-2,5)$ are three vertices of a parallelogram $ABCD$.
Find the coordinates of D.

(b) Show that the points $A(3,-1)$, $B(7,2)$ and $C(1, 10)$ are the vertices of a right-angled triangle.
Also find the area of ΔABC.

(a) $(-4,6)$ (b) 25 units^2

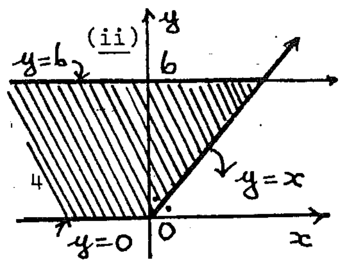
Question 4 :

(a) Find the equation of the line passing through the point $(2, 7)$ and parallel to the line $2x - 3y = 8$.

(b) On a sketch indicate, by suitable shading and labelling, the region $\{(x,y) : y \geq x\} \cap \{(x,y) : 0 \leq y \leq 6\}$.

(a) $2x - 3y + 17 = 0$

(b)



Question 5 :

(a) The three lines $3x - y = 6$, $2x + y = 14$ and $y = 0$ enclose a triangle. Find its area.

(b) The two perpendicular lines $3x + 2y = 12$, $2x + ay = b$ intersect at the point $(2, 3)$. Find the values of a and b.

(c) Show that the points $(2, 7)$, $(5, 13)$, $(-4, -5)$ are collinear.

(a) 15 units^2 (b) $a = -3, b = -5$

Question 6 :

(a) Give three inequalities satisfied by every point in the interior of the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$ and such that no point outside the triangle satisfies all three inequalities.

(b) R is the foot of the perpendicular from the point $P(-5, 10)$ to the line $4x - 3y = 0$.
Find the coordinates of R.

(a) $x > 0, y > 0, x + y < 1$ (b) $R(3,4)$

Question 1

Without using square paper, plot on the Cartesian plane the three points A, B, C, whose coordinates are $(-5,3)$, $(1,-5)$, $(2,2)$, respectively.

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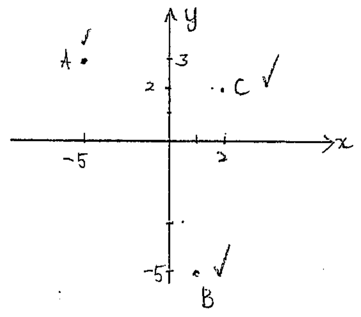

(a) $d_{AB} = \sqrt{6^2 + 8^2}$

$= \sqrt{100} = 10$ units ✓

(distance is positive)

(b) $m_{AB} = \frac{-8}{6} = \frac{-4}{3}$

$\therefore y + 5 = \frac{-4}{3}(x - 1)$ ✓

$3y + 15 = -4x + 4$

$4x + 3y + 11 = 0$ ✓

(c) $m_{\perp} = \frac{3}{4}$ $y - 2 = \frac{3}{4}(x - 2)$

$4y - 8 = 3x - 6$ ✓

$3x - 4y = -2$ — ①

$4x + 3y = -11$ — ②

① × 3 : $9x - 12y = -6$ — ③

② × 4 : $16x + 12y = -44$ — ④ ✓

③ + ④ : $25x = -50$

$x = -2$. sub in ①

$-6 - 4y = -2$

$4y = -4$

$y = -1$ ✓

$\therefore (-2, -1)$ is N

(d) $d_{CN} = \sqrt{16 + 9}$

$= 5$ units ✓

$\therefore$ A of $\Delta ABC = \frac{1}{2} \cdot 5 \cdot 10$

$= 25$ units^2

(a) 10 units (b) $4x + 3y + 11 = 0$ (c) $(-2, -1)$ (d) 25 units^2

Question 2 :

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$9x - 2y + 20 \geq 0$, $3x + y - 10 \leq 0$, $x + 2y \geq 0$.

(d) Find, as a surd, the perpendicular distance from P to m.

(e) Hence, or otherwise, find the exact value of the area of the triangle bounded by the three lines k, l and m.

(a) $9x - 2y = -20$ — ①

$3x + y = 10$ — ②

② × 2 : $6x + 2y = 20$. — ③

③ + ① : $15x = 0$

$x = 0$ ✓

$y = 10$ $\therefore (0, 10)$

$\therefore$ k and l intersects at a point P $(0, 10)$ on the y-axis

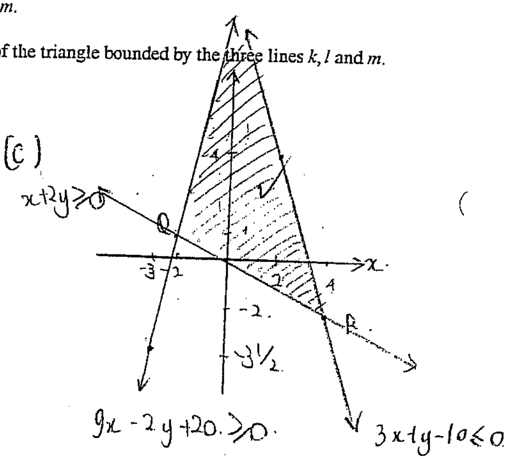
(b) $m_{QR} = \frac{-3}{6} = \frac{-1}{2}$.

$y - 1 = -\frac{1}{2}(x + 2)$ ✓

$2y - 2 = -x - 2$

$\therefore x + 2y = 0$ ✓

(c)



(d) P $(0, 10)$ m : $x + 2y = 0$

$\therefore d = \frac{|20|}{\sqrt{1+4}} = \frac{20}{\sqrt{5}}$

$= \frac{20\sqrt{5}}{5}$ ✓

$= 4\sqrt{5}$ units

(e) $d_{QR} = \sqrt{36 + 9}$

$= \sqrt{45} = 3\sqrt{5}$ units

$\therefore A = \frac{1}{2} \times 3\sqrt{5} \times 4\sqrt{5}$ ✓

$= 30$ units^2

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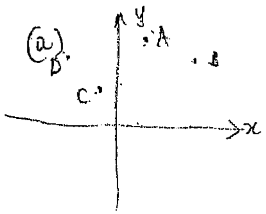
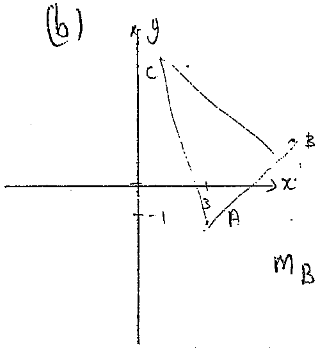
(a)



$D(-4, 6)$ ✓

(b)



$m_{BC} = \frac{8}{-6} = -\frac{4}{3}$

$m_{AB} = \frac{3}{4}$

since $m_{BC} \times m_{AB} = -\frac{4}{3} \times \frac{3}{4} = -1$ ✓

$AB \perp BC$

∴ A, B, C are the vertices of a right-angled Δ ✓

$d_{BC} = \sqrt{36+64} = \sqrt{100} = 10$ units

$d_{AB} = \sqrt{16+9} = \sqrt{25} = 5$ units

∴ A is 25 units² ✓

(a) (−4, 6) (b) 25 units²

Question 4 :

(a) Find the equation of the line passing through the point (2, 7) and parallel to the line $2x - 3y = 8$.

(b) On a sketch indicate, by suitable shading and labelling, the region $\{(x, y) : y \geq x\} \cap \{(x, y) : 0 \leq y \leq 6\}$.

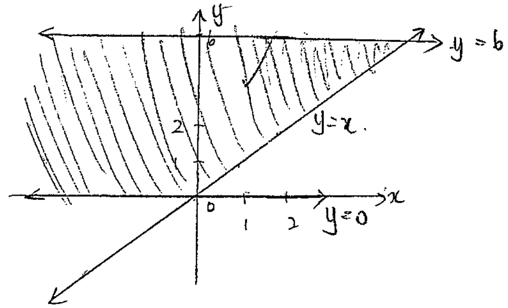
(a) $3y = 2x - 8$

$m = \frac{2}{3}$ ✓ ∴ $y - 7 = \frac{2}{3}(x - 2)$

$3y - 21 = 2x - 4$

∴ $2x - 3y + 17 = 0$ ✓

(b)



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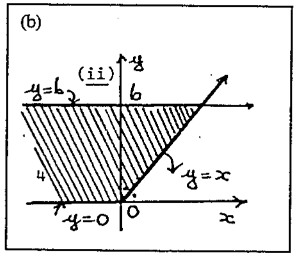

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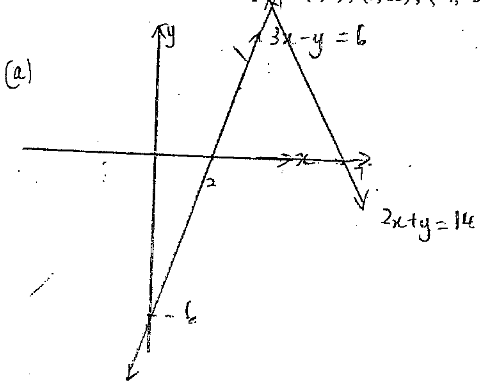
(a)



$3x - y = 6$ —①
$2x + y = 14$ —②
① + ②: $5x = 20$
$x = 4$ sub in ①
$12 - y = 6$
$6 = y$ ∴ $(4, 6)$

$d = 6$ units.

∴ $A = 6 \times 5 \times \frac{1}{2}$
$= 15$ units^2

(b) $2y = -3x + 12$
$y = \frac{-3}{2}x + 12$
$m = \frac{-3}{2}$
∴ $m_\perp = \frac{2}{3}$

∴ $ay = -2x + b$
$y = \frac{-2}{a}x + \frac{b}{a}$
∴ $\frac{-2}{a} = \frac{2}{3}$
$2a = -6$
$a = -3$

$2(2) - 3(3) = b$
$4 - 9 = b$
$b = -5$
∴ $a = -3$
$b = -5$

(a) 15 units^2 (b) $a = -3, b = -5$

Question 6 :

(a) Give three inequalities satisfied by every point in the interior of the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$ and such that no point outside the triangle satisfies all three inequalities.

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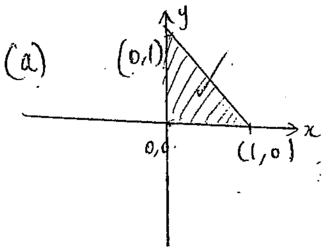
(a)



$y > 0 \cap x > 0 \cap y < -x + 1$
interior only.

(b) $3y = 4x$
$m = \frac{4}{3}$
$m_\perp = \frac{-3}{4}$
∴ $y - 10 = \frac{-3}{4}(x + 5)$
$4y - 40 = -3x - 15$
∴ $3x + 4y = 25$ —①
$4x - 3y = 0$ —②
① × 3: $9x + 12y = 75$ —③
② × 4: $16x - 12y = 0$ —④
③ + ④: $25x = 75$
$x = 3$ sub in ②
$12 - 3y = 0$
∴ $y = 4$

∴ $R = (3, 4)$.

(a) $x > 0, y > 0, x + y < 1$ (b) $R(3, 4)$