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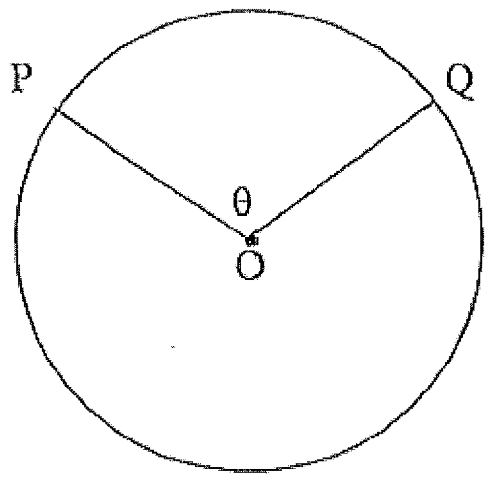
YEAR 12 – MATHEMATICS

SPECIMEN PAPER 2

TOPIC : PROBLEMS INVOLVING MAXIMA & MINIMA

ASCHAM 2001 Q8



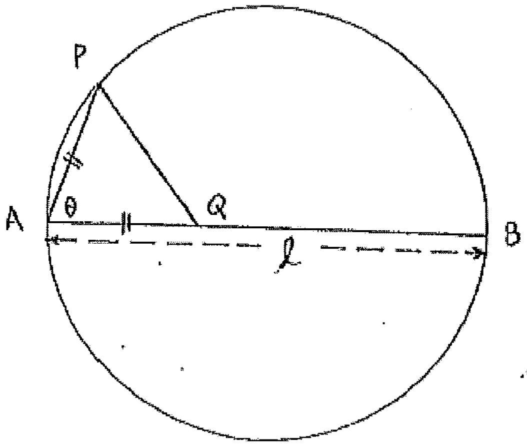
a) Arc PQ subtends an angle of θ radians at the centre.
The radius is r cm and the perimeter of the minor sector is 10 cm.

i) Show that the area of the sector is $A = 5r - r^2$ (3)

ii) Hence find the radius of the sector of maximum area when the perimeter of the sector is 10 cm. (2)

SYDNEY GRAMMAR 2000 Q10

(a)



In the diagram above, P is a point on the circle with diamater $AB = \ell$. The point Q is on the diameter such that $AP = AQ$. Let $\angle PAQ = \theta$ and let S be the area of $\triangle PAQ$.

Marks

3 (i) Show that $S = \frac{\ell^2}{2}\cos^2\theta\sin\theta$.

[3] (ii) Find the maximum area of $\triangle APQ$ as P moves along the circumference of the circle.

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(b) A cylindrical can of radius r centimetres and height h centimetres is to be made from a sheet of metal with area 300π centimetres^2. There is 10% wastage of the sheet in manufacturing the can. **6**

(i) Show that $h = \dfrac{135 - r^2}{r}$.

(ii) Find an expression for the volume V as a function of r.

(iii) Find the value of r which gives the maximum volume.

(iv) Calculate the maximum volume.

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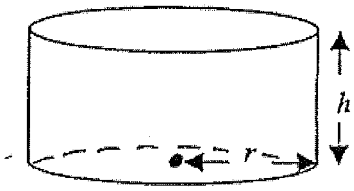
(c)



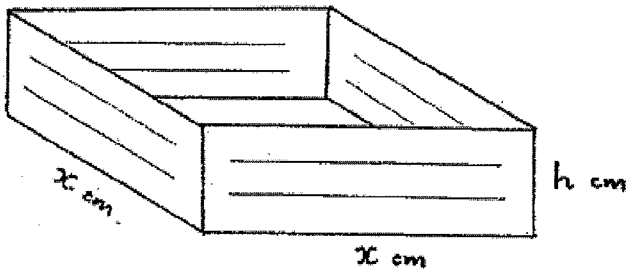
Given $V = \pi r^2 h$, and $r + h = 12cm$

Show that $\frac{dv}{dr} = 3\pi r(8 - r)$

and hence show that the maximum volume of the cylinder is $256\pi\ cm^3$

SYDNEY GRAMMAR 2002 Q10

(a) A metal tray, in the shape of a rectangular prism with a square base, is made out of 108 square centimetres of sheet metal. The tray is open at the top.



Let x centimetres be the side length of the base, and let h centimetres be the height.

Marks

(i) Show that $h = \dfrac{108 - x^2}{4x}$. **1**

(ii) Show that the volume V of the tray is given by **1**

$$V = 27x - \frac{x^3}{4}.$$

(iii) Find the maximum volume of the tray. 3

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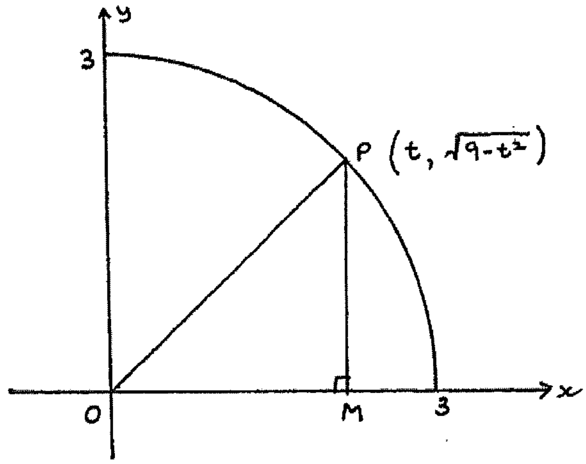
b) The diagram shows the curve $y=\sqrt{9-x^2}$ for $x \geq 0$. P is the point $(t, \sqrt{9-t^2})$ on the graph and M is the foot of the perpendicular from P to the x axis.



(i) Write down an expression in terms of t for the area A of the triangle OPM. 1

(ii) Find the coordinates of the point P which gives triangle OPM a maximum area. 5

SOLUTIONS

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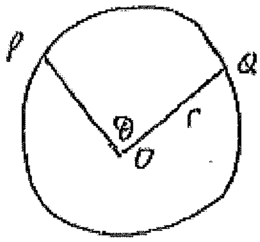
a)



$P = 10\text{cm}$

i) $r\theta + 2r = 10$ ✓

$\theta = \frac{10-2r}{r}$

Area $= \frac{1}{2}r^2\theta$

$A = \frac{1}{2}r^2\left(\frac{10-2r}{r}\right)$ ✓

$= \frac{1}{2}r(10-2r)$

$= r(5-r)$ ✓ (3)

$= 5r - r^2$

ii) $A' = 5-2r$ ✓

$A'' = -2$

Max A if $A' = 0$ & $A'' < 0$

$5-2r = 0$

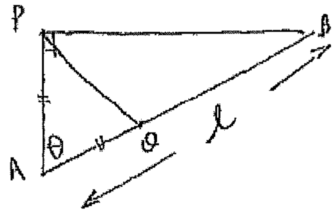
$r = 2\frac{1}{2}$ ½

$A'' = -2 < 0$ ½ (2)

$\therefore$ If radius is $2\frac{1}{2}$cm, area is maximum

SYDNEY GRAMMAR 2000 Q10

a)



(i)

$\cos\theta = \frac{AP}{l}$

$\therefore AP = l.\cos\theta$ ✓

$S = \frac{1}{2}.l\cos\theta.l\cos\theta.\sin\theta$ ✓

$S = \frac{l^2}{2}.\cos^2\theta.\sin\theta$, as required. ✓

$= \frac{l^2}{2}.(1-\sin^2\theta)(\sin\theta)$

$= \frac{l^2}{2}(\sin\theta - \sin^3\theta)$

(ii) $\frac{dS}{d\theta} = \frac{l^2}{2}\left(\cos\theta - 3\sin^2\theta\cos\theta\right) = \frac{l^2}{2}\cos\theta\left(1-3\sin^2\theta\right)$ ✓

$\frac{dS}{d\theta} = 0$ when $\cos\theta = 0$ or $\sin\theta = \pm\frac{1}{\sqrt{3}}$

since $0 < \theta < 90°$, $\sin\theta = \frac{1}{\sqrt{3}}$ is only possible solution.

$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{3}{4}$
$\frac{dS}{d\theta}$	+	0	−

$\therefore$ When $\sin\theta = \frac{1}{\sqrt{3}}$, S is a maximum. ✓

$S = \frac{l^2}{2}.\left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}\right)$

$= \frac{l^2}{2}.\left(\frac{3-1}{3\sqrt{3}}\right)$

$= \frac{l^2}{3\sqrt{3}}$

$= \frac{\sqrt{3}l^2}{9}$. is the maximum area. ✓

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(b) (i) $270\pi = 2\pi r^2 + 2\pi rh$

$135 = r^2 + rh$

$h = \dfrac{135 - r^2}{r}$

(ii) $V = \pi r^2 \left(\dfrac{135 - r^2}{r}\right)$

$V = 135\pi r - \pi r^3$

(iii) $V' = 135\pi - 3\pi r^2 = 0$

$r^2 = 45$

$r = 3\sqrt{5}$ $(r > 0)$

$V'' = -6\pi r < 0$ Max

When $r = 3\sqrt{5}$ cm

(iv) $h = \dfrac{135 - (3\sqrt{5})^2}{3\sqrt{5}}$

$= 30/\sqrt{5}$

$\therefore$ Max $V = \pi \cdot (3\sqrt{5})^2 \cdot 30/\sqrt{5}$

$= \dfrac{1350\pi}{\sqrt{5}}$ cm^3

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c) $h = 12 - r$

$V = \pi r^2 h$

$= \pi r^2(12 - r)$

$= 12\pi r^2 - \pi r^3$ ✓

$\therefore \frac{dV}{dr} = 24\pi r - 3\pi r^2$

$3\pi r(8 - r)$ ✓

to find stationary points

let $\frac{dV}{dr} = 0$

$\therefore 3\pi r(8 - r) = 0$ ✓

$3\pi r = 0, \quad 8 - r = 0$

$r = 0 \qquad r = 8$

$r \neq 0$ as it is a length

$\therefore r = 8$ cm ✓

$\frac{dV}{dr} = 24\pi r - 3\pi r^2$

$\frac{d^2V}{dr^2} = 24\pi - 6\pi r$ ✓

at $r = 8$, $\frac{d^2V}{dr^2} = -75.39... < 0$ $\therefore$ at $r = 8$ it is a maximum.

$V = \pi r^2 h \qquad h + r = 12 \quad h = 4$

$\therefore V = \pi r^2 h$

$= \pi \times 8 \times 8 \times 4$

$= 256\pi \text{ cm}^3$ ✓

SYDNEY GRAMMAR 2002 Q10

(10)

(a) (i) $SA = x^2 + 4xh$

$108 = x^2 + 4xh$

$108 - x^2 = 4xh$

$h = \dfrac{108 - x^2}{4x}$ ✓

(ii) $V = lbh$

$V = x^2 h$

$= x^2\left(\dfrac{108 - x^2}{4x}\right)$

$= \dfrac{108x}{4} - \dfrac{x^3}{4}$

$= 27x - \dfrac{x^3}{4}$ cm³ ✓

(iii) $\dfrac{dV}{dh} = 27 - \dfrac{3x^2}{4}$

when $\dfrac{dV}{dh} = 0$, $\dfrac{3x^2}{4} = 27$

$x^2 = 36$

$x = 6$ ✓

$h = \dfrac{108 - 36}{24}$

$h = 3$

$\dfrac{d^2V}{dh^2} = -\dfrac{3x}{2}$

when $x = 6$, $\dfrac{d^2V}{dh^2} = -9$

< 0 ✓

So the volume is a maximum when $x = 6$

maximum volume $= 27(6) - \dfrac{216}{4}$ ✓

$= 108$ cm³

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(b) (i) $A = \frac{1}{2}$ base $\times \perp$ height

$= \frac{1}{2} \cdot t \cdot \sqrt{9-t^2}$

$= \frac{t}{2}(9-t^2)^{\frac{1}{2}}$

(ii) $\frac{dA}{dt} = (9-t^2)^{\frac{1}{2}} \cdot \frac{1}{2} + \frac{t}{2} \cdot \frac{1}{2}(9-t^2)^{-\frac{1}{2}} \cdot (-2t)$

$= \frac{\sqrt{9-t^2}}{2} - \frac{t^2}{2\sqrt{9-t^2}}$

stationary point occurs when $\frac{dA}{dt} = 0$

ie $\frac{\sqrt{9-t^2}}{2} - \frac{t^2}{2\sqrt{9-t^2}} = 0$

ie $\frac{\sqrt{9-t^2}}{2} = \frac{t^2}{2\sqrt{9-t^2}}$

$\Rightarrow 2(9-t^2) = 2t^2$

$18 - 2t^2 = 2t^2$

$18 = 4t^2$

$t^2 = \frac{18}{4}$

$\therefore t = \frac{3\sqrt{2}}{2}$ (since $t \geq 0$)

TEST:

t	2.1	$\frac{3\sqrt{2}}{2}$	2.2
$\frac{dA}{dt}$	0.6	0	-0.3

NOTE: must give values for this test!!

$\therefore$ maximum area occurs at $t = \frac{3\sqrt{2}}{2}$

$\therefore$ Co-ordinates of P are $\left(\frac{3\sqrt{2}}{2}, \sqrt{9-\frac{9}{2}}\right)$

$= \left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$