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YEAR 11 – MATHEMATICS

SPECIMEN PAPER 1
TOPIC : QUADRATIC EQUATIONS

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Question 1:

(a) Show that $x^2 + 4x + 7 = (x + 2)^2 + a$ where a is to be determined. [2]

(b) Sketch the graph of $y = x^2 + 4x + 7$, giving the equation of the axis of symmetry and the co-ordinates of its vertex [4]

Question 2:

A quadratic function is defined by $f(x) = x^2 + 8x - 3$

(a) Find values of p and q such that $f(x) = (x - p)^2 + q$ [3]

(b) Hence find the value of x for which $f(x)$ has a minimum value. State this minimum value. [2]

Question 3:

(a) Write $x^2 + 4x + 14$ in the form $(x + p)^2 + q$, where p and q are integers to be found [2]

(b) Find the minimum value of $x^2 + 4x + 14$. State the value of x for which this minimum occurs. [2]

(c) Find the value of the constant k for which $x^2 + 4x + k = 0$ has equal roots [2]

Question 4:

Given a quadratic function $f(x) = 12 - 8x - x^2$

(a) Find values of p and q such that $f(x) = q - (x + p)^2$ [2]

(b) Hence find the value of x for which $f(x)$ has a maximum value. State this maximum value [2]

(c) Find the minimum value of $\dfrac{1}{12 - 8x - x^2}$ [1]

Question 5:

(a) Determine the values of a and b such that $x^2 - 8x - 1 \equiv (x + a)^2 + b$.
Hence find the minimum value of $x^2 - 8x - 1$ and state the value of x for which this occurs. [4]

(b) Write down the maximum value of $\dfrac{1}{x^2 - 8x - 1}$ [1]

(c) Sketch the graph of $y = x^2 - 8x - 1$.
On your graph mark the co-ordinates of the points where the curve cuts the co-ordinate axes. What is the equation of the line of symmetry of the curve? [3]

Question 6:

(a) By completing the square, find the maximum value of the quadratic function $5 + 8x - 2x^2$.
State the value of x at which this occurs. [4]

(b) Sketch the function $y = 5 + 8x - 2x^2$, giving the equation of the axis of symmetry of the curve. Show on your sketch the co-ordinates of the points where the curve cuts the co-ordinate axes. [3]

(c) Write down the minimum value of $\dfrac{1}{5+8x-2x^2}$ [1]

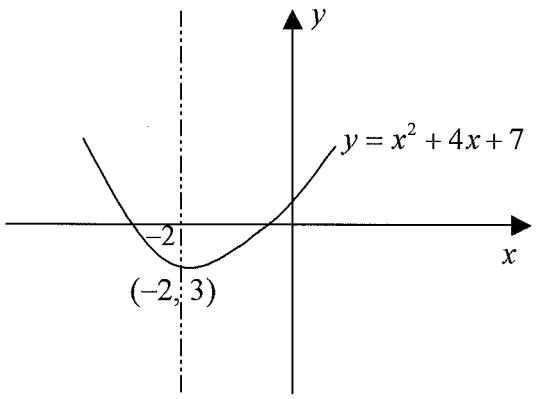
Solutions to Quadratic Equations – Specimen paper 1:

Question 1:

(a) By completing the square

$$x^2 + 4x + 7 \equiv (x+2)^2 - 2^2 + 7$$
$$\equiv (x+2)^2 - 4 + 7$$
$$\equiv (x+2)^2 + 3$$
$$\Rightarrow \quad a = 3$$

(b)
$$y = x^2 + 4x + 7$$
$$\Rightarrow \quad y = (x+2)^2 + 3$$
$\Rightarrow$ The function has a minimum of 3 where $x = -2$



The dashed line is the axis of symmetry which has equation $x = -2$

Question 2:

(a)
$$f(x) = x^2 + 8x - 3$$

By completing the square

$$f(x) = (x+4)^2 - 4^2 - 3$$
$$= (x+4)^2 - 19$$
$$= (x-(-4))^2 + (-19)$$
$$\Rightarrow \quad p = -4 \quad , \quad q = -19$$

(b) When $x = -4$, the function will have a minimum value of -19

Question 3:

(a) By completing the square

$$x^2 + 4x + 14 = (x + 2)^2 - 2^2 + 14$$
$$= (x + 2)^2 + 10$$

$\Rightarrow \quad p = 2 \qquad q = 10$

(b) $x^2 + 4x + 14 = (x - (-2))^2 + 10$

$\Rightarrow$ Minimum value of $x^2 + 4x + 14$ is 10 which occurs when $x = -2$

(c) $x^2 + 4x + k = 0$

$a = 1 \qquad b = 4 \qquad c = k$

For equal roots

$$b^2 - 4ac = 0$$
$$4^2 - 4 \times 1 \times k = 0$$
$$16 = 4k$$

$\Rightarrow \quad k = 4$

Question 4:

(a) $f(x) = 12 - 8x - x^2$

By completing the square

$$f(x) = -\left[x^2 + 8x - 12\right]$$
$$= -\left[(x+4)^2 - 4^2 - 12\right]$$
$$= -\left[(x+4)^2 - 28\right]$$
$$= 28 - (x + 4)^2$$

$\Rightarrow \quad p = 4 \quad , \quad q = 28$

(b) $f(x)$ has a maximum value of 28 which occurs when $x = -4$

(c) Minimum value of $\dfrac{1}{12 - 8x - x^2}$ occurs when

$12 - 8x - x^2$ has a maximum value.

$\Rightarrow$ Minimum value of $\dfrac{1}{12 - 8x - x^2} = \dfrac{1}{28}$

Question 5:

(a) By completing the square

$$x^2 - 8x - 1 \equiv (x-4)^2 - 4^2 - 1$$
$$\equiv (x + (-4))^2 + (-17)$$

$\Rightarrow$ a $= -4$, $b = -17$

Minimum value of $x^2 - 8x - 1$ is -17 which occurs when $x = 4$

(b) Maximum value of $\dfrac{1}{x^2 - 8x - 1}$ occurs when $x^2 - 8x - 1$ has a minimum value

$$\Rightarrow \text{Maximum value of } \frac{1}{x^2 - 8x - 1} = \frac{1}{-17}$$

$$\text{Maximum value of } \frac{1}{x^2 - 8x - 1} = -\frac{1}{17}$$

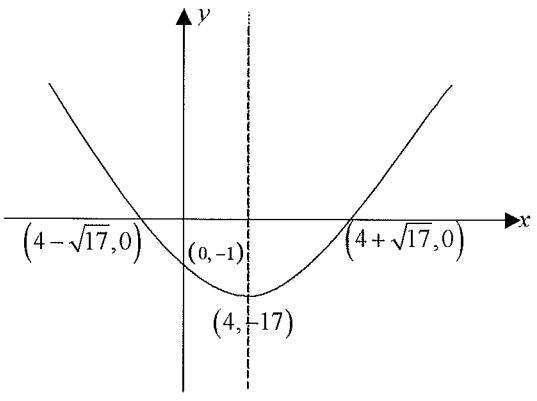
(c) To cut y axis $x = 0$

$\Rightarrow$ Curve cuts y axis at $(0, -1)$

To cut x axis $y = 0$

$$x^2 - 8x - 1 = 0$$
$$(x-4)^2 - 17 = 0$$
$$(x-4)^2 = 17$$
$$x - 4 = \pm\sqrt{17}$$
$$x = 4 \pm \sqrt{17}$$

$\Rightarrow$ Curve cuts x axis at $\left(4-\sqrt{17}, 0\right), \left(4+\sqrt{17}, 0\right)$



The line of symmetry (dashed line) is $x = 4$

Question 6:

(a) By completing the square

$$\begin{aligned} 5 + 8x - 2x^2 &= -2\left[x^2 - 4x - \frac{5}{2}\right] \\ &= -2\left[(x-2)^2 - 2^2 - \frac{5}{2}\right] \\ &= -2\left[(x-2)^2 - \frac{13}{2}\right] \\ &= 13 - 2(x-2)^2 \end{aligned}$$

$\Rightarrow$ The maximum value of the function is 13 which occurs when $x = 2$

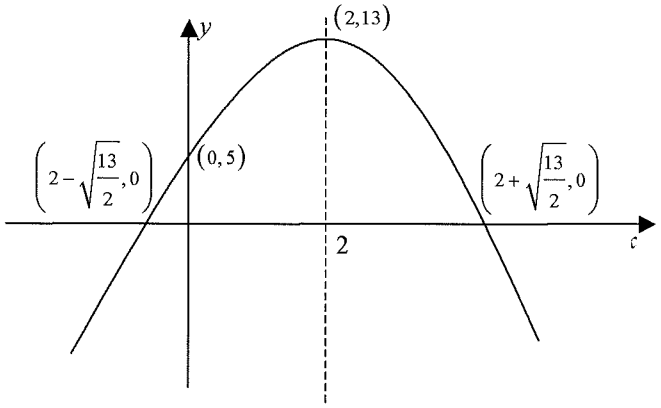
(b) To cut x axis

$$\begin{aligned} y &= 0 \\ 5 + 8x - 2x^2 &= 0 \\ 13 - 2(x-2)^2 &= 0 \\ (x-2)^2 &= \frac{13}{2} \\ x - 2 &= \pm\sqrt{\frac{13}{2}} \\ x &= 2 \pm \sqrt{\frac{13}{2}} \end{aligned}$$

$\Rightarrow$ Curve cuts x axis at $\left(2 - \sqrt{\frac{13}{2}}, 0\right)$ and $\left(2 + \sqrt{\frac{13}{2}}, 0\right)$

To cut y axis $x = 0$

$\Rightarrow$ Curve cuts y axis at (0, 5)



Equation of axis of symmetry is $x = 2$

(c) Minimum value of $\frac{1}{5+8x-2x^2}$ occurs when $5 + 8x - 2x^2$ is a maximum.

$\Rightarrow$ Minimum value $= \frac{1}{13}$