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YEAR 12 – MATHEMATICS

REVIEW TOPIC (PAPER 1):

RATES OF CHANGE

SBHS 04

(6) The rate at which people, N, are admitted to Homebake, a music festival in the Domain, is given by

$$\frac{dN}{dt} = 450t(8-t)$$

where t is measured in hours.

(i) Find the maximum rate of people being admitted to the festi 1

(ii) If initially $N = 0$, find an expression for the amount of people present at time t. 2

(iii). The festival *lasted* as long as there was a person there. 1
How long did the festival last for?

JAMES RUSE 03

(4) The rate of water flowing, R litres per hour, into a pond is given by

$$R = 65 + 4t^{\frac{1}{3}}$$

(i) Calculate the initial flow rate 1

(ii) Find the volume of water in the pond when 8 hours have elapsed, if initially there was 15 litres in the pond. 2

PYMBLE LADIES 03

(4) The rate at which a reservoir is being filled is given by

$$V'(t) = 150\, t^{-\frac{1}{4}} + 10 \quad \text{litres/sec.}$$

(i) Find $V(t)$, the volume of water in the reservoir at time t secs, given that the reservoir holds 3 000 litres after 16 seconds. 2

(ii) How much more will be added to the reservoir, to the nearest 100 litres, by the end of 15 minutes? 2

CSSA 09

(9) The equation below refers to the filtering cycle of a pump in Helen's garden.

The flow rate of the volume of water that the filter pumps water into and out of a pond in litres per minute, is given by

$$\frac{dV}{dt} = 20\sin\frac{\pi}{35}t.$$

(i) If the pump started at 8.55 pm, what is the first time after 8.55 pm at which the flow rate is zero? 2

(ii) If the pond is initially empty find an expression for the volume, V, of water in the pond after t minutes. 3

(iii) Find the maximum volume of water in the pond during the filtering cycle. Leave your answer in terms of π. 2

GRAMMAR 10

(8) Sophie has a toy that she uses to blow spherical bubbles. The rate of change of the volume V cm^3 of a bubble is given by

$$\frac{dV}{dt} = \frac{6t}{t^2 + 1} \text{ cm}^3/\text{s}.$$

(i) Find the equation for the volume V of a bubble t seconds after Sophie starts blowing. Assume that the initial volume of a bubble is zero. 2

(ii) A bubble will burst when its radius exceeds 1·5 cm. Sophie takes a deep breath and blows a bubble. After how many seconds of blowing will it burst? Give your answer correct to one decimal place. 3

GRAMMAR 09

(8) A swimming pool is being emptied. The volume of water L litres in the pool after t minutes is given by the equation

$$L = 1000(20 - t)^3.$$

(i) Find the rate at which the pool is emptying after 10 minutes. [2]

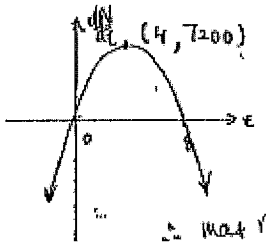
(ii) When is the pool emptying at a maximum rate? [1]

Solutions

SBHS 04

(6)

$\therefore \frac{dN}{dt} = 450t(8-t)$



$\therefore$ max rate 7200 ppl/hr ✓

(ii) $N = 1800t^2 - 150t^3 + C$ ✓

when $t=0$, $N=0$

$\therefore C=0$ ✓

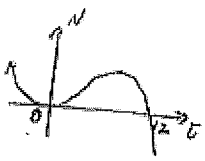
$\therefore N = 1800t^2 - 150t^3$

(iii) when $N=0$,

$150t^2(12-t) \geq 0$

$\therefore t = 12$

$\therefore$ 12 hours. ✓



JAMES RUSE 03

(4) (d)

(i) $R = 65 + 4t^{\frac{1}{3}}$

when $t = 0$, $R = 65 + 4(0)^{\frac{1}{3}} = 65$

(ii) now $R = \frac{dv}{dt} = 65 + 4t^{\frac{1}{3}}$

$\therefore V = 65t + 3t^{\frac{4}{3}} + C$

when $t = 0$, $V = 15$, $\therefore C = 15$

$\therefore V = 65t + 3t^{\frac{4}{3}} + 15$

when $t = 0$, $V = 583$ litres

PYMBLE LADIES 03

(4)

(b) (i) $V'(t) = 150t^{-\frac{1}{4}} + 10$

$V(t) = 150 \cdot \frac{4}{3} t^{\frac{3}{4}} + 10t + C$

$V(16) = 3000$ (2)

$\therefore 3000 = 200(16)^{\frac{3}{4}} + 160 + C$

$= 1600 + 160 + C \quad \therefore C = 1240$

$V(t) = 200t^{\frac{3}{4}} + 10t + 1240$

(ii) 15 mins → $t = 900$

$V(900) = 200 \cdot 30\sqrt{30} + 9000 + 1240$

So $V(900) - V(16) = 6000\sqrt{30} + 7240$ (2)

$\approx 40\,100$ L.

CSSA 09

(9) When $\frac{dV}{dt} = 0$

$20\sin\frac{\pi}{35}t = 0$

$\sin\frac{\pi}{35}t = 0$

$\frac{\pi}{35}t = \pi$

$\therefore t = 35$ minutes

Therefore the first time the flow rate is zero after 8.55 pm is 8.55 + 35 minutes = 9.30 pm

(a) (ii) (3 marks)

$\frac{dV}{dt} = 20\sin\frac{\pi}{35}t$

$V = 20\int\left(\sin\frac{\pi}{35}t\right)dt$

$= 20\left(\frac{-\cos\frac{\pi}{35}t}{\frac{\pi}{35}}\right) + C \quad = \frac{-700}{\pi}\cos\frac{\pi}{35}t + C$

At $t = 0, V = 0 \quad \therefore 0 = \frac{-700}{\pi}\cos\frac{\pi}{35}(0) + C$

$\therefore C = \frac{700}{\pi}$

$\therefore$ Volume $V = \frac{700}{\pi} - \frac{700}{\pi}\cos\frac{\pi}{35}t$

(a) (iii) (2 marks)

From part (i), the filtering cycle is 35 minutes.

$\therefore V = \frac{700}{\pi} - \frac{700}{\pi}\cos\frac{\pi}{35}\times 35 \quad = \frac{700}{\pi} + \frac{700}{\pi} \quad = \frac{1400}{\pi}$ litres

GRAMMAR 10

(8) b) (i) $\frac{dV}{dt} = \frac{6t}{t^2+1}$

$V = 3\int \frac{2t}{t^2+1}\,dt$

$= 3\ln(t^2+1) + C$ ✓

$t = 0 \quad V = 0 \quad C = 0$

$\therefore \quad V = 3\ln(t^2+1)$ ✓

(ii) $V = \frac{4}{3}\pi r^3$, $r = 1.5$

$= \frac{4}{3} \times \pi \times \left(\frac{3}{2}\right)^3$ ✓

$= \frac{9\pi}{2}$

$\frac{9\pi}{2} = 3\ln(t^2+1)$ ✓

$\ln(t^2+1) = \frac{3\pi}{2}$

$t^2+1 = e^{\frac{3\pi}{2}}$

$t^2 = e^{3\pi/2} - 1$

$t \doteqdot 10.5\text{s}$ ✓

GRAMMAR 09

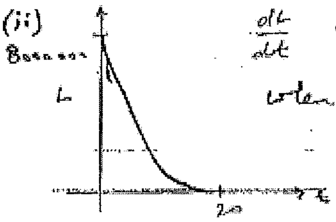
(8) (a) $L = 1000(20-t)^3$

(i) $\frac{dL}{dt} = -3000(20-t)^2$ ✓

at $t = 10$. $\frac{dL}{dt} = -300000$

After 10 minutes the pool is being emptied at 300 000 L/minute. ✓

(ii)



$\frac{dL}{dt}$ is a max. when $t = 0$. ✓

The pool is being emptied at a maximum rate initially.