NAME :

Centre of Excellence in Mathematics
S201 / 414 GARDENERS RD. ROSEBERY 2018
www.cemtuition.com.au

MOBILE 0412 880 475

PHONE 9666 3331

YEAR 12 – MATHEMATICS

SPECIMEN PAPER 2
TOPIC : RATES OF CHANGE

JAMES RUSE AHS 2002 Q7

(a) Water flows into and out of a tank at a rate (in litres/hour) given by $R = 2\pi \sin \pi t$. If the tank was initially empty at 10am, find:

(i) The first time (after 10am) when the tank is filling at its greatest rate. **2**

(ii) An expression for the volume (V litres) of water in the tank after t hours. **2**

(iii) The maximum volume of water in the tank. **1**

SYDNEY GRAMMAR 2000 Q9

(b) Concrete is pumped from a truck into a building foundation. The rate $R\,\text{m}^3/\text{hour}$ at which the concrete is flowing is given by the expression $R = 9t^2 - t^4$ for $0 \leq t \leq 3$, where t is the time measured in hours after the concrete begins to flow.

1 (i) Find the rate of flow at time $t = 2$.

1 (ii) Explain why t is restricted to $0 \leq t \leq 3$.

3 (iii) Find the maximum flow rate of concrete.

2 (iv) When the concrete begins to flow, the foundation has $1000\,\mathrm{m}^3$ already in place. Find an expression for the amount of concrete in the foundation at time t.

INDEPENDENT 2002 Q10

(c) A bottle had 500 millilitres of water in it. More water was poured into the bottle for 10 seconds until it was full. During this time the volume flow rate of water, in millilitres per second, was given by the formula

$$\frac{dV}{dt} = 2(10 - t)$$

(i) Find a formula for the volume of water V in the bottle after t seconds where $t \leq 10$. **2**

(ii) How many millilitres of water were in the bottle when it was full? **1**

(iii) What was the initial flow rate? **1**

ST GEORGE GIRLS 2002 Q8

b) The rate at which gas escapes from a balloon is given by $\frac{dG}{dt} = \frac{-3}{t+1}$ where the gas is measured in cm^3 and time in seconds.

(i) Find G as a function of time if the initial amount of gas in the balloon is $10cm^3$. **2**

(ii) How long before all the gas has escaped? **3**

CSSA 2001 Q8

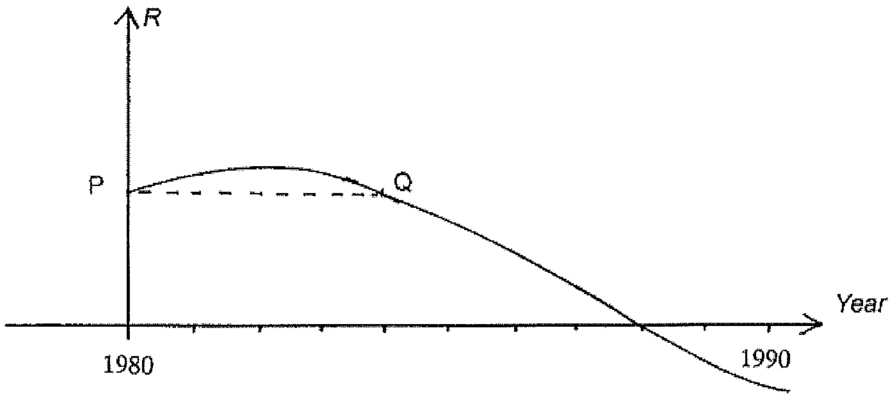
(c) Liquid petroleum is pumped out of a 25 000 litre storage container through a valve such that the volume flow rate of petroleum in litres per second is given by $\frac{dV}{dt} = -1 \cdot 92t$ $(t \geq 0)$.

(i) Show that if the tank was initially full, then $V = 25\,000 - 0.96t^2$. **2**

(ii) How long before the tank is only 40% full? **2**

INDEPENDENT 2000 Q2

(b) The number of sheep in Australia has gone up and down several times during the 20th Century. The rate of change, R, of the number of sheep during the 1980's is shown in the graph below. **3**



(i) In which year of the 1980's, did the number of sheep begin to decrease?

(ii) Using points P and Q, compare the rate of increase in 1980 with the rate of increase in 1984.

SOLUTIONS

JAMES RUSE 2002 Q7

QUESTION 7

(a) (i) $R = 2\pi \sin \pi t$

max R when $\pi t = \frac{\pi}{2}$

$t = \frac{1}{2}$

$\therefore$ time $= 10{\cdot}30$ am

(ii) $V = \int 2\pi \sin \pi t \, dt$

$= -2\cos \pi t + c$

$t = 0,\ V = 0 \Rightarrow 0 = -2 + c$

$c = 2$

$\therefore V = 2 - 2\cos \pi t$ ℓ

(iii) max vol. $= 4\ \ell$ (when $\cos \pi t = -1$)

SYDNEY GRAMMAR 2000 Q9

b)(i) $R = 9t^2 - t^4$

When $t = 2$; $R = 9.4 - 2^4$

$= 20 \text{ m}^3/\text{h}$ ✓

(ii) When $t = 3$, $R = 0$ and when $t > 3$, $R < 0$ which suggests that concrete is going back into the truck! Since $t \geq 0$ we have $0 \leq t \leq 3$ ✓

(iii) $\frac{dR}{dt} = 18t - 4t^3$ and $\frac{d^2R}{dt^2} = 18 - 12t$

$= 2t(9 - 2t^2)$ ✓

$\frac{dR}{dt} = 0$ when $t = 0$ or $\frac{3}{\sqrt{2}}$ or $-\frac{3}{\sqrt{2}}$

Since $t \geq 0$, ignore $t = -\frac{3}{\sqrt{2}}$. When $t = 0$ $\frac{d^2R}{dt^2} > 0$ and when $t = \frac{3}{\sqrt{2}}$, $\frac{d^2R}{dt^2} < 0$.

✓ So maximum flow-rate occurs when $t = \frac{3}{\sqrt{2}}$. i.e. $R = 9 \cdot \frac{9}{2} - \frac{81}{4}$

$= \frac{81}{4} \text{ m}^3/\text{h}$ ✓

(iv) $A = \int 9t^2 - t^4 \, dt$

$A = 3t^3 - \frac{t^5}{5} + c$ ✓

When $t = 0$, $A = 1000$ ∴ $c = 1000$.

∴ $A = 3t^3 - \frac{t^5}{5} + 1000$. ✓

(Be generous in part (ii).

INDEPENDENT 2002 Q10

(c)(i) $V = 20t - t^2 + c$

$t = 0 \quad V = 500 = c$

$\therefore V = 20t - t^2 + 500$

(ii) $t = 10$

$V = 20(10) - 10^2 + 500$

$= 600\,mL$

(iii) $t = 0$

$\frac{dV}{dt} = 2(10)$

$= 20\,mLs^{-1}$

ST GEORGE GIRLS 2002 Q8

(b) (i) $\frac{dG}{dt} = -\frac{3}{t+1}$

$\Rightarrow G = \int -\frac{3}{t+1}\,dt$

$G = -3\ln(t+1) + c$

at $t = 0$, $G = 10$

$\therefore 10 = -3\ln 1 + c$

$\therefore c = 10$

i.e. $G = 10 - 3\ln(t+1)$

(ii) Gas escapes $\Rightarrow G = 0$

i.e. $0 = 10 - 3\ln(t+1)$

$3\ln(t+1) = 10$

$\ln(t+1) = \frac{10}{3}$

$t+1 = e^{10/3}$

$t = e^{10/3} - 1$

$= 27.03\ldots$

$\therefore$ Time is 27 seconds (correct to nearest second.)

CSSA 2001 Q8

(c) (i) $\frac{dv}{dt} = -1.92t \ (t \geq 0)$

$v = \frac{-1.92t^2}{2} + C$

$= -0.96t^2 + C$

when $t = 0, V = 25000$

$\Rightarrow 25000 = C$

$\Rightarrow V = 25000 - 0.96t^2$

(ii) When 40% full the container holds 0.4 x 25000 = 10000 litres

Solve $25000 - 0.96t^2 = 10000$

$\Rightarrow 0.96t^2 = 15000$

$\Rightarrow t^2 = 15625$

$t = 125s \ (t > 0)$

INDEPENDENT 2000 Q2

(b) (i) 1988

(ii) Rates same but increasing 1980 and decreasing 1984