

Review of 2 Unit Sequences & Series – Specimen Paper 1

1. Arithmetic Progressions (AP)

The n th term of an AP : $T_n = a + (n-1)d$

Partial sums of APs : $S_n = \frac{1}{2}n(a+l)$
 $S_n = \frac{1}{2}n(2a + (n-1)d)$

2. Geometric Progressions (GP)

The n th term of a GP : $T_n = ar^{n-1}$

Partial sums of GPs : $S_n = \frac{a(r^n - 1)}{r - 1}$

[$S_n = \frac{a(1 - r^n)}{1 - r}$ is the same thing as $(x - c) = -(c - x)$]

2.1 Limiting Sums:

Limiting sums of GPs (when $-1 < r < 1$) : $S_\infty = \frac{a}{1 - r}$

(it's just $S_n = \frac{a(r^n - 1)}{r - 1}$ with $n \rightarrow \infty$)

3. Arithmetic and Geometric Means of Two Numbers

The arithmetic mean (AM) of a and b is the number x such that a, x, b forms an AP.

$$AM = \frac{1}{2}(a + b)$$

A geometric mean (GM) of a and b is a number x such that a, x, b forms a GP.

$$GM = \sqrt{ab} \quad \text{or} \quad -\sqrt{ab} \quad (\text{proof: condition for a GP is } b/x = x/a)$$

However, statements such as “Insert three geometric means between 10 and 40” simply mean “Find three numbers between 10 and 40 so that the five numbers form a GP”.

4. **Sigma Notation** : $\sum_{n=k}^l T_n = T_k + T_{k+1} + T_{k+2} + \dots + T_l$

5. **Finding T_n given S_n** : Sample question: Find T_n if $S_n = n^2$

For $n \geq 2$: $T_n = S_n - S_{n-1} = n^2 - (n-1)^2 = 2n - 1$. We know that $T_1 = S_1$ so $T_1 = 1$. Let's try the formula $T_n = 2n - 1$ when $n = 1$ anyway in case it works too: $T_1 = 2(1) - 1 = 1$. Hence, $T_n = 2n - 1$ for $n \geq 1$.

Exercises:

1) Find the next 3 terms in the series $8 + 5 + 2 + \dots$

$-1, -4, -7$

2) Find the next 3 terms in the series $3 + 12 + 48 + \dots$

$192 + 768 + 3072$

3) Find the 20th term of the series with nth term given by $T_n = 5n - 4$

96

4) Is 102 a term of the series with nth term $T_n = 3n + 1$? Give a reason for your answer.

No

5) If $3 + x + 19 + \dots$ form an arithmetic series, find the value of x .

$x = 11$

6) Find the 16th term of the series $6 + 10 + 14 + \dots$

7) Which term of $2 + 5 + 8 + \dots$ is equal to 320?

66

8) How many terms are there in the series $3 + 10 + 17 + \dots + 157$?

T_{107}

9) Find the first value of n for which the series $98 + 93 + 88 + \dots$ becomes negative.

23

$n = 21$

- 10) The 1st term of an arithmetic series is 7 and the 5th term is 23. Find the 20th term of the series.

- 11) The 5th term of an arithmetic series is 14 and the 10th term is 59. Find the first term and the common difference of the series. 83

- 12) Find the sum of the first 25 terms of the series $6 + 10 + 14 + \dots$ $a = -22; d = 9$

- 13) Evaluate $9 + 14 + 19 + \dots + 494$. 1350

24647

14) Find the sum of the first 50 terms of the series $100 + 97 + 94 + \dots$

15) How many terms of the series $6 + 8 + 10 + \dots$ give a sum of 2064?

1325

16) How many terms of the series $53 + 49 + 45 + \dots$ give a sum of 378?

43

14

- 17) The sum of the first 5 terms of an arithmetic series is 20 and the 8th term is 19.
(a) Find the values of a and d .

$$a = -2; d = 3$$

- (b) Find the sum of the first 50 terms of the series.

$$3575$$

- 18) Evaluate $\sum_{n=4}^{25} 6n - 5$

$$1804$$

- 19) The positive multiples of 9 are 9, 18, 27, 36, ...
(a) What is the largest multiple of 9 less than 500?

495

- (b) Find the sum of all the multiples of 9 that are less than 500.

13860

- 20) (a) The series $5 + y + 20 + \dots$ is geometric. Find the value of y .

$y = \pm 10$

- (b) Find the 10th term for each of the above series.

± 2560