Review of 2 Unit Sequences & Series - Specimen Paper 1

1. Arithmetic Progressions (AP)

The *n*th term of an AP :
$$T_n = a + (n-1)d$$

Partial sums of APs : $S_n = \frac{1}{2}n(a+l)$
: $S_n = \frac{1}{2}n(2a+(n-1)d)$

2. Geometric Progressions (GP)

The *n*th term of a GP:
$$T_n = ar^{n-1}$$

Partial sums of GPs: $S_n = \frac{a(r^n - 1)}{r - 1}$

$$[S_n = \frac{a(1 - r^n)}{1 - r} \text{ is the same thing as } (x - c) = -(c - x)]$$

2.1 Limiting Sums:

Limiting sums of GPs (when
$$-1 < r < 1$$
) : $S_{\infty} = \frac{a}{1-r}$
(it's just $S_n = \frac{a(r^n - 1)}{r - 1}$ with $n \to \infty$)

3. Arithmetic and Geometric Means of Two Numbers

The arithmetic mean (AM) of a and b is the number x such that a, x, b forms an AP. $AM = \frac{1}{2}(a+b)$

A geometric mean (GM) of a and b is a number x such that a, x, b forms a GP. $\overline{GM = \sqrt{ab} \quad or \quad -\sqrt{ab}} \text{ (proof: condition for a GP is } b/x = x/a \text{)}$

However, statements such as "Insert three geometric means between 10 and 40" simply mean "Find three numbers between 10 and 40 so that the five numbers form a GP".

4. Sigma Notation:
$$\sum_{n=k}^{l} T_n = T_k + T_{k+1} + T_{k+2} + ... + T_l$$

5. Finding Tn given Sn: Sample question: Find T_n if $S_n = n^2$

For $n \ge 2$: $T_n = S_n - S_{n-1} = n^2 - (n-1)^2 = 2n-1$. We know that $T_1 = S_1$ so $T_1 = 1$. Let's try the formula $T_n = 2n-1$ when n = 1 anyway in case it works too: $T_1 = 2(1)-1=1$. Hence, $T_n = 2n-1$ for $n \ge 1$.

1

Exercises:

Find the next 3 terms in the series 8 + 5 + 2 + ...

-1, -4, -7

2) Find the next 3 terms in the series 3 + 12 + 48 + ...

192 + 768 + 3072

3) Find the 20th term of the series with nth term given by $T_n = 5n - 4$

96

4) Is 102 a term of the series with nth term $T_n = 3n + 1$? Give a reason for your answer.

No

5) If 3 + x + 19 + ... form an arithmetic series, find the value of x.

6) Find the 16th term of the series 6 + 10 + 14 + ...

7) Which term of 2 + 5 + 8 + ... is equal to 320?

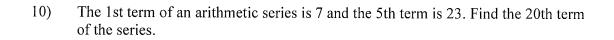
 T_{107}

66

8) How many terms are there in the series 3 + 10 + 17 + ... + 157?

9) Find the first value of n for which the series 98 + 93 + 88 + ... becomes negative.

n = 21



83

11) The 5th term of an arithmetic series is 14 and the 10th term is 59. Find the first term and the common difference of the series.

$$a = -22; d = 9$$

12) Find the sum of the first 25 terms of the series 6 + 10 + 14 + ...

1350

13) Evaluate 9 + 14 + 19 + 494.

24647

14) Find the sum of the first 50 terms of the series 100 + 97 + 94 + ...

15)

How many terms of the series $6 + 8 + 10 + \dots$ give a sum of 2064?

How many terms of the series $53 + 49 + 45 + \dots$ give a sum of 378?

43

17) The sum of the first 5 terms of an arithmetic series is 20 and the 8th term is 19. (a) Find the values of a and d.

$$a = -2; d = 3$$

(b) Find the sum of the first 50 terms of the series.

18) Evaluate $\sum_{n=4}^{25} 6n - 5$

19) The positive multiples of 9 are 9, 18, 27, 36,...
(a) What is the largest multiple of 9 less than 500?

495

(b) Find the sum of all the multiples of 9 that are less than 500.

13860

20) (a) The series 5 + y + 20 + ... is geometric. Find the value of y.

 $y = \pm 10$

(b) Find the 10th term for each of the above series.

±2560