C.E.M.TUITION

Name :________________________________

Review Topic : Trapezoidal & Simpson's Rules

(HSC Course - Paper 3)

Year 12 - Mathematics

13.

t	0	1	2	3	4
$v(t)$	0	20	50	30	10

(TREMAINING)

$$\int_0^4 v(t)\,dt = \frac{h}{2}[\text{first} + \text{last} + 2(\text{remaining})] \qquad (FLR)$$

$$= \frac{1}{2}\left[f(0) + f(4) + 2\left[f(1) + f(2) + f(3)\right]\right]$$

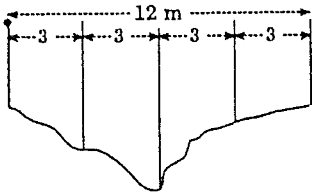
$$= \frac{1}{2}\left[0 + 10 + 2(20 + 50 + 30\right]$$

$$= \frac{1}{2}[210]$$

$$= 105. \qquad \therefore \int_0^4 v(t)\,dt = 105.$$

$\int_0^4 v(t)\,dt$ represents the distance travelled by the particle in the first 4 seconds, that is, distance travelled by the particle in the first 4 seconds is 105 metres.

14. Illustrated is a ditch of width 12 metres with vertical banks. The depth (in metres) is measured at 3 metre intervals across a cross-section and the measurements recorded in this table.



Distance from *A*	0	3	6	9	12
Depth	0·95	1·25	1·70	1·05	0·85

(a) Calculate the area of this cross-section using Simpson's Rule with 5 function values.

(b) Assuming this cross-section is typical of the ditch, calculate the volume of fill required for a 50 m section of the ditch.

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$$= \frac{1}{2}[f(0)+f(4)+2[f(1)+f(2)+f(3)]]$$

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common
end - point
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14.

d	0	3	6	9	12
D	0·95	1·25	1·70	1·05	0·85

← subinterval 1 →

← subinterval 2 →

(a) For first subinterval,

$$A_1 = \frac{1}{6}(b-a)\left[f(a)+f(b)+4f\left(\frac{a+b}{2}\right)\right]$$

$$= \frac{1}{6}(6-0)[0{\cdot}95+1{\cdot}70+4(1{\cdot}25)]$$

$$= 7{\cdot}65.$$

$a=0,\ b=6,\ \frac{a+b}{2}=3.$
$f(a)=0{\cdot}95,$
$f(b)=1{\cdot}70,$
$f\left(\frac{a+b}{2}\right)=1{\cdot}25.$

For 2nd subinterval,

$$A_2 = \frac{1}{6}(12-6)[f(6)+f(12)+4f(9)]$$

$$= \frac{1}{6}(6)[1{\cdot}70+0{\cdot}85+4(1{\cdot}05)]$$

$$= 6{\cdot}75.$$

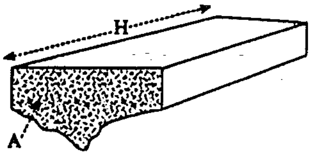
Area $= 7{\cdot}65 + 6{\cdot}75 = 14{\cdot}4$.
Cross-sectional area is $14{\cdot}4\ \text{m}^2$.

$a=6,\ b=12,$
$\frac{a+b}{2}=\frac{6+12}{2}=9.$
$f(a)=f(6),$
$f(b)=f(12),$
$f\left(\frac{a+b}{2}\right)=f(9).$

(b) $V = A \times H$
$= 14{\cdot}4 \times 50$
$= 720.$

$A = 14{\cdot}4$
$H = 50$

Volume of fill needed is $720\ \text{m}^3$.



15. Four-function values allow for 3 subintervals with Trapezoidal Rule 1 – 2, 2– 3, 3– 4.

Put $\mathbf{I} = \int_1^4 x \log_e x\,dx$.

For subinterval 1, then $\mathbf{I}_1 = \frac{1}{2}(b-a)[f(a)+f(b)]$

$$= \frac{1}{2}(2-1)[f(1)+f(2)]$$

$$= \frac{1}{2}[0+1{\cdot}39]$$

13.

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(TREMAINING)

$$\int_0^4 v(t)\,dt = \frac{h}{2}[\text{first} + \text{last} + 2(\text{remaining})] \qquad (FLR)$$

$$= \frac{1}{2}\left[f(0)+f(4)+2\left[f(1)+f(2)+f(3)\right]\right]$$

$$= \frac{1}{2}\left[0+10+2(20+50+30\right]$$

$$= \frac{1}{2}[210]$$

$$= 105. \qquad \therefore \int_0^4 v(t)\,dt = 105.$$

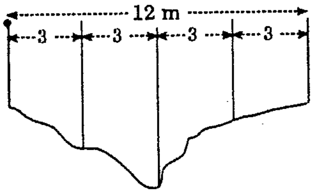
$\int_0^4 v(t)\,dt$ represents the distance travelled by the particle in the first 4 seconds, that is, distance travelled by the particle in the first 4 seconds is 105 metres.

13. Utilise the given table to evaluate $\int_0^4 v(t)\,dt$, using the Trapezoidal Rule.

t	0	1	2	3	4
$v(t)$	0	20	50	30	10

If t represents time in seconds and $v(t)$ velocity in ms^{-1} after t seconds, describe a possible physical interpretation of $\int_0^4 v(t)\,dt$.

14. Illustrated is a ditch of width 12 metres with vertical banks. The depth (in metres) is measured at 3 metre intervals across a cross-section and the measurements recorded in this table.



Distance from *A*	0	3	6	9	12
Depth	0·95	1·25	1·70	1·05	0·85

(a) Calculate the area of this cross-section using Simpson's Rule with 5 function values.

(b) Assuming this cross-section is typical of the ditch, calculate the volume of fill required for a 50 m section of the ditch.

15. This is a table of values for $f(x) = x\log_e x$. Use the Trapezoidal Rule with all 4 function values given to calculate the approximate value of

x	1	2	3	4
$f(x)$	0	1·39	3·30	5·55

$$\int_1^4 x\log_e x\,dx.$$

13.

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(TREMAINING)

$$\int_0^4 v(t)\,dt=\frac{h}{2}[\text{first}+\text{last}+2(\text{remaining})] \qquad (FLR)$$

$$=\frac{1}{2}\big[f(0)+f(4)+2[f(1)+f(2)+f(3)]\big]$$

$$=\frac{1}{2}[0+10+2(20+50+30]$$

$$=\frac{1}{2}[210]$$

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common
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$$=\frac{1}{6}(6-0)[0{\cdot}95+1{\cdot}70+4(1{\cdot}25)]$$

$$=7{\cdot}65.$$

$a=0,\ b=6,\ \frac{a+b}{2}=3.$
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$a=6,\ b=12,$
$\frac{a+b}{2}=\frac{6+12}{2}=9.$
$f(a)=f(6),$
$f(b)=f(12),$
$f\left(\frac{a+b}{2}\right)=f(9).$

Area $=7{\cdot}65+6{\cdot}75=14{\cdot}4.$

Cross-sectional area is 14·4 m^2.

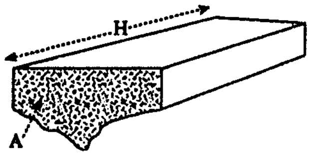
(b) $V=A\times H$

$=14{\cdot}4\times 50$

$=720.$

$A=14{\cdot}4$
$H=50$

Volume of fill needed is 720 m^3.



15. Four-function values allow for 3 subintervals with Trapezoidal Rule 1 – 2, 2– 3, 3– 4.

Put $I=\int_1^4 x\log_e x\,dx.$

For subinterval 1, then $I_1=\frac{1}{2}(b-a)[f(a)+f(b)]$

$$=\frac{1}{2}(2-1)[f(1)+f(2)]$$

$$=\frac{1}{2}[0+1{\cdot}39]$$

$$= 0\cdot695.$$

$$I_2 = \frac{1}{2}(3-2)[f(2)+f(3)] = \frac{1}{2}[1\cdot39 + 3\cdot30] = 2\cdot345.$$

$$I_3 = \frac{1}{2}(4-3)[f(3)+f(4)] = \frac{1}{2}[3\cdot30 + 5\cdot55] = 4\cdot425.$$

Then $I = 0\cdot695 + 2\cdot345 + 4\cdot425 = 7\cdot465$.

$$\int_0^4 x\log_e x\,dx \approx 7\cdot465.$$

OR alternate method. (TREMAINING)

x	1	2	3	4
$f(x)$	0	1·39	3·30	5·55

$h = 2 - 1 = 1$

↑ F ↑ ↑ REMAINING ↑ L

(FLR)

$$\int_1^4 f(x)\,dx = \frac{h}{2}[FIRST + LAST + 2(REMAINING)]$$

$$= \frac{1}{2}\left[f(1) + f(4) + 2\left(f(2) + f(3)\right)\right]$$

$$= \frac{1}{2}\left[0 + 5\cdot55 + 2\left(1\cdot39 + 3\cdot30\right)\right]$$

$$= 7\cdot465.$$

$$\therefore \int_1^4 x\log_e x\,dx = 7\cdot465.$$