C.E.M.TUITION

Student Name :________________________________

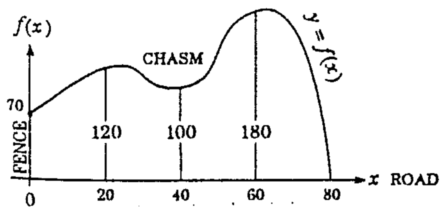
Review Topic : Trapezoidal and Simpson's Rule

(HSC Course - Paper 1)

Year 12 - 2 Unit

1995

1.



The diagram represents a scale drawing of an area of parkland bounded by a chasm on one side, a fence, and a road perpendicular to the fence on the other side.

x	0	20	40	60	80
$f(x)$					

Perpendicular distances have been found from the road to the chasm at 20 m intervals.

All distances are in metres.

(a) Read the distances from the diagram and complete the table.

(b) Estimate the area of the parkland using Simpson's Rule with five function values. Answer to the nearest m^2.

2. Use the Trapezoidal Rule with five equal subintervals to estimate

$\int_1^6 \log_e\left(\frac{1}{x}\right)\, dy$ correct to two decimal places.

3. The function $f(t)$ is defined as $f(t) = te^{-t}$.

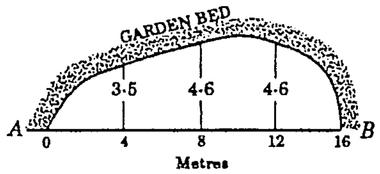
(a) Complete this table of values for $f(t)$, writing values of $f(t)$ correct to 3 decimal places.

t	0	1	2	3	4	5
$f(t)$						

(b) Use the Trapezoidal Rule and 6 function values to calculate an approximate area under the curve $f(t) = te^{-t}$ between $t = 0$ and $t = 5$.

4. The curve $y = \log x$ is rotated about the x axis between $x = 1$ and $x = 3$. By using Simpson's Rule and three function values, calculate the volume of the solid so formed.

5.



A section of grass between a path (AB) and a curved garden bed is to be removed, and concrete to a depth of 10 cm is to be laid.

(a) Use Simpson's Rule with 2 subintervals to calculate the area to be concreted. Answer correct to three significan figures.

(b) Calculate the volume of concrete required to the nearest $0{\cdot}1\ \text{m}^3$.

6. Some values of a function $\phi(x)$, continuous over $5 \le x \le 25$, are listed in this table. Using the Trapezoidal Rule and four subintervals, estimate the value of $\int_5^{25} \phi(x)\,dx$.

x	5	10	15	20	25
$\phi(x)$	0·03	0·24	0·76	1·8	0·87

1. (a)

x	0	20	40	60	80
$f(x)$	70	120	100	180	0

↑ common (under 40)

(b) Five function values implies 2 strips. The first subinterval uses the first 3 values, the second, the last 3 values, with the middle value common to both.

$$a = 0, \quad b = 40, \quad \frac{a+b}{2} = \frac{0+40}{2} = 20$$

$$A_1 = \frac{1}{6}(b-a)\left[f(a)+f(b)+4f\left(\frac{a+b}{2}\right)\right]$$

$$= \frac{1}{6}(40-0)[70+100+4(120)]$$

$$= 4333 \cdot 333\,3\ldots$$

Similarly with 2nd subinterval $a = 40$, $b = 80$, $\frac{a+b}{2} = 60$.

$$A_2 = \frac{1}{6}(80-40)[100+0+4(180)]$$

$$= 5466 \cdot 666\,6\ldots$$

Total area = 9800·0. Area of park is 9800 m^2.

OR Again using alternate method, TODD Simpson Rule

x	0	20	40	60	80
$f(x)$	70	120	100	180	0
	↑ F	↑ even	↑ odd	↑ even	↑ L

[Interval width = 20]

$(FLOE)$ $\quad h = \frac{80-0}{4}$ $\quad \therefore h = 20$

> OR, as h is really just the width of the uniform section, i.e. 0 – 20, width 20; 20 – 40, width 20, etc.

$$A = \frac{h}{3}[f(\text{first}) + f(\text{last}) + 2(\text{odd}) + 4(\text{even})]$$

$$= \frac{20}{3}[f(0)+f(80)+2f(40)+4\{f(20)+f(60)\}]$$

$$= \frac{20}{3}[70+0+2(100)+4(120+180)]$$

$$= \frac{20}{3}[70+200+1200]$$

$$= 9800.$$

Area is 9800 m^2.

2. $f(x) = \log_e\left(\frac{1}{x}\right) = \log x^{-1} = -\log x$ $\quad$ $\boxed{\log a^n = n\log a}$

x	1	2	3	4	5	6
$f(x)$	0	−0·693	−1·099	−1·386	−1·609	−1·792

Now, using 5 separate subintervals,

$$\int_1^6 f(x)\ dx = \frac{1}{2}(2-1)[f(1)+f(2)] + \frac{1}{2}(3-2)[f(2)+f(3)]$$

$$+\frac{1}{2}(4-3)[f(3)+f(4)] + \frac{1}{2}(5-4)[f(4)+f(5)]$$

$$+\frac{1}{2}(6-5)[f(5)+f(6)]$$

$$= \frac{1}{2}(1)[f(1)+2f(2)+2f(3)+2f(4)+2f(5)+f(6)]$$

$= \frac{1}{2}[0 + 2(-0\cdot693) + 2(-1\cdot099) + 2(-1\cdot386)$
$+ 2(-1\cdot609) + -1\cdot792]$

$= \frac{1}{2}[-11\cdot366]$

$= -5\cdot683$

$\approx -5\cdot68$ (2 dec. places), $\therefore \int_1^6 \log\left(\frac{1}{x}\right) dx = -5\cdot68.$

OR using alternate Trapezoidal Rule formula [TREMAINING]

x	1	2	3	4	5	6
$f(x)$	0	$-0\cdot693$	$-1\cdot099$	$-1\cdot386$	$-1\cdot609$	$-1\cdot792$

↑ F ↑ ↑ ↑ ↑ REMAINING ↑ L (*FLR*)

h = width of uniform section: $2 - 1 = 1$, $3 - 2 = 1$, etc.

$\int_1^6 f(x)\ dx = \frac{h}{2}[f(F) + f(L) + 2(REMAINING)]$

$= \frac{1}{2}[0 + -1\cdot792 + 2(-0\cdot693 + -1\cdot099 + -1\cdot386 + -1\cdot609)]$

$= -5\cdot683$

$\approx -5\cdot68$ (2dp) $\therefore \int_1^6 \log\left(\frac{1}{x}\right) dx = -5\cdot68$

3. (a)

t	0	1	2	3	4	5
$f(t)$	0	$0\cdot368$	$0\cdot271$	$0\cdot149$	$0\cdot073$	$0\cdot034$

(b) $f(t) = te^{-t}$, using $A_a^b = \frac{1}{2}(b-a)[f(a) + f(b)]$
and strips $0-1$, $1-2$, $2-3$, $3-4$, $4-5$,

$A_0^5 = A_0^1 + A_1^2 + A_2^3 + A_3^4 + A_4^5$

$A_0^5 = \frac{1}{2}(1)[f(0) + 2f(1) + 2f(2) + 2(3) + 2f(4) + f(5)]$

$= 0\cdot5[0 + 2(0\cdot368) + 2(0\cdot271) + 2(0\cdot149) + 2(0\cdot073) + 0\cdot034]$

$= 0\cdot878.$

Area under the curve is $0\cdot878$ units^2.

OR

t	0	1	2	3	4	5
$f(t)$	0	$0\cdot368$	$0\cdot271$	$0\cdot149$	$0\cdot073$	$0\cdot034$

↑ F ↑ ↑ ↑ ↑ REMAINING ↑ L

h = width of uniform section, i.e. $h = 1 - 0 = 1$.

$A_0^5 = \frac{h}{2}[f(F) + f(L) + 2f(REMAINING)]$ (*FLR*)

$= \frac{1}{2}[f(0) + f(5) + 2(f(1) + f(2) + f(3) + f(4))]$

$= \frac{1}{2}[0 + 0\cdot034 + 2(0\cdot368 + 0\cdot271 + 0\cdot149 + 0\cdot073)]$

$= 0\cdot878.$

Area under the curve is $0\cdot878$ units^2.

4. $y = \log x \qquad \therefore \quad y^2 = [\log_e x]^2$

Volume $= \pi \int_1^3 y^2 \, dx = \pi \int_1^3 (\log_e x)^2 \, dx$

Put $f(x) = (\log_e x)^2$

x	1	2	3
$f(x)$	0	0·480 453	1·206 949

Evaluate $\int_1^3 (\log_e x)^2 \, dx$

Now $\int_1^3 (\log_e x)^2 \, dx = \frac{1}{6}(b-a)\left[f(a)+f(b)+4f\left(\frac{a+b}{2}\right)\right]$

where $a = 1, \quad \therefore \quad f(a) = f(1)$

$b = 3, \quad \therefore \quad f(b) = f(3)$

$\frac{a+b}{2} = \frac{3+1}{2} = 2$

$\therefore \quad f\left(\frac{a+b}{2}\right) = f(2) = 0{\cdot}480\,453$

$(b-a) = 3 - 1 = 2$

$$\therefore \int_1^3 (\log_e x)^2 \, dx = \frac{1}{6}(3-1)[f(1)+f(3)+4f(2)]$$

$$= \frac{1}{3}[0 + 1{\cdot}206\,949 + 4(0{\cdot}480\,453)]$$

$$= 1{\cdot}042\,920\,3.$$

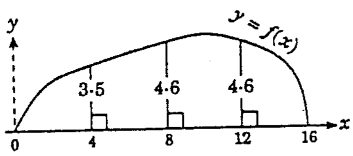
Then $V = \pi \int_1^3 (\log_e x)^2 \, dx = \pi \times 1{\cdot}042\,920\,3$

$= 3{\cdot}276\,430\,9$

$\approx 3{\cdot}28$ (2 dec. places).

Volume is $3{\cdot}28$ units^3.

5.



Call curve $y = f(x)$.
Set up table of values.

x	0	4	8	12	16
$f(x)$	0	3·5	4·6	4·6	0

(a) Subintervals will be 0 – 8, and 8 – 16.
Midpoints are thus 4 and 12, i.e., if $a = 0$ and $b = 8$,

then $\frac{a+b}{2} = \frac{0+8}{2} = 4$.

$$\text{Area} = \frac{1}{6}(8-0)[f(0)+f(8)+4f(4)] + \frac{1}{6}(16-8)[f(8)+f(16)+4f(12)]$$

$$= \frac{4}{3}[0 + 4{\cdot}6 + 4(3{\cdot}5)] + \frac{4}{3}[4{\cdot}6 + 0 + 4(4{\cdot}6)]$$

$$= 24{\cdot}8 + 30{\cdot}666\,67$$

$$\approx 55{\cdot}466\,67.$$

Area to be concreted is 55·5 m^2.

OR TODD Simpson method

x	0	4	8	12	16
$f(x)$	0	3·5	4·6	4·6	0

$h = 4 - 0 = 8 - 4$, etc.
$= 4$

↑ ↑ ↑ ↑ ↑
F *ODD* *L*
EVEN *EVEN*

$$\text{Area} = \frac{h}{3}\left[f(F) + f(L) + 2(ODD) + 4(EVEN)\right] \qquad (FLOE)$$

$$= \frac{4}{3}\left[0 + 0 + 2(4\cdot6) + 4(3\cdot5 + 4\cdot6)\right]$$

$$= \frac{4}{3}[41\cdot6]$$

$$= 55\cdot466\,67$$

$$\approx 55\cdot5 \text{ m}^2.$$

(b) Volume = Area × depth $= 55\cdot466\,667 \times 0\cdot1$
$= 5\cdot546\,666\,7$
$\approx 5\cdot5$ (1 dec. place).

$V = Ah$

Volume of concrete required is 5·5 m^3.

6.

x	5	10	15	20	25
$\phi(x)$	0·03	0·24	0·76	1·8	0·87

(TREMAINING)
$h = 10 - 5$
$= 5$

$$\int_5^{25} \phi(x)\,dx = \frac{h}{2}\left[FIRST + LAST + 2(REMAINING)\right] \qquad (FLR)$$

$$= \frac{5}{2}\left[f(5) + f(25) + 2\big(f(10) + f(15) + f(20)\big)\right]$$

$$= 2\cdot5\left[0\cdot03 + 0\cdot87 + 2(0\cdot24 + 0\cdot76 + 1\cdot8)\right]$$

$$= 16\cdot25.$$

Value of integral $\int_5^{25} \phi(x)\,dx$ is 16·25.