NAME :

Centre of Excellence in Mathematics
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YEAR 12 – EXT. 1 MATHS

REVIEW TOPIC (SP4)

PARAMETRIC REPRESENTATION OF THE PARABOLA

EXERCISES:

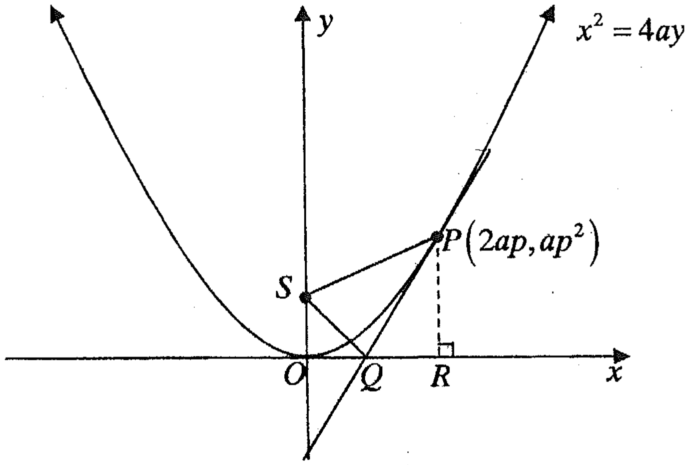
(1)(a) Find the Cartesian equation to each of these parametric equations:

(i) $x = 2t, y = 4t^2$ *(2m)*

(ii) $x = 4\cos\theta, y = 3\sin\theta$ *(2m)*

(2)

(a)



P is a point on $x^2 = 4ay$. The tangent at P meets the x-axis at Q, R is the foot of the ordinate from P, S is the focus and O the vertex.
Prove that :

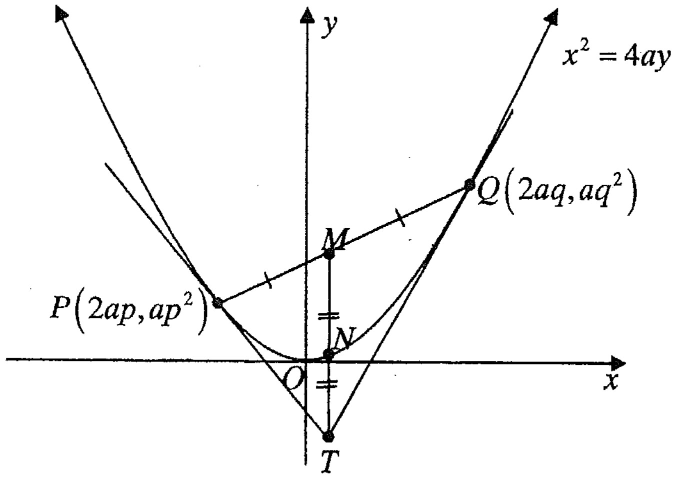
(i) Q is the midpoint of OR. *(3m)*

(ii) PQ is perpendicular to SQ. *(2m)*

(iii) $(SQ)^2 = OS \times SP$. *(2m)*

(3)

(a)



In the diagram, $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are distinct variable points on the parabola $x^2 = 4ay$.

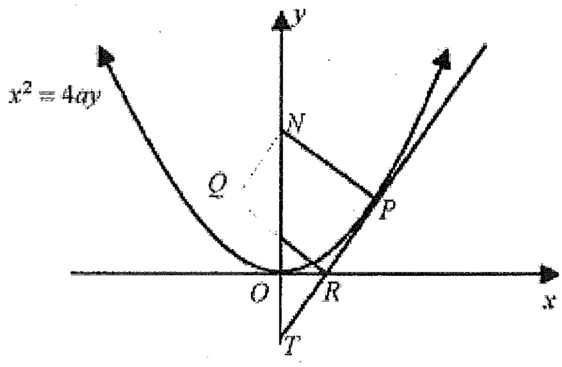
(i) Show that the tangent at P has equation $y = px - ap^2$. *(2m)*

(ii) The tangents at P and Q meet at T.
Show that T is the point $(a(p+q), apq)$. *(2m)*

(iii) M is the midpoint of the chord PQ. Show that MT is parallel to the axis of symmetry of the parabola. *(2m)*

(iv) N is the midpoint of MT. Show that as P and Q vary on the parabola $x^2 = 4ay$, N also varies on the parabola $x^2 = 4ay$. *(2m)*

(4)



(i) Prove that RS is parallel to NP. **4**

(ii) Prove that S is the mid-point of NT. **4**

(iii) Show that the locus of the point Q is a horizontal line and state its position. **4**

(5) Two points $P\left(6p, 3p^2\right)$ and $Q\left(6q, 3q^2\right)$ lie on a parabola.

(a) Find the equation of this parabola.

$\boxed{x^2 = 12y}$

(b) Derive the equation of the tangent at *P*.

$\boxed{y = px - p^2}$

(c) Find the coordinates of the point of intersection *M*, of the tangents at *P* and *Q*.

$\boxed{\left(a\left(p+q\right), apq\right)}$

(d) The tangents at P and Q intersect at an angle of 45^0 at M.

Show that $p-q=1+pq$

*(e) Find the equation of the locus of M.

$x^2-y^2+18y-9=0$

(6) The straight line $y = mx + b$ meets the parabola $x = 2at,\ y = at^2$ in distinct points P and Q with parameters p and q.

*(a) Prove that $p^2 + q^2 = 4m^2 + \frac{2b}{a}$

*(b) Prove that $pq = -\frac{b}{a}$

(c) Show that the equation of the normal at P is $x + py = 2ap + ap^3$.

(d) The point N is the point of intersection of the normals at P and Q. Show that the coordinates of N are $\left(-apq\left(p+q\right), a\left(2+p^2+pq+q^2\right)\right)$ and express these coordinates in terms of a, m and b.

$\left(2bm, 2a+b+4am^2\right)$