Review Exercises 5 – Differential Calculus

Name: ______________________

1. If $f(x) = 3x^2 - 5x$ find:

   (a) f(4) =

   (b) f(2a) =

   (c) f(x+2) =

2. Differentiate the following functions:

   (a) $y = 4x^3 - 3x + 7$

   (b) $y = 3x^{-4}$

   (c) $y = 6x^{½}$

   (d) $y = \frac{3x}{4}$

   (e) $y = (2x+3)^2$

   (f) $y = \frac{1}{6x^3}$

   (g) $y = \frac{4}{x}$

   (h) $y = \frac{1}{\sqrt{x}}$

3. (a) If $f(x) = 5x - x^3$ find:-

   (i) $f'(x)$

   (ii) $f'(3)$

   (iii) $f''(1)$

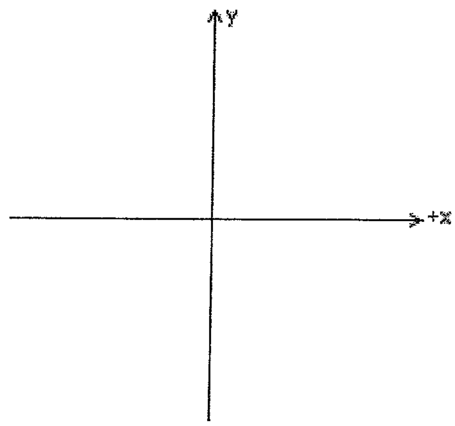
   (b) Given $\frac{dy}{dx} = 6x^2 - 5$ find $y = f(x)$ if it passes through the point P (2,0)

4. (a) Find the gradient of $y = 2x^2(3x - 5)$ at the point where x=2.

(b) Find the co-ordinates of the point(s) on the curve $y = x^3 - 11x + 2$ where the gradient is 1 (one).

(c) Find the stationary points of the curve $y = x^3 - 12x$

(d) Determine their type (nature) and sketch the curve.



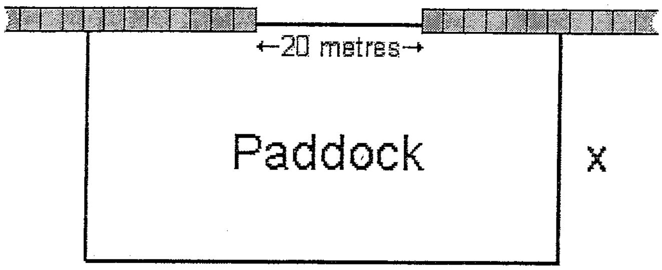
5. For the curve $y = x^3 - 12x$

(a) Find the equation of the tangent at the point P(3,-9)

(b) Find the gradient of the Normal at P.

(e) Where does the curve cut the x-axis ?

6. (a) 200 metres of fencing is to be used to form the paddock shown below:-



An existing brick wall is used with a 20 meter gap that must be fenced.
Show that if the depth of the paddock is x meters then its area is:-
$A = 180x - 2x^2$

(b) Find the maximum possible area of the paddock.

SOLUTIONS — Review Ex's — Differentiation.

1.(a) $f(4) = 28$ (b) $f(2a) = 12a^2 - 10a$

(c) $f(x+2) = 3(x+2)^2 - 5(x+2) = 3x^2 + 7x + 2$

2.(a) $y' = 12x^2 - 3$ (b) $y' = -\frac{12}{x^5}$

(c) $y' = \frac{3}{\sqrt{x}}$ (d) $y' = \frac{3}{4}$

(e) $y' = 8x + 12$ (f) $y' = \frac{-1}{2x^4}$

(g) $y' = \frac{-4}{x^2}$ (h) $y' = -\frac{1}{2\sqrt{x^3}}$

3.(a)

i) $f'(x) = 5 - 3x^2$

ii) $f'(3) = 5 - 27 = -22$

iii) $f''(x) = -6x \longrightarrow f''(1) = -6$

(b) $y = 2x^3 - 5x + c \leftarrow$ subs. $(2, 0)$

$0 = 16 - 10 + c \rightarrow c = -6$

$\therefore y = 2x^3 - 5x - 6$

4.(a) $y = 6x^3 - 10x^2$

$y' = 18x^2 - 20x$

$y'(2) = 18(2)^2 - 20(2) = 32$

(b) $y' = 3x^2 - 11 = 1 \Rightarrow 3x^2 = 12$

$x = \pm 2$.

$\therefore$ Points are $(2, -12)$ and $(-2, 16)$

5.(a) $y' = 3x^2 - 12$ $[y = x^3 - 12x]$

$y'(3) = 15 \leftarrow m$

Equ: $y + 9 = 15(x - 3)$

$\Rightarrow y = 15x - 54$

(b) $m_\perp = -\frac{1}{5}$

$\therefore$ Equ: $y + 9 = -\frac{1}{15}(x - 3)$

$\Rightarrow x + 15y + 132 = 0$

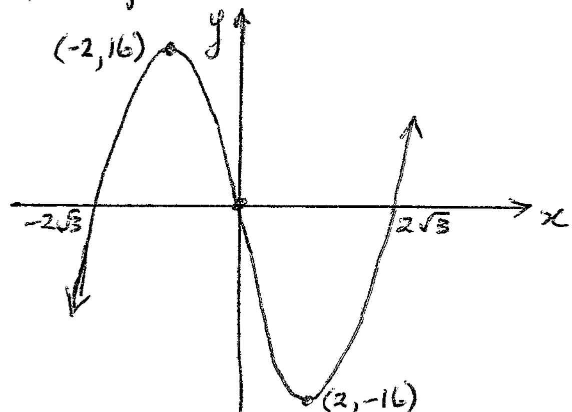
(c) $y' = 0$ when $3(x+2)(x-2) = 0$

$\therefore$ Stat. pts. at $(2, -16)$ and $(-2, 16)$

(d) $y'' = 6x$

At $(2, -16)$ $y''(2) = 12 > 0 \Rightarrow$ Local Min.

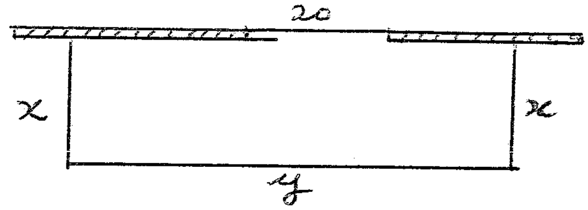
At $(-2, 16)$ $y''(-2) = -12 < 0 \Rightarrow$ Local Max.



(e)

Note $y = 0 \rightarrow x(x - 2\sqrt{3})(x + 2\sqrt{3})$

6.



(a) $2x + y + 20 = 200$

$\therefore 2x + y = 180 \longrightarrow y = 180 - 2x$

Then $A = xy$ ← subs

$= x(180 - 2x)$

$\therefore A = 180x - 2x^2$

(b) $A'(x) = 180 - 4x$

$= 0$ when $x = 45$

and when $x = 45$

$A = 180(45) - 2(45)^2$

$= 4050\ m^2$