REVISION OF 2 UNIT INTEGRATION THEORY

There is a table of integrals on the back cover of this document

269. Evaluate each of the following indefinite integrals:

a) $\int x^2 - 3x + 4\, dx$ b) $\int \sqrt{t} - \frac{1}{t^3} dt$

c) $\int (3x+5)^{17} dx$ d) $\int \frac{4}{x} dx$

e) $\int e^{8x} dx$ f) $\int \cos(4x) dx$

g) $\int \sec^2(\frac{x}{3}) dx$ h) $\int \sin(x) - x\, dx$

i) $\int \frac{2}{4x+3} dx$ j)(*) $\int \tan(x) dx$

270. Evaluate each of the following definite integrals:

a) $\int_0^1 x - 3x^2 + 5x^4 dx$ b) $\int_0^9 \frac{1}{\sqrt{t}} dt$

c) $\int_{\ln(3)}^{\ln(4)} e^{2x} dx$ d) $\int_{e^7}^{e^8} \frac{2}{x} dx$

e) $\int_0^{\frac{\pi}{4}} \cos(2x) dx$ f) $\int_0^{\frac{\pi}{2}} \sec^2(\frac{x}{2}) dx$

g)(*) $\int_{-2}^{1} |x| dx$ h)(*) $\int_{-1}^{1} 2^x dx$

271. (a) Verify that $\frac{d}{dx}(e^{x^2}) = 2xe^{x^2}$.

(b) Hence or otherwise find $\int xe^{x^2} dx$.

272. Professor Dolittle has two pet functions; $y = \text{heckle}(x)$ and $y = \text{jeckle}(x)$. It is known that

$$\frac{d}{dx}(\text{heckle}(x)) = \text{jeckle}(x) - 3x^2.$$

What is $\int \text{jeckle}(x) dx$?

273. Find the area of the region bounded by the graph of $y = 4x^3 + 8x$ and the x -axis from $x = 0$ to $x = 2$.

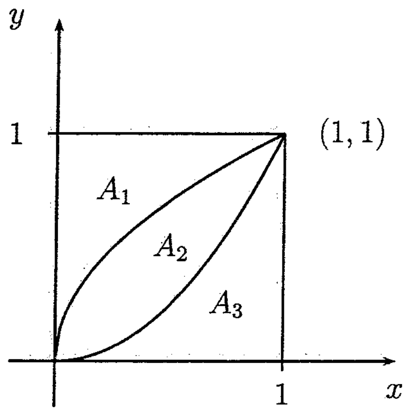
274. Determine the area of the region bounded by the graph of $y = \cos(\frac{x}{2})$ and the x -axis from $x = 0$ to $x = \pi$.

275. Find (correct to 2 decimal places) the area of the region bounded by the graphs of $y = e^{2x}$ and $y = e^{5x}$ from $x = 1$ to $x = 3$.

276. (a) Evaluate $\int_{-2}^{2} x^3 dx$.

(b) What is the area of the region bounded by the graph of $y = x^3$ and the x -axis, from $x = -2$ to $x = 2$?

277. Professor Pepperoni has baked a square pizza with an area of 1 square unit and wishes to share it with two friends. He places the pizza in the first quadrant as shown and makes two cuts, one along the curve $y = x^2$ and the other along $y = \sqrt{x}$, producing three pieces A_1, A_2 and A_3. Find the area of each of these regions and hence verify that the three pieces are of equal area.



278. Find the area of the region bounded by the graphs of $y = x^2$ and $y = 8 - x^2$ from $x = 0$ to $x = 3$.

279. (*) (a) Find the two values of k for which $\int_{0}^{k} (4 - 2x)dx = 3$.

(b) By considering the graph of $y = 4 - 2x$ explain why two values of k exist.

280. (*) (a) Sketch $y = \sin(x)$ and $y = \cos(x)$ on the same set of axes for $0 \leq x \leq \pi$.

(b) Find the area of the region bounded by the two graphs from $x = 0$ to $x = \pi$.

SOLUTIONS

Revision of 2 unit integration theory

269. (a) $\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x + C$ (b) $\frac{2}{3}t^{\frac{3}{2}} + \frac{1}{2t^2} + C$

(c) $\frac{(3x+5)^{18}}{54} + C$ (d) $4\ln|x| + C$

(e) $\frac{1}{8}e^{8x} + C$ (f) $\frac{1}{4}\sin(4x) + C$

(g) $3\tan\left(\frac{x}{3}\right) + C$ (h) $-\cos(x) - \frac{x^2}{2} + C$

(i) $\frac{1}{2}\ln|4x+3| + C$ (j) $-\ln|\cos(x)| + C$

270. (a) $\frac{1}{2}$ (b) 6 (c) $\frac{7}{2}$ (d) 2 (e) $\frac{1}{2}$ (f) 2 g) $\frac{5}{2}$ (h) $\frac{3}{2\ln(2)}$

271. (a) Proof (b) $\frac{e^{x^2}}{2} + C$

272. heckle $(x) + x^3 + C$

273. 32 units ^{2}

274. 2 units ^{2}

275. 653575.77

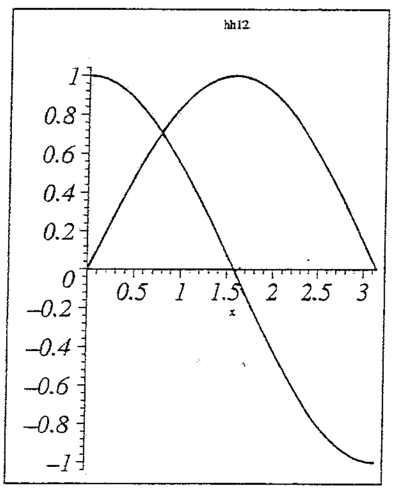
276. (a) 0 (b) 8 units ^{2}

277. Proof.

278. $15\frac{1}{3}$ units ^{2}

279. (a) $k = 1, 3$ (b) The integral is negative for $x > 2$.

280. (a)



(b) $2\sqrt{2}$ units ^{2}