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YEAR 12 – MATHS EXT. 1

REVIEW TOPIC (SP1)

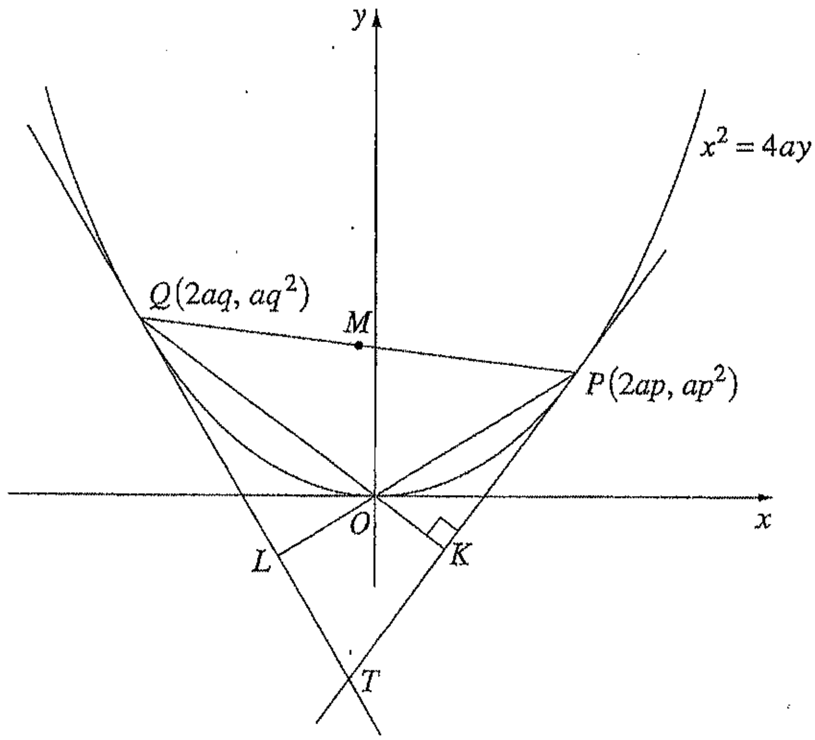
PARAMETRIC EQUATIONS

PAST HSC QUESTIONS :

HSC 08

(4)

(c)



The points $P(2ap, ap^2)$, $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The tangents to the parabola at P and Q intersect at T. The chord QO produced meets PT at K, and $\angle PKQ$ is a right angle.

(i) Find the gradient of QO, and hence show that $pq = -2$. **2**

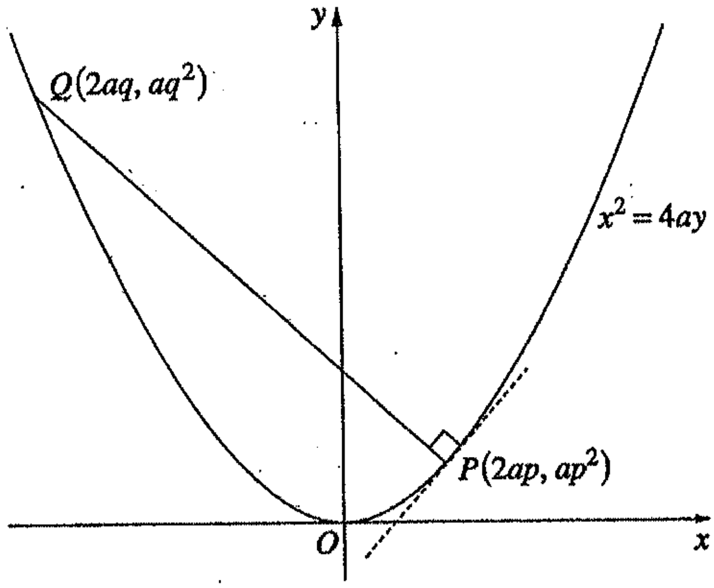
(ii) The chord PO produced meets QT at L. Show that $\angle PLQ$ is a right angle. **1**

(iii) Let M be the midpoint of the chord PQ. By considering the quadrilateral $PQLK$, or otherwise, show that $MK = ML$. **2**

HSC 07

(5)

(d)



The diagram shows a point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$. The normal to the parabola at P intersects the parabola again at $Q(2aq, aq^2)$.

The equation of PQ is $x + py - 2ap - ap^3 = 0$. (Do NOT prove this.)

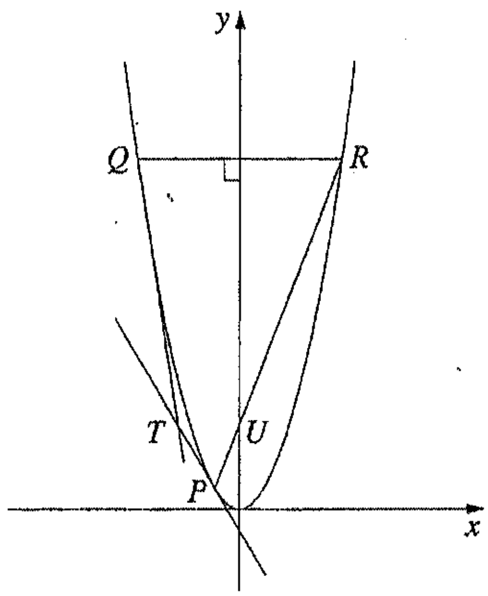
(i) Prove that $p^2 + pq + 2 = 0$. **1**

(ii) If the chords OP and OQ are perpendicular, show that $p^2 = 2$. **2**

HSC 06

(2)

(c)



The points $P(2ap, ap^2)$, $Q(2aq, aq^2)$ and $R(2ar, ar^2)$ lie on the parabola $x^2 = 4ay$. The chord QR is perpendicular to the axis of the parabola. The chord PR meets the axis of the parabola at U.

The equation of the chord PR is $y = \frac{1}{2}(p+r)x - apr$. (Do NOT prove this.)

The equation of the tangent at P is $y = px - ap^2$. (Do NOT prove this.)

(i) Find the coordinates of U. **1**

$U(0, -apr)$

(ii) The tangents at P and Q meet at the point T. Show that the coordinates of T are $(a(p+q),\ apq)$. **2**

(iii) Show that TU is perpendicular to the axis of the parabola. **1**

HSC 05

(4)

(c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The equation of the normal to the parabola at P is $x + py = 2ap + ap^3$ and the equation of the normal at Q is similarly given by $x + qy = 2aq + aq^3$.

(i) Show that the normals at P and Q intersect at the point R whose coordinates are **2**

$$\left(-apq[p+q],\ a\left[p^2 + pq + q^2 + 2\right]\right).$$

(ii) The equation of the chord PQ is $y = \frac{1}{2}(p+q)x - apq$. (Do NOT show this.) **1**

If the chord PQ passes through $(0, a)$, show that $pq = -1$.

(iii) Find the equation of the locus of R if the chord PQ passes through $(0, a)$. **2**

$$\boxed{x^2 = a(y - 3a)}$$

HSC 04

(b) The two points $P(2ap,\ ap^2)$ and $Q(2aq,\ aq^2)$ are on the parabola $x^2 = 4ay$.

(i) The equation of the tangent to $x^2 = 4ay$ at an arbitrary point $(2at,\ at^2)$ on the parabola is $y = tx - at^2$. (Do not prove this.) **2**

Show that the tangents at the points P and Q meet at R, where R is the point $(a(p+q),\ apq)$.

(ii) As P varies, the point Q is always chosen so that $\angle POQ$ is a right angle, where O is the origin. **2**

Find the locus of R.

$\boxed{y = -4a}$

HSC 03

(1)

(d) A curve has parametric equations $x = \frac{t}{2}$, $y = 3t^2$. Find the Cartesian equation for this curve. **2**

$y = 12x^2$

HSC 02

(1)

(e) The variable point $(3t, 2t^2)$ lies on a parabola. Find the Cartesian equation for this parabola. **2**

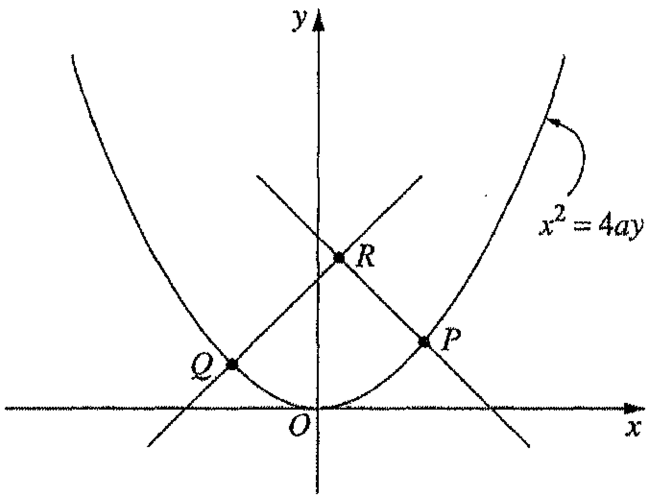
$2x^2 = 9y$

HSC 01

(6) (b) Consider the variable point $P\left(2at, at^2\right)$ on the parabola $x^2 = 4ay$.

(i) Prove that the equation of the normal at P is $x + ty = at^3 + 2at$. **2**

(ii) Find the coordinates of the point Q on the parabola such that **1** the normal at Q is perpendicular to the normal at P.



$Q\left(-\frac{2a}{t}, \frac{a}{t^2}\right)$

(iii) Show that the two normals of part (ii) intersect at the point R, whose coordinates are **4**

$$x = a\left(t - \frac{1}{t}\right),\ y = a\left(t^2 + 1 + \frac{1}{t^2}\right).$$

(iv) Find the equation in Cartesian form of the locus of the point R given in part (iii). **2**

$$\boxed{x^2 = a\left(y - 3a\right)}$$