Randwick Girls High School

NAME:- _____

CLASS:- _____

2006 HSC Assessment

Year 12 Mathematics

Task 2

Time Allowed: 1 Hour

- **Start each question on a new page.**
- **Label question parts clearly.**
- **Attempt all questions.**
- **Approved calculators may be used.**

Question	Marks
Q1. Logarithms and Exponential Functions /13	13
Q2. Areas and Integration /16	16
Q3. Simpson's Rule and Volume 12 /12	12
Total /43	41 /41

QUESTION 1:(13 marks)

a) Evaluate:

i) $\log_3 81$ (1)

ii) $\log_3(\frac{1}{9})$ (1)

b) Given that $\log a = 0.86$ and $\log b = 0.42$, find the values of:

i) $\log(\frac{a}{b})$ (2)

ii) $\log\sqrt{ab}$ **(2)**

c) Differentiate, with respect to x:

i) $y = e^{2x+1}$ (1)

ii) $y = \frac{e^x}{x}$ **(2)**

iii) $y = \frac{\ln x}{x}$ **(2)**

d) Find the primitive of $\int e^{x/2} dx$ (2)

QUESTION 2:(16 MARKS)

a) Find the area between the curve $y = x^3$, the x-axis and The lines at $x = -2$ and $x = 4$. (2)

b) Find the primitives of:

i) $\int (x^4+3x^2)dx$ (1)

ii) $\int (5x-3)^5 dx$ (2)

iii) $\int \frac{2x}{x^2+1}dx$ (1)

c) Find the value of:

i) $\int_1^{16} \frac{1}{\sqrt{x}}dx$ (2)

ii) $\int_{-1}^{0} \frac{dx}{2x+3}$ (3)

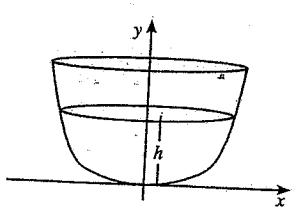
d)

i) Sketch the graphs of $y=x+1$ and $y=x^2-x-2$, Showing their point(s) of intersection. (2)

ii) Find the area enclosed by the graphs in (i). (3)

QUESTION 3:(14 marks)

a) A glass is formed by rotating part of the curve $16y=x^4$ about the y-axis. The scales on both axes being 1cm= 1 unit.



i) If the depth of water in the glass is h cm, show that the volume of water is $\frac{8\pi h^{3/2}}{3}$ ML. (3)

ii) If the volume of water is 250mL, find h, correct to one decimal place. (1)

b) i) Copy and complete the table of values for $y=\ln(\frac{1+x}{1-x})$ **(2)**

on your writing paper.

x	0	0.2	0.4	0.6	0.8
y	0		0.847		2.197

ii) By using Simpson's Rule with 5 function values, estimate the area described by the integral $\int_0^{0.8} \ln(\frac{1+x}{1-x})dx$. Give your answer correct to 2 significant figures. (2)

iii) Prove that the derivative of $y=\ln(\frac{1+x}{1-x})$ is $\frac{2}{1-x^2}$ (3)

iv) Show that $y=\ln(\frac{1+x}{1-x})$ has no stationary points. (1)

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Question 1

a) i) $\log_3 81 = \frac{\ln 81}{\ln 3}$

$= 4$ ✓

ii. $\log_3 1 - \log_3 9$

$= -2$. ✓

b) i) $\log a - \log b$

$= 0.86 - 0.42$

$= 0.44$. ✓

ii) $\log (ab)^{1/2}$

$= \frac{1}{2}[\log a + \log b]$

$= \frac{1}{2}[0.86 + 0.42]$

$= 0.64$. ✓

c) i) $y = e^{2x+1}$

$\frac{dy}{dx} = 2e^{2x+1}$ ✓

ii) $y = \frac{e^x}{x}$ (u / v) $\quad \frac{VU' - UV'}{V^2}$

$\frac{dy}{dx} = \frac{x e^x - e^x}{x^2}$

$= \frac{e^x (x-1)}{x^2}$ ✓

iii) $y = \frac{\ln x}{x}$ (u / v)

$\frac{dy}{dx} = \frac{\frac{x}{x} - \ln x}{x^2}$

$= \frac{1 - \ln x}{x^2}$ ✓

d) $\int e^{x/2} \, dx$

$= 2 \int \frac{1}{2} e^{x/2} \, dx$

$= 2 e^{x/2} + C$ ✓

16/16

Lovely work Hui Ling.

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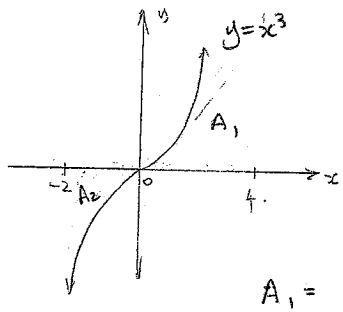
Question 2.

a)



$y = (4)^3$

$= 64$

$(4, 64)$

$y = (-2)^3$

$= -8$.

$(-2, -8)$

$A_1 = \int_0^4 x^3 \, dx$

$= \left[\frac{x^4}{4}\right]_0^4$

$= \frac{1}{4}\left[(4)^4\right]$

$= 64$ units^2

$A_2 = \left| \int_{-2}^{0} x^3 \, dx \right|$

$= \left| \left[\frac{x^4}{4}\right]_{-2}^{0} \right|$

$= \left| \frac{1}{4}\left[(0)^4 - (-2)^4\right] \right|$

$= 4$ units^2

$A_1 + A_2$ = Area under the curve

$64 + 4 = 68$ units^2. ✓

b) i) $\int (x^4 + 3x^2) \, dx$

$= \frac{x^5}{5} + \frac{3x^3}{3} + C$

$= \frac{x^5}{5} + x^3 + C$. ✓

ii) $\int (5x-3)^5 \, dx$

$= \frac{(5x-3)^6}{6 \times 5} + C$

$= \frac{(5x-3)^6}{30} + C$. ✓

iii) $\int \frac{2x}{x^2+1} \, dx$

$= \ln (x^2+1) + C$. ✓

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Q2. Part II

c) i $\int_1^{16} \frac{1}{\sqrt{x}}\,dx$

$= \int_1^{16} x^{-\frac{1}{2}}\,dx$

$= \left[\frac{2x^{\frac{1}{2}}}{1}\right]_1^{16}$

$= 2\left(\sqrt{16} - \sqrt{1}\right)$

$= 2(3)$

$= 6.$ ✓

2

ii $\int_{-1}^{0} \frac{dx}{2x+3}$

$= \frac{1}{2}\int_{-1}^{0} \frac{2}{2x+3}\,dx$

$= \frac{1}{2}\left[\ln(2x+3)\right]_{-1}^{0}$

$= \frac{1}{2}(\ln 3 - \ln 1)$

$= \frac{\ln 3}{2}$

$= 0.55$ (2.d.p). ✓

3

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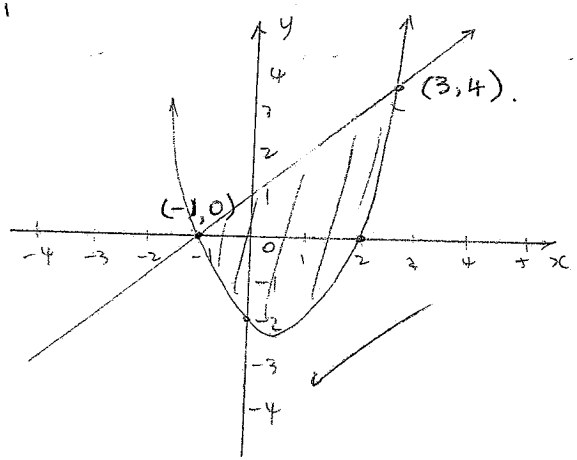
Q2 part III

d) $y = x+1$

$y = x^2 - x - 2.$ $(x-2)(x+1)$

i



✓

2

$x+1 = x^2 - x - 2$

$= x^2 - 2x - 3$

$= (x-3)(x+1)$

$x = -1 \quad x = 3.$

$y = 0 \quad y = 4$

ii.

$A = \int_{-1}^{3} x+1 - (x^2 - x - 2)\,dx$

$= \int_{-1}^{3} x+1 - x^2 + x + 2\,dx$

$= \int_{-1}^{3} 2x + 3 - x^2\,dx$

$= \left[\frac{2x^2}{2} + 3x - \frac{x^3}{3}\right]_{-1}^{3}$

$= 3^2 + 9 - \frac{3^3}{3} - \left((-1)^2 + 3(-1) - \frac{(-1)^3}{3}\right)$

$= 9 + 1\frac{2}{3}.$

$= \frac{32}{3}$ units^2 ✓

3

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Question 3.

a) i $V = \pi \int_0^h x^2 \, dy$ $\quad 16y = x^4$

$= \pi \int_0^h 4y^{1/2} \, dy$ $\quad \sqrt{16y} = x^2$

$= \pi \left[\frac{(2)4y^{3/2}}{3} \right]_0^h$ $\quad 4y^{1/2} = x^2$

$= \pi \left[\frac{8y^{3/2}}{3} \right]_0^h$

$= \pi \left(\frac{8(h)^{3/2}}{3} - \frac{8(0)^{3/2}}{3} \right)$

$= \frac{8\pi h^{3/2}}{3}$ units^3

Unit^3 = cm^3 = ml.

$= \frac{8\pi h^{3/2}}{3}$ ml. ✓ 3

II $\frac{8\pi h^{3/2}}{3} = 250$

$\pi h^{3/2} = \frac{750}{8}$

$\sqrt[2]{h^3} = \frac{93.75}{\pi}$

$h = 9.62$

$= 9.6$ (1 d.p). units. ✓ 1

Q3 part II

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b) i

	Y_0	Y_1	Y_2	Y_3	Y_n
x	0	0.2	0.4	0.6	0.8
y	0	ln 1.5 = 0.4055 (3.dp). ✓	0.347	ln 4 = 1.386 (3d.p) ✓	2.197

II $\frac{h}{3} \left[Y_0 + Y_n + 4(Y_1 + Y_3) + 2(Y_2) \right]$

$= \frac{0.2}{3} \left[0 + 2.197 + 4(0.4055 + 1.386) + 2(0.347) \right]$

$= \frac{1}{15} \left[11.055 \right]$

$= 0.737$ units^2

$= 0.74$ units^2 (2 sig. fig) ✓ 4

III $y = \ln \left(\frac{1+x}{1-x} \right)$

$= \ln(1+x) - \ln(1-x)$

$\frac{dy}{dx} = \frac{1}{1+x} - \frac{1}{1-x} \times -1$

$= \frac{1}{1+x} + \frac{1}{1-x}$

$= \frac{1-x + (1+x)}{(1+x)(1-x)}$

$= \frac{2}{1-x^2}$ ✓ 3

Q3 part III.

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IV $\frac{dy}{dx} = \frac{2}{1-x^2}$

Stat pts occur when $\frac{dy}{dx} = 0$.

$\frac{2}{1-x^2} \neq 0.$ ✓1