



Randwick Girls' High School

**2006  
TRIAL HIGHER SCHOOL CERTIFICATE  
EXAMINATION**

# Mathematics Extension 1

**General Instructions**

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (84)

- Attempt Questions 1-7
- All questions are of equal value

Question	Marks
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
Total	/84

**STANDARD INTEGRALS**

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE :  $\ln x = \log_e x, \quad x > 0$

**Total Marks – 84**

**Attempt Questions 1-7**

**All Questions are of equal value**

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

QUESTION 1	(12 MARKS)	Begin a NEW sheet of writing paper.	Marks
a)	Divide the interval A (-2, 7) B (12, 0) <u>internally</u> in the ratio 4:3		2
b)	Find $\int_{-2}^2 \frac{dx}{4+x^2}$		2
c)	Factorise $27x^6 + \frac{1}{8}$		2
d)	Solve $3^{2x} - (1 + \sqrt{3}) \times 3^x + \sqrt{3} = 0$		2
e)	On the separate page provided you are given $y = f(x)$ Sketch $y = f'(x)$ immediately below.		2
f)	For what values of $x$ is $ 2x - 5  < -1$		2

**QUESTION 2** (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

\* (a) A point P (x, y) moves so that its distance from A(8, -2) is equal to twice its distance from B (-1, 4). 3  
Find its locus in algebraic form and describe the locus geometrically.

(b) (i) Show that  $\frac{d}{dx}(\operatorname{cosec}(x^2)) = -2x \operatorname{cosec}(x^2) \cot(x^2)$  2

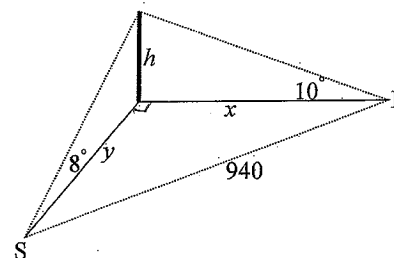
(ii) Hence evaluate  $\int_{\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{\sqrt{2}}} x \operatorname{cosec}(x^2) \cot(x^2) dx$  2

(c) A surveyor who is y metres south of a tower sees the top of it with an angle of elevation  $8^\circ$ . A second surveyor is x metres east of the tower. From his position the angle of elevation is  $10^\circ$  to the top of the tower.

The two surveyors are 940m apart.

(i) Show that  $y = h \cot 8^\circ$  1

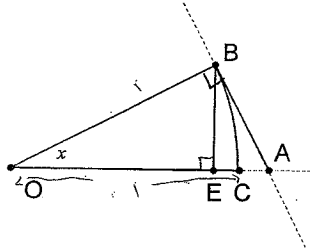
(ii) Find the height of the tower to the nearest metre. 4



QUESTION 3 (12 MARKS) Begin a NEW sheet of writing paper.

Marks

(a)



The diagram shows a part of a unit circle  
i.e.  $OB = OC = 1$ .  $BE \perp OE$   $\angle OBA = 90^\circ$   
O, E, C, A are collinear.

- (i) Define  $x$ ,  $\tan x$  and  $\sin x$  in terms of lengths shown on the diagram. 1
- (ii) Use the diagram to show that  $\sin x < x < \tan x$  1

- (b) (i) Two lines with gradients  $m_1$  and  $m_2$  intersect on the Cartesian Plane. If the acute angle between the lines is  $\theta$ , write the formula for  $\tan \theta$ . 1

- (ii) If  $m_1 = 2$ , find the exact value(s) of  $m_2$  if  $\theta = 60^\circ$  2

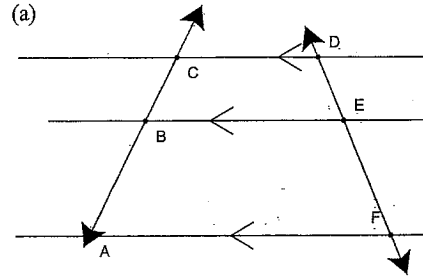
- (c) Evaluate  $\int_0^3 x\sqrt{9-x^2} dx$  using the substitution  $u = 9 - x^2$  4

- (d) Use the principal of Mathematical Induction to prove that  $7^n - 3^n$  is a multiple of 4 for all positive whole numbers. 3

QUESTION 4 (12 MARKS) Begin a NEW sheet of writing paper.

Marks

(a)



The diagram shows 3 parallel lines;  $CD \parallel BE \parallel AF$   
and 2 transversals AC and DF

- (i) Copy the diagram onto your answers and draw the parallel to AC through E. 1
- (ii) Prove  $\frac{BC}{BA} = \frac{DE}{EF}$  2

(You may assume that the opposite sides of a parallelogram are equal)

- (b) The polynomial  $px^3 - qx + r = 0$  has a double root at  $x = \alpha$ . 3

Show that  $\alpha = \sqrt[3]{\frac{r}{2p}}$

- (c) Find the domain and range for the function  $y = 5\sin^{-1}\left(\frac{x}{\pi}\right)$ . 2

- (d) Evaluate  $\int_0^{\frac{\pi}{8}} \cos^2 2x dx$  3

- (e) Write the sum  $5 + 9 + 13 + \dots + 4n - 3$  in Sigma Notation 1

QUESTION 5 (12 MARKS) Begin a NEW sheet of writing paper.

Marks

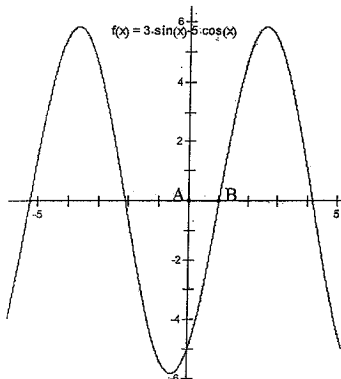
- (a) P  $(2ap, ap^2)$  and Q  $(2aq, aq^2)$  are distinct points on the parabola  $x^2 = 4ay$  such that  $\angle PSQ = 90^\circ$  and S is the focus.

★ (i) Show that  $(pq + 1)^2 = (p - q)^2$ . 4

ii) The tangents at P and Q intersect at T.

Show that the figure PSQT cannot be a cyclic quadrilateral. 2

- (b) The curve  $y = 3 \sin x - 5 \cos x$  is shown
- (i) State the amplitude of the function in exact terms. 1
- (ii) Calculate the distance AB correct to 3 decimal places. 2



- (c) Consider the equation  $x^2 + 2x + 4y^2 - 16y + 13 = 0$  which represents an ellipse. 3

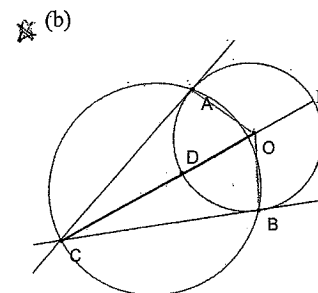
The line  $y = mx$  meets the ellipse.

Show that  $3m^2 - 4m - 3 \geq 0$ . What value(s) of  $m$  make the line a tangent to the ellipse?

QUESTION 6 (12 MARKS) Begin a NEW sheet of writing paper.

Marks

- (a) 250ml of coffee at  $60^\circ$  is poured into each of 2 containers. 5  
 The surrounding temperature is  $24^\circ$ . The temperature of the coffee after 5 minutes is  $50^\circ$  in the normal cup and  $58^\circ$  in the insulated cup.  
 Find the temperature difference after a further 25 minutes have elapsed.  
 You may use  $T = A + Ce^{kt}$  to model this situation.



O is the centre of the small circle. CA and CB are tangents to this circle.

The large circle is drawn through A, B and C. CO is produced to E and intersects the small circle at D.

**Note:**  $\overline{CD}$  means the length of CD

- (i) Show that O lies on the large circle. 2
- (ii) Show that  $\overline{CD} = \frac{\overline{CB} \times \overline{CA}}{\overline{CD} + 2\overline{DO}}$  2
- (c) Prove that the equation of motion  $x = 10 \cos(2t - \frac{\pi}{3}) + 1$  3  
 specifies Simple Harmonic Motion and find the initial position of the particle.

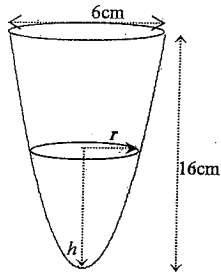
Not to Scale

**QUESTION 7 (12 MARKS)**

Begin a NEW sheet of writing paper.

**Marks**

(a) A wine glass is formed by rotating  $y = ax^2$  around the  $y$  axis.



The depth of liquid in the glass is  $h$  and the radius at the top of the liquid is  $r$ .

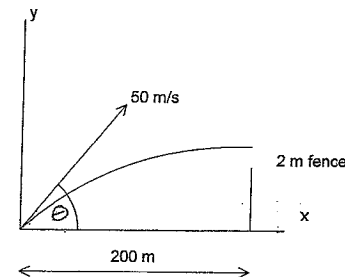
- (i) Find the value of  $a$  1
- (ii) Write an expression for  $h$  in terms of  $r$ . 1
- (iii) Show that the volume of liquid in the glass 1  
when the depth is  $h$  cm is  $\frac{8\pi r^4}{9}$
- (iv) Liquid is being added to the glass at the 3  
rate  $3(15 - h)$  ml per second. Find the rate  
at which the radius of the surface is increasing  
when  $h = 10$  cm.

**Question 7 is continued on page 9**

**QUESTION 7 CONTINUED**

**Marks**

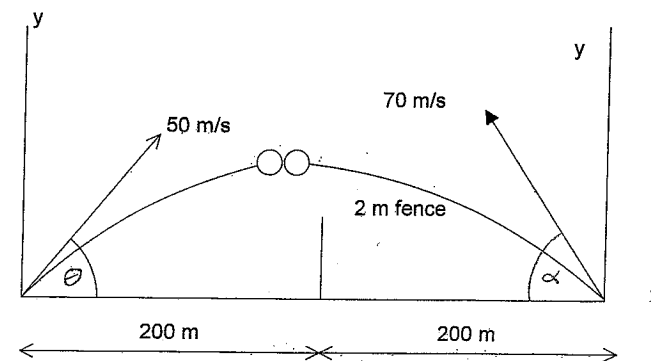
(b) A method to score a home run in baseball is to hit the ball over the boundary fence on the field.



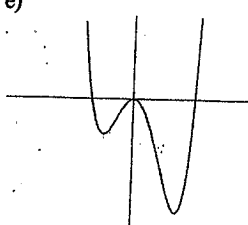
A ball is hit at  $50 \text{ m s}^{-1}$ . The fence 200 metres away is 2 metres high. You may neglect air resistance and acceleration due to gravity can be taken as  $10 \text{ m s}^{-2}$  and you may assume the following equations of motion

$$x = 50 t \cos \theta \text{ and } y = 50 t \sin \theta - 5 t^2$$

- (i) Show that if the ball just clears the 2 m boundary fence then 2  
 $80 \tan^2 \theta - 200 \tan \theta + 82 = 0$
- (ii) In what range of values must  $\theta$  lie to score a home run by this method? 2
- (iii) In an adjacent field another ball is hit at the same instant at  $70 \text{ m s}^{-1}$  and the balls collide. Assume that  $\theta = 30^\circ$ . Find the angle of projection  $\alpha$  of the second ball and the time and position where the balls collide. 2



**End of Examination**

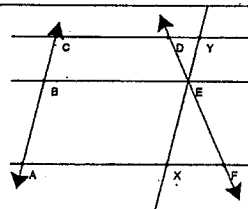
Solutions	Marks/Comments
1 a) $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right) = \left(\frac{4 \times 12 + 3 \times -2}{4+3}, \frac{4 \times 0 + 3 \times 7}{4+3}\right)$ $\left(\frac{42}{7}, \frac{21}{7}\right) = (6, 3)$	1
b) Using the standard integrals $I = \frac{1}{2} \left[\tan^{-1}\left(\frac{x}{2}\right)\right]_2^4$ $= \frac{1}{2} [\tan^{-1} 2 - \tan^{-1}(1)] = \frac{1}{2} \left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$	1
c) Of the form $(a+b)(a^2 - ab + b^2)$ $\left(3x^2 + \frac{1}{2}\right)\left(9x^4 - \frac{3}{2}x^2 + \frac{1}{4}\right)$	1 for significant progress 1 for completion
d) $(3^x - 1)(3^x - \sqrt{3}) = 0$ so $3^x = 1, x = 0$ or $3^x = \sqrt{3}, x = \frac{1}{2}$	1
e) 	1 min in correct location 1 max in correct location
f) Solving $2x - 5 = -1, x = 2$ and $5 - 2x = -1, 3 = x$ gives critical points. Testing $x = 0$ $5 < -1$ false. $x = 2.5$ $0 < -1$ false, $x = 4$ $3 < -1$ false No solution	1 1 Allow 1 for $x < 2$ OR $x > 3$

/12

Solutions	Marks/Comments
2 a) $\sqrt{(x-8)^2 + (y+2)^2} = 2 \times \sqrt{(x+1)^2 + (y-4)^2}$ $x^2 - 16x + 64 + y^2 + 4y + 4 = 4(x^2 + 2x + 1 + y^2 - 8y + 16)$ $x^2 - 16x + 64 + y^2 + 4y + 4 = 4x^2 + 8x + 4 + 4y^2 - 32y + 64$ $0 = 3x^2 + 24x - 60 + 3y^2 - 36y + 60$ $0 = x^2 + 8x - 20 + y^2 - 12y + 20$ $36 + 16 = x^2 + 8x + 16 + y^2 - 12y + 36$ $52 = (x+4)^2 + (y-6)^2$ Which is a circle centre $(-4, 6)$ radius $2\sqrt{13}$	1
b) let $u = x^2 \quad \frac{du}{dx} = 2x \quad y = \operatorname{cosec} u = (\sin u)^{-1}$ i) $\frac{dy}{du} = -1 \cos u (\sin u)^{-2}$ $\frac{dy}{du} \times \frac{du}{dx} = -2x \frac{\cos(x^2)}{(\sin(x^2))^2} = -2x \operatorname{cosec}(x^2) \cot(x^2)$ as req'd	1
ii) $I = \frac{1}{2} \left[ \operatorname{cosec}(x^2) \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{1}{2} (\operatorname{cosec} \frac{\pi}{2} - \operatorname{cosec} \frac{\pi}{4}) = \frac{1 - \sqrt{2}}{2}$	2 (1 for substantial progress)
c) i) $\frac{h}{y} = \tan 8^\circ$ so $\frac{y}{h} = \cot 8^\circ$ ie $y = h \cot 8^\circ$ ii) likewise $x = h \cot 10^\circ$ By Pythagoras $x^2 + y^2 = 940^2$ i.e. $h^2 \cot^2 8^\circ + h^2 \cot^2 10^\circ = 883600$ $h^2 (\cot^2 8^\circ + \cot^2 10^\circ) = 883600$ $h^2 = \frac{883600}{\cot^2 8^\circ + \cot^2 10^\circ} = \frac{883600}{82.7919} = 10672.54$ $= 103 \text{ metres (to nearest metre)}$	1 1 1 1 1

/12

Solutions	Marks/Comments
3 a) i) $x = \text{arc length BC}$ $\sin x = \frac{BE}{BC}$ $\tan x = \frac{BE}{BA}$	1
ii) $\frac{BE}{BC} < \frac{BE}{BA}$ so $\sin x < x < \tan x$	1
b) i) $\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$	1
ii) $\left  \frac{2 - m_2}{1 + 2m_2} \right  = \sqrt{3}$	
Then $\frac{2 - m_2}{1 + 2m_2} = \sqrt{3}$ OR $\frac{2 - m_2}{1 + 2m_2} = -\sqrt{3}$	1
$2 - m_2 = \sqrt{3} + 2\sqrt{3}m_2$ $2 - m_2 = -\sqrt{3} - 2\sqrt{3}m_2$	
$2 - \sqrt{3} = m_2(1 + 2\sqrt{3})$ $2 + \sqrt{3} = (1 - 2\sqrt{3})m_2$	
$m_2 = \frac{2 - \sqrt{3}}{1 + 2\sqrt{3}}$ OR $m_2 = \frac{2 + \sqrt{3}}{1 - 2\sqrt{3}}$	1
c) let $u = 9 - x^2$ $\frac{du}{dx} = -2x$ $x = 0, u = 9$ $x = 3, u = 0$	1
$I = -\frac{1}{2} \int_0^3 2x(9 - x^2)^{\frac{1}{2}} dx = -\frac{1}{2} \int_9^0 \frac{du}{dx} u^{\frac{1}{2}} dx = \frac{1}{2} \left[ \frac{2}{3} u^{\frac{3}{2}} \right]_0^9$	2
$= 9$	1
c) If $n = 1$ $7^1 - 3^1 = 7 - 3 = 4$ (Which is a multiple of 4)	
Assume true for $n = k$ i.e. $7^k - 3^k = 4p$	1
Show that $7^{k+1} - 3^{k+1} = 4q$	
LHS = $7^k - 3^k = 7^k \times 7 - 3^k \times 3$	
$= 4 \times 7^k + 3 \times 7^k - 3 \times 3^k$	
$= 4 \times 7^k + 3(7^k - 3^k)$	
$= 4 \times 7^k + 3(4p)$	
$= 4(7^k + 3p)$	
$= 4q$	
$\therefore$ if true for $n = k$ also true for $n = k + 1$ but since true for $n = 1$ , by induction also true for all positive whole numbers.	1
	/12

Solutions	Marks/Comments
4 a) i) 	1
ii) $\triangle DEY \parallel \triangle FEX$ since $\angle DEY = \angle FEX$ vertically opposite & $\angle EDY = \angle EFX$ z angles parallel lines	1
$\frac{EY}{EX} = \frac{ED}{EF}$ corresponding sides in similar triangles	
but $EY = BC$ and $EX = AB$ Opposite sides in parallelogram	
so $\frac{BC}{BA} = \frac{DE}{EF}$ as required	1
4 b) $px^3 - qx + r = 0$ has roots $\alpha, \alpha$ and $\beta$	
Sum of the roots = $2\alpha + \beta = -\frac{b}{a} = 0$	1 for sum
$\therefore \beta = -2\alpha$	
Product of the roots = $\alpha^2 \beta = -\frac{d}{a} = -\frac{r}{p}$	1 for product
Substituting $\alpha^2 \beta = \alpha^2(-2\alpha) = -\frac{r}{p}$	
$-2\alpha^3 = -\frac{r}{p} \Rightarrow \alpha^3 = \frac{r}{2p} \Rightarrow \alpha = \sqrt[3]{\frac{r}{2p}}$	1 for result
c) $5 \sin^{-1}\left(\frac{x}{3}\right)$ Domain $-1 \leq \frac{x}{3} \leq 1$ so $\pi \leq x \leq \pi$	1 for each of domain and range
Range $-\frac{5\pi}{2} \leq y \leq \frac{5\pi}{2}$	1
$\cos^2 x = \frac{1}{2}(1 + \cos 2x) \therefore \cos^2 2x = \frac{1}{2}(1 + \cos 4x)$	1
$\int_0^{\frac{\pi}{8}} \cos^2 2x dx = \frac{1}{2} \left[ x + \frac{1}{4} \sin 4x \right]_0^{\frac{\pi}{8}} = \frac{\pi + 2}{16}$	1 + 1
d) $\sum_{r=2}^n 4r - 3$	1 /12

Solutions	Marks/Comments
5 a) i) $m_{pq} = \frac{ap^2 - a^2}{2ap}$	
$= \frac{p^2 - 1}{2p}$ similarly $m_{qr} = \frac{q^2 - 1}{2q}$	1 for gradients
Note $(pq + 1)^2 = p^2 q^2 + 2pq + 1$	1 for product = -1
$m_1 \times m_2 = -1$	
$\frac{p^2 - 1}{2p} \times \frac{q^2 - 1}{2q} = -1$	1 for simplifying
$p^2 q^2 - p^2 - q^2 + 1 = -4pq$	
Re arranging and adding $2pq$ to both sides	
$p^2 q^2 + 2pq + 1 = p^2 + q^2 - 2pq$	1 for result
$(pq + 1)^2 = (p - q)^2$	
ii) For cyclic quadrilateral opposite angles are supplementary thus $\angle PTQ$ must equal $90^\circ$ so $pq$ must equal $-1$ . From (i) $(pq+1)^2 = (p-q)^2$ this is impossible as $p$ and $q$ are distinct points (they would need to be equal for this condition to apply).	1 for property 1 for correct conclusion
(b) ) i) $R = \sqrt{3^2 + 5^2} = \sqrt{34}$	1
ii) $\frac{3}{\sqrt{34}} \sin x - \frac{5}{\sqrt{34}} \cos x$ is of the form $\sin x \cos \alpha - \cos x \sin \alpha$	1
i.e. $y = 3 \sin x - 5 \cos x = \sqrt{34} \sin(x - \tan^{-1} \frac{5}{3})$	
$= \sqrt{34} \sin(x - 1.030)$	
$AB = 1.030$ units correct to 3 d.p.	1
c) $x^2 + 2x + 4y^2 - 16y + 13 = 0$	
$(x+1)^2 + 4(y-2)^2 - 4 = 0$	
$(x+1)^2 + 4(mx-2)^2 + 13 = 0$	1
$x^2 + 2x + 4m^2 x^2 - 16mx + 13 = 0$	
$(4m^2 + 1)x^2 + (2 - 16m)x + 13 = 0$	
$\Delta = b^2 - 4ac = 4 - 64m + 256m^2 - 52(4m^2 + 1)$	
$= 48m^2 - 64m - 48$	
Line intersects if this expression $\geq 0$ ie if $3m^2 - 4m - 3 \geq 0$	1
For the line to be tangent $3m^2 - 4m - 3 = 0$	
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 + 36}}{6} \approx 1.868... \text{ or } -0.535...$	1

Solutions	Marks/Comments
6 a) The equation for the two cups is $T = A + Ce^{kt}$ observe that $A = 24$ and $C = 36$ since $T = 60$ when $t = 0$	1
In the first cup $50 = 24 + 36e^{5k}$ so $\frac{26}{36} = e^{5k}$	
$k_1 = \ln\left(\frac{26}{36}\right) + 5 = -0.06508448$ and likewise...	1
$k_2 = \ln\left(\frac{34}{36}\right) + 5 = -0.01143168277$	1
When $t = 30$ $T_1 = 24 + 36e^{30 \times -0.06508448} \approx 29.1^\circ$	1
$T_2 = 24 + 36e^{30 \times -0.01143168277} \approx 49.5^\circ$ and so the difference is $20.4^\circ$	1
b) i) $\angle OAC = \angle OBC = 90^\circ$ radius meets tangent at right angles. Now OACB is a cyclic quadrilateral since two opposite angles are supplementary. So O is on the circle.	1
ii) Produce CO till it meets the circle again at E.	1
The square on the tangent equals the product of the intercepts of the secant i.e. $CA^2 = CD \times CE = CD(CD + 2DO)$	1 reasons req <sup>d</sup> in full
Also $CA = CB$ tangents from an external point are equal	
so $CD(CD + 2OD) = CA \times CB$ i.e. $CD = \frac{CA \times CB}{CD + 2OD}$	1
as req <sup>d</sup>	
c) $x = 10 \cos\left(2t - \frac{\pi}{3}\right) + 1$ $\ddot{x} = -20 \sin\left(2t - \frac{\pi}{3}\right)$	
$\ddot{x} = -40 \cos\left(2t - \frac{\pi}{3}\right)$	1
Now $-4x = -40 \cos\left(2t - \frac{\pi}{3}\right) + 4$	
$-4x - 4 = -40 \cos\left(2t - \frac{\pi}{3}\right)$	
$-4(x - 1) = \ddot{x}$	1 not req <sup>d</sup> (x = 1)
$\therefore$ Motion can be expressed in the form $\ddot{x} = -n^2 x$	
so motion is simple harmonic	
$\ddot{x} = -2^2(x - 1)$	
SHM with centre of $x = 1$	
Initially $x = 10 \cos\left(-\frac{\pi}{3}\right) + 1 = 6$	1



Solutions	Marks/Comments
7 a) i) $y = ax^2$ $16 = a \times 3^2$ $a = \frac{16}{9}$	1
ii) Substituting $(r, h)$ into $y = ax^2$ gives $h = \frac{16}{9}r^2$	1
iii) $V = \pi \int r^2 . dh = \pi \int \frac{9}{16} h . dh = \frac{9\pi}{32} h^2 = \frac{9\pi}{32} \left( \frac{16r^2}{9} \right)^2 = \frac{8\pi r^4}{9}$	1
iv) $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$ and	1
$r = \frac{3}{4}\sqrt{h} = \frac{3}{4}\sqrt{10}$ when $h = 10$	1
$\frac{dr}{dt} = 3(15-h) \times \frac{9}{32\pi^3} = \frac{135}{32 \times \pi \frac{270\sqrt{10}}{64}} = 0.100658 \text{cms}^{-1}$	1
	continued

	Marks /Comments
7b)i) (200,2) satisfies $x = 50t \cos \theta$ and $y = 50t \sin \theta - 5t^2$	
$x = 50t \cos \theta$	
$200 = 50t \cos \theta$	
$\therefore t = \frac{4}{\cos \theta}$	1 for t
Substituting I into II gives	
$50 \left( \frac{4}{\cos \theta} \right) \sin \theta - 5 \left( \frac{4}{\cos \theta} \right)^2 - 2 = 0$	
$200 \tan \theta - 80 \sec^2 \theta - 2 = 0$	
$200 \tan \theta - 80(1 + \tan^2 \theta) - 2 = 0$	
$80 \tan^2 \theta - 200 \tan \theta + 82 = 0$	
ii)	1 for simplifying
$\tan \theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
$\tan \theta = \frac{200 \pm \sqrt{13760}}{160}$	1 for substitution
$\theta = 63^\circ 14'$ or $27^\circ 20'$	
$\therefore$ Range is $27^\circ 20' \leq \theta \leq 63^\circ 14'$	1 for range
iii)	
For ball 1	For ball 2
$x_1 = 50t \cos 30^\circ$	$x_2 = 70t \cos \alpha$
$= 25\sqrt{3} t$	
$y_1 = 50t \sin 30^\circ - 5t^2$	$y_2 = 70t \sin \alpha - 5t^2$
$= 25t - 5t^2$	
Now $x_1 + x_2 = 400$	
$\therefore x_1 = 400 - x_2$	
$= 400 - 70t \cos \alpha$	
For the balls to collide $y_1 = y_2$	
$25t - 5t^2 = 70t \sin \alpha - 5t^2$	
$\sin \alpha = \frac{25}{70}$	
$\alpha = 20^\circ 55'$	
Sub $\alpha = 20^\circ 55'$ into $x_1 = 400 - 70t \cos \alpha$	
$25\sqrt{3} t = 400 - 70t \cos 20^\circ 55'$	
$\therefore t = \frac{400}{25\sqrt{3} + 70 \cos 20^\circ 55'} = 3.68 \text{ seconds}$	1 for time
Position	
$x_1 = 25\sqrt{3} t$	$y_1 = 25t - 5t^2$
$x_1 = 25\sqrt{3} \times 3.68 = 159.36m$	$y_1 = 25 \times 3.68 - 5(3.68)^2 = 24.28m$
	1 for position