Randwick Girls High School

2005
Half Yearly Examination

Hui Ling Lim.

Year 11

Mathematics

General Instructions

- Reading Time- 5 minutes
- Working Time – 1 hour
- Write using a blue or black pen
- Approved calculators may be used
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**40**)

- Attempt Questions 1-4
- All questions are of equal value

Q1 — 8
Q2 — 10
Q3 — 8
Q4 — 7

33.

Question 1.

Marks

a). Factorize $8+x^3$ — 2

b). Solve $3x^2-4x-4=0$ by first factorizing. — 2

c). Solve for x

$$\frac{2x-1}{2}-\frac{3x-4}{5}=3$$ — 2

d). Find the value of e in the formula $b^2=a^2\left(1-e^2\right)$ given that $a=13$ *and* $b=12$. — 2

e). Evaluate correct to 2 decimal places. $\dfrac{4.23}{\sqrt{6.14-1.78}}$ — 2

Question 2.

Marks

a). Solve $|x-1|=4$ — 2

b). Rationalize the denominator $\dfrac{1}{3-\sqrt{2}}$ — 2

c). Solve $|x+1|\leq 3$ and plot your solution on a number line. — 2

d). Solve the pair of simultaneous equations

$$2x+y=7$$
$$x-2y=3$$ — 2

e). Write $0.\dot{2}\dot{1}\dot{4}$ as a simple fraction. — 2

Question 3.

a). The approximate distance ***d*** kilometers, to the visible horizon from a height ***h*** metres above sea level is given by the formula $d = 5\sqrt{\frac{h}{2}}$. Use the formula to find

Marks

i) the approximate distance to the horizon when standing 45 metres above sea level. — 1

ii) the height necessary (to the nearest metre) to see a distance of 30 kilometres. — 2

b). Write $\frac{a^{-1}+b^{-1}}{a+b}$ in simplest form without the use of negative indices. — 2

c). Show that $f(x) = x^3 + 3x$ is an odd function — 2

d). Use the method of completing the square to find the center and radius of the circle given by $x^2 + 6x + y^2 - 2y + 7 = 0$ — 3

Question 4.

Marks

a). For the function defined as $f(x) = \sqrt{x-1}$. Find

i) the domain — 1

ii) the range — 1

iii) draw a neat sketch of the function — 2

b). Using the domain $-4 \le x \le 4$ on the same set of co ordinate axes neatly graph — 4

i) $y = x^2 - 3x - 2$ and

ii) $y = 5 - x$.

iii) Find algebraically the coordinates of the points of intersection of $y = x^2 - 3x - 2$ and $y = 5 - x$. — 2

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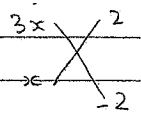
Ms Jagger
Huilung Lim

Question 1. 8/10

a) $8+x^3$
$= (2+x)(4-2x+x^2)$ ✓ — 2

b) $3x^2-4x-4=0$
$(3x+2)(x-2)=0$ — 1



$x = ?$

c) $\frac{2x-1}{2} - \frac{3x-4}{5} = 3$

$\frac{5(2x-1) - 3(3x-4)}{10} = 3$ — 2

$10x-5-6x+8 = 30$
$4x+3 = 30$
$4x = 30-3$
$x = \frac{27}{4}$
$= 6.75.$ ✓

d) $b^2 = a^2(1-e^2)$
$12^2 = 13^2(1-e^2)$
$\frac{12^2}{13^2} = 1-e^2$ — 2
$\frac{144}{169} = 1-e^2$
$\frac{144}{169} - 1 = -e^2$ ✓
$-\left(\frac{144}{169}-1\right) = e^2$ ✗ — 1
$e = \pm\sqrt{+\left(\frac{144}{169}-1\right)}$
$\frac{5}{13}$

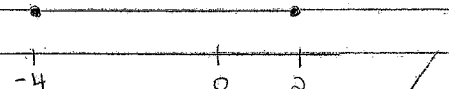
e) $\frac{4.23}{\sqrt{6.14-1.78}}$
$= \frac{4.23}{\sqrt{4.36}}$
$= 2.03.$ ✓

Question 2 10/10

a) $|x-1| = 4$ or $-x+1=4$
$x = 4+1$ $-x = 4-1$
$x = 5$ ✓ $-x = 3$
$x = -3$ ✓ — 2

b) $\frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$
$= \frac{3+\sqrt{2}}{9-2}$ — 2
$= \frac{3+\sqrt{2}}{7}$ ✓

c) $|x+1| \leq 3.$ $-x-1=3$
$x = 3-1$ $-x = 3+1$
$x = 2$ ✓ $x = -4$ ✗
— 2



Soln $-4 \leq x \leq 2$ ✓

d) $2x+y=7$ — ①
$x-2y=3$ — ②
take ①
$y = 7-2x.$ — ③
— 2
Sub ③ into ②.
$x-2(7-2x)=3$
$x-14+4x=3$
$5x = 3+14$
$x = \frac{17}{5}$

$y = 2\left(\frac{17}{5}\right)+y=7$
$y = 7-2\left(\frac{17}{5}\right)$
$= \frac{1}{5}$

e) $0.\dot{2}1\dot{4} = x$.

$10x = 2.1\dot{4}$

$100x = 21.\dot{4}$

$1000x = 214.\dot{2}14$

$1000x - x = 214.\dot{2}14 - 0.\dot{2}14$

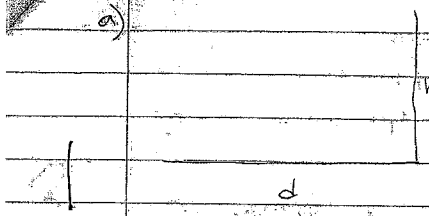
$999x = 214$

$x = \frac{214}{999}$ ✓

2

Question 3.

a)



$d = 5\sqrt{\frac{h}{2}}$

i) $d = 5\sqrt{\frac{45}{2}}$

$= 23.7$ (1.d.p). ✓

ii) $30 = 5\sqrt{\frac{h}{2}}$

$\frac{30}{5} = \sqrt{\frac{h}{2}}$

$6 = \sqrt{\frac{h}{2}}$

$6^2 = \frac{h}{2}$

$36 \times 2 = h$

$h = 72$ m. ✓

2

8/10.

b) $\frac{a^{-1}+b^{-1}}{a+b}$.

$\frac{\frac{a+b}{a} + \frac{1}{a+b} + \frac{a+b}{b}}{a+b}$ $\quad \frac{1}{ab}$

$\frac{\frac{1}{a(a+b)} + \frac{1}{a+b} + \frac{1}{b(a+b)}}{a+b}$

$= \frac{b+1+a}{a+b}$ $\quad = \frac{b+ab+a}{ab(a+b)}$

$= \frac{b+1+a}{a+b}$.

2

c) $f(-x) = -f(x)$.

$f(-x) = (-x)^3 + 3(-x)$ $\quad -f(x) = -(x^3+3x)$

$= -x^3 - 3x$ $\quad = -x^3 - 3x$.

$f(-x) = -f(x)$

Therefore its an odd function. ✓

2

d) $x^2+6x+y^2-2y+7=0$

$x^2+6x+9+y^2-2y+1=-7+9+1$

$(x+3)^2+(y-1)^2=3$

3

The centre is $(-3, 1)$

The radius is $\sqrt{3}$

Question 4. 7/10

a) i) $f(x)=\sqrt{x-1}$ II) Range $y \geq 0$.

$y=\sqrt{x-1}$

$D = x \geq 1$

1½

$y^2=x-1$

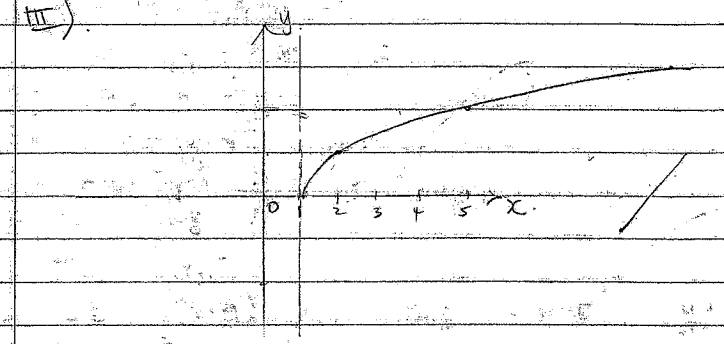
$y^2-x=-1$

$-x=-1+y^2$

$x=+1-y^2$

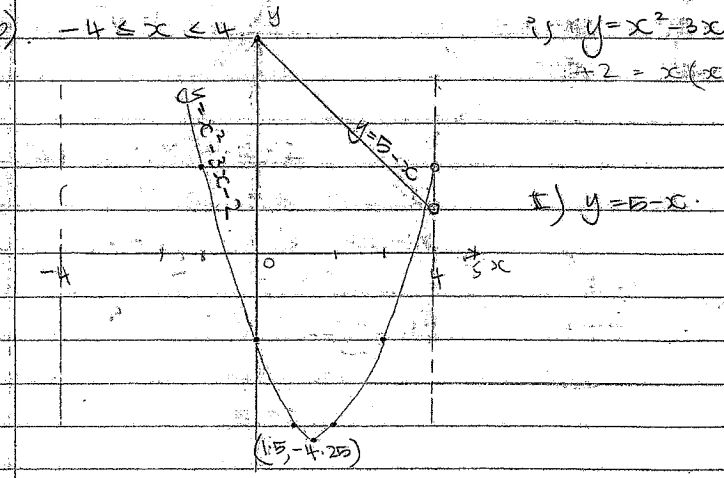
$x=1-y^2$

III)



2

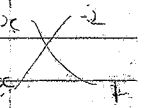
b) $-4 \leq x < 4$



3

i) $y=x^2-3x-2$

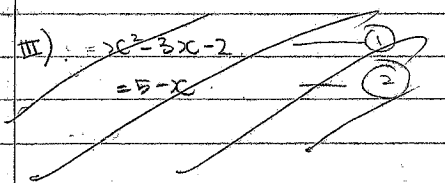
$+2=x(x-3)$



ii) $y=5-x$.

iii) $=x^2-3x-2$

$=5-x$



$y = x^2 - 3x - 2$ — ①

$y = 5 - x$ — ②

$2.25 + 2 = x^2 - 3x + 2.25$

$4.25 = (x - 1.5)^2$

$\sqrt{4.25} = x - 1.5$

$\pm\sqrt{4.25} + 1.5 = x$

$y = 5 - (-\sqrt{4.25} + 1.5)$

$y = 5.56$

$y = 5 - (+\sqrt{4.25} + 1.5)$

$= 1.43$

$(3.56, 1.43)$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{-3^2 - 4 \times 1 \times -2}}{2 \times 1}$

$= \frac{3 \pm \sqrt{9 + 8}}{2}$

$= \frac{3 + \sqrt{17}}{2} = x$

$x = 1 \pm \sqrt{8}$

$y = 4 \pm \sqrt{8}$