HL
Hui Ling Lim.

RANDWICK GIRLS' HIGH SCHOOL

YEAR 11
MATHEMATICS
EXTENSION 1

Time Allowed: 1 hour

Examiner: D. Posener

Examination Date: 17 May 2005

INSTRUCTIONS:

- All questions may be attempted.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work.
- Approved calculators may be used.
- Start each question on a new page.

QUESTION	MARK	
1		/20
2		/22
3		/16
4		/30
TOTAL		**/88**

Question 1:

(a) Write $\frac{8}{5+\sqrt{3}}$ with a rational denominator in simplest form. **2**

(b) Find a and b given $a+\sqrt{b}=\left(3\sqrt{2}+1\right)^2$ **2**

(c) Solve: (i) $3-5x<13$ **3**

(ii) $|2x+6|<4$

(d) $\frac{a^3-27}{a^2-9}$ **3**

(e) Simplify $48^{1-n}\times 12^{2n}\times 3^{-n}$ **3**

(f) Solve $(x+4)(x-2)(x-3)>0$ **3**

(g) Solve $\frac{x-3}{1-x}>2$ **4**

Question 2:

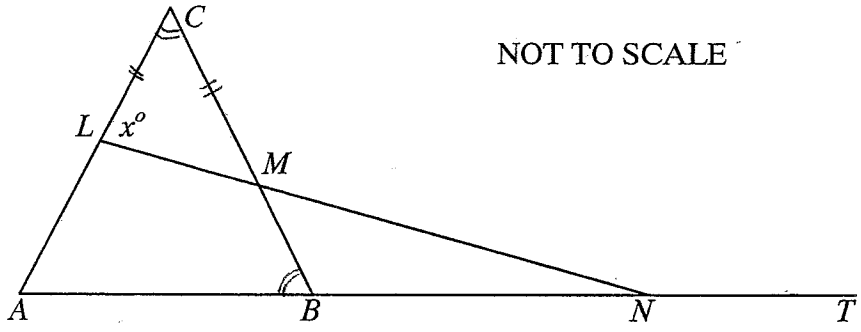
(a) Expand and simplify $\sqrt{(a-4)(a+4)+16}$, $a>0$ **2**

(b) Factorise $x^2+4x+4-y^2$ **3**

(c) Express $2\cdot 3\dot{0}\dot{1}$ as a rational number. **3**

(d) Simplify $\frac{3}{x+3}-\frac{6x}{x^2-9}$ **3**

(e) **4**



NOT TO SCALE

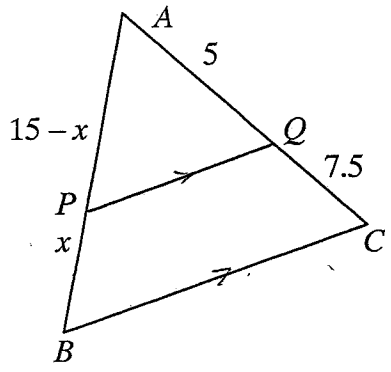
In the diagram ABC is an isosceles triangle with $\angle ABC = \angle ACB$. The line LMN is drawn as shown so that $CL = CM$ and $\angle CLM = x^o$.

(i) Show $\angle ABC = (180-2x)^o$

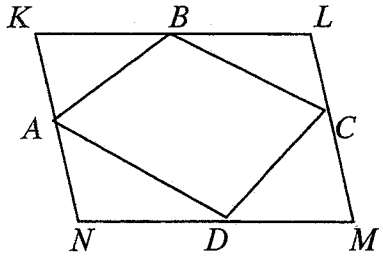
(ii) Hence show $\angle TNL = 3x^o$

Question 2(Continued):

(f) In the figure $PQ \parallel BC$. Find the value of x giving reasons. All measurements are in cm. **3**



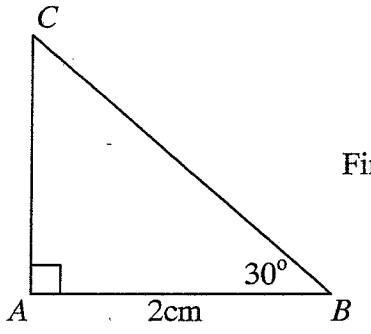
(g) **4**



KLMN is a parallelogram. *A, B, C, D* are the midpoints of the sides.
Prove *ABCD* is a parallelogram.

Question 3:

(a) **2**



Find the exact value of BC

(b) Solve $\cos\theta = \dfrac{-\sqrt{3}}{2}$ for $0° \le \theta \le 360°$ **2**

(c) Find the exact value of $\tan\theta$ given that θ is a reflex angle and $\cos\theta = \dfrac{2}{5}$ **3**

(d) Evaluate $\sec^2 81°47'$ **2**

(e) Evaluate $\dfrac{\sec 315° + \sin 270°}{\cos ec\, 585° + \cot^2(-225°)}$ **5**

(f) Sketch the graph of $y = \sin(90° - x)$ for $-360° \le x \le 360°$ **2**

Question 4:

(a) Given $f(x) = x^3$, find $\dfrac{f(x+h) - f(x)}{h}$ **4**

(b) Write down the natural domain of: (i) $y = \sqrt{x-3}$ **2**

(ii) $y = \dfrac{1}{\sqrt{3-x}}$

(c) State the domain and range of $y = x^2 - 4$ **2**

(d) On separate diagrams sketch: **8**

(i) $xy = 4$

(ii) $y = -2^{-x}$

(iii) $y = (x-1)^3$

(iv) $y = |2x-1|$

(v) $y = -\sqrt{2-x^2}$

(vi) $x = y^2$

(vii) $(x-3)^2 + (y+1)^2 = 9$

(viii) $y = \dfrac{1}{x-2} + 9$

(e) Explain why $f(x) = \sin x$ represents an odd function **2**

(f) A function $f(x)$ is odd and it is known that $f(3) = 7$ **3**

(i) What is the value of $f(-3)$

(ii) Sketch a possible graph of $y = f(x)$

(g) Write down the equation of a non-linear function $f(x)$ such that $f(x) = f(-x)$ **2**

Question 4 (Continued):

(h) A function is defined as: **4**

$$f(x) = \begin{cases} 3 \text{ if } x \le -1 \\ -2x+b \text{ if } -1 < x < 0 \\ 2^x \text{ if } x \ge 0 \end{cases}$$, where b is a constant

(i) Find (A) $f(-2)$ **4**
(B) $f(o)$
(C) $f(c^2)$

(ii) Given that the graph of $y = f(x)$ is continuous everywhere, determine the value of b. **4**

(i) For the function $f(x) = \dfrac{1}{(x+1)(1-3x)}$ **3**

Write down the (i) equations of the vertical asymptotes
(ii) $y-$ intercept

Ms Jagger
HuiLing Lim.

Question 1. /16

a) $\frac{8}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$

$= \frac{40-8\sqrt{3}}{25-3}$

$= \frac{8(5-\sqrt{3})}{22}$

$= \frac{4(5-\sqrt{3})}{11}$ ✓

b) $a+\sqrt{b} = (3\sqrt{2}+1)^2$

$(3\sqrt{2})^2 + 2(3\sqrt{2}\times 1) + 1$

$= 9\times 2 + 6\sqrt{2} + 1$

$= 19 + 6\sqrt{2}$

$a = 19 \quad b = 36\times 2 = 72.$

c) i) $3-5x < 13$

$-5x+3 < 13$

$-5x < 13-3$

$-10 < 5x$

$\frac{-10}{5} < x$

$-2 < x$

$x > -2.$ ✓

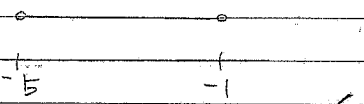
ii) $|2x+6| < 4$

$-4 < 2x+6 < 4$

$-4-6 < 2x < 4-6$

$-10 < 2x < -2$

$-5 < x < -1$ ✓



$-5 < x < -1$ ✓

d) $\frac{(a-3)(a^2+3a+9)}{(a-3)(a+3)}$

$= \frac{a^2+3a+9}{a+3}$ ✓

e) $48^{1-n} \times 12^{2n} \times 3^{-n}$

$= (48\times 12\times 3)^{1-n+2n-n}$

$= 1728^1$ ✗

*change base - same.

$(16\times 3)^{1-n} \times (3\times 4)^{2n} \times 3^{-n}$

$= 4^{2(1-n)} \times 3^{1-n} \times 3^{2n} \times 4^{2n} \times 3^{-n}$

$= 4^{2-2n} \times 3^{1-n+2n-n} \times 4^{2n}$

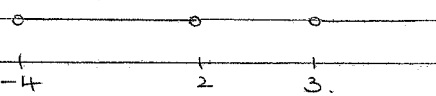
$= 4^2 \times 3 = 48.$

f) $(x+4)(x-2)(x-3) > 0$

$x = -4$

$x = +2$

$x = 3.$



Soln = $-4 < x < 2 \quad x > 3.$ ✓

Ms Jaggar
HuiLing Lim

g) $\frac{x-3}{1-x} > 2.$

$x \neq 1$ ✓

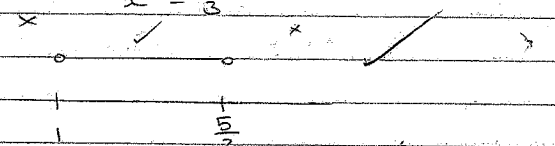
$x-3 = 2(1-x)$

$x-3 = 2-2x$

$x+2x = 2+3$

$3x = 5$

$x = \frac{5}{3}$



Soln = $1 < x < \frac{5}{3}$ ✓

Ms. Jagger
Hui Ling Lim

Question 2.

a) $\sqrt{(a-4)(a+4)+16}$

$= \sqrt{a^2-4^2+16}$

$= a$ ~~$(a+4)$~~

$= a+2$ ✗

b) $x^2+4x+4-y^2$

$= x^2-y^2+4x+4$

$= (x-y)(x+y)+4(x+1)$

$=$

$x(x+2)^2-y^2$

$(x+2-y)(x+2+y)$

c) $x = 2.30\dot{1}$

$10x = 23.\dot{0}\dot{1}$

$100x = 230.\dot{1}$

$1000x = 2301.\dot{0}\dot{1}$

$1000x - 10x = 2301.\dot{0}\dot{1} - 23.\dot{0}\dot{1}$

$990x = 2278$

$x = \frac{2278}{990}$ ✓

$x = \frac{1139}{495}$.

d) $\frac{3}{x+3} - \frac{6x}{x^2-9}$

$= \frac{3}{x+3} - \frac{6x}{(x+3)(x-3)}$

$= \frac{3(x-3) - 6x}{(x+3)(x-3)}$

$= \frac{3x-9-6x}{(x+3)(x-3)}$

$= \frac{-3x-9}{(x+3)(x-3)}$

$= \frac{-3(x+3)}{(x+3)(x-3)} = \frac{-3}{(x-3)}$ ✓

e) i) In $\triangle CLM$, $\angle CLM = \angle CML = x°$ (given)

$\therefore \angle LCM = (180-2x)°$

$\angle LCM = \angle ABC = (180-2x)°$ ($\triangle ABC$ is isosceles $\triangle$)

ii) $\angle CML = x°$

$\therefore \angle NMB = x°$ (vertical opp. $\angle$ are =).

$\angle MBN = 180 - (180-2x)°$ (supplementary $\angle$).

$= 180 - 180 + 2x$

$= 2x$

$\therefore \angle TNL = 2x + x°$ (ext $\angle$ = ~~internal~~ opposite interm $\angle$ sum)

$= 3x°$

f) $\frac{7.5}{12.5} = \frac{x}{15-x}$ ~~(intercepts in ratio)~~ ~~(ratio of intercepts are equal)~~ similar $\triangle$.

$\frac{3}{5} = \frac{x}{15-x}$ * similar $\triangle$

* set up ratio

$\frac{3}{5}(15-x) = x$

$x = 9 - \frac{3x}{5}$

$x = \frac{45-3x}{5}$

$5x = 45-3x$

$5x+3x = 45$

$8x = 45$

$= 5\frac{5}{8}$.

$\frac{x}{15-x} = \frac{7.5}{5}$

$5x = 7.5 \times 15 - 7.5x$

$12.5x = 7.5 \times 15$

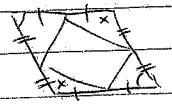
$x = \frac{7.5 \times 15}{12.5} = 9.$

g) $\angle ABC = \angle ADC$ (midpoints intercept's =) SAS vu

$\therefore \angle BAD = \angle BCD$ (midpoints intercept's =) (congruent

angles sum of parallelogram $= 360°$ similarly.

$\therefore$ ABCD is a // gram (opposite sides =)



Ms Jagger
Hui Ling Lim

Question 3

a) $BC = \cos = \frac{adj}{hyp}$

$\cos 30° = \frac{2}{BC}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{2}{BC}$

$\frac{2}{\sqrt{3}} = \frac{BC}{2}$

$\frac{4}{\sqrt{3}} = BC$ $\quad \frac{4\sqrt{3}}{3}$

2

b) $\cos\theta = -\frac{\sqrt{3}}{2}$ $\quad 0° \leq \theta \leq 360°$

S	A
T	C

cos is (−) in quad 2, 3.

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30°$

quad 2 $= 180 - 30°$ $\quad$ quad 3 $= 180 + 30°$

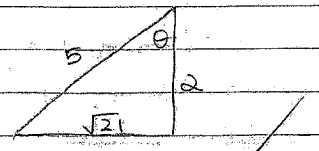
$= 150°$ $\quad = 210°$

Soln − $\theta = 150°, 210°$

2

c) $\tan\theta = \frac{opp}{adj}$ $\quad \cos\theta = \frac{2}{5}$

$\tan\theta = \frac{\sqrt{21}}{2}$



$c^2 = a^2 + b^2$

$5^2 = 2^2 + b^2$

$b = \sqrt{21}$

2

$\tan\theta = -\frac{\sqrt{21}}{2}$

S	A
T	C

4th quad.

d) $\sec^2 81°47'$

$= \frac{1}{\cos^2 81°47'}$

$= \left(\frac{1}{\cos 81°47'}\right)^2$

1

$= 49.959.$ $\quad 49.959$

e) $\frac{\sec 315° + \sin 270°}{\operatorname{cosec} 585° + \cot^2(-225)°}$ $\quad \frac{\frac{1}{\cos 45°} + -(\sin 90°)}{\sin 45° \quad \tan^2 135°}$ - quad 3

$= \frac{\cos 45° + \sin 270°}{\sin 45° + \tan^2 135°}$

$= \frac{\sin(45°) + \sin 270°}{\sin 45° + \frac{\sin^2}{\cos}(135°)}$

$= \frac{\sin 315}{\sin 45° + \left(\frac{\sin}{\cos}\right)^2(135°)}$

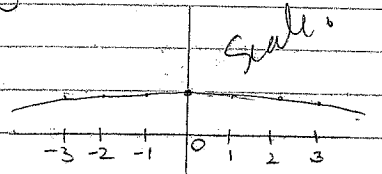
$= \frac{\sqrt{2}-1}{-\sqrt{2}+1}$ (rati

$= -1.$

f) $y = \sin(90° - x)$ $\quad -360° \leq x \leq 360°$

$y = \cos x$

Graph



x	-3	-2	-1	0	1	2	3
y	0.99						

Ms Jagger
Huiling Lim

Question 4

a) $\frac{(x+h)^3 - x^3}{h}$

$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$

$= \frac{h(3x^2 + 3xh + h^2)}{h}$

$= 3x^2 + 3xh + h^2$.

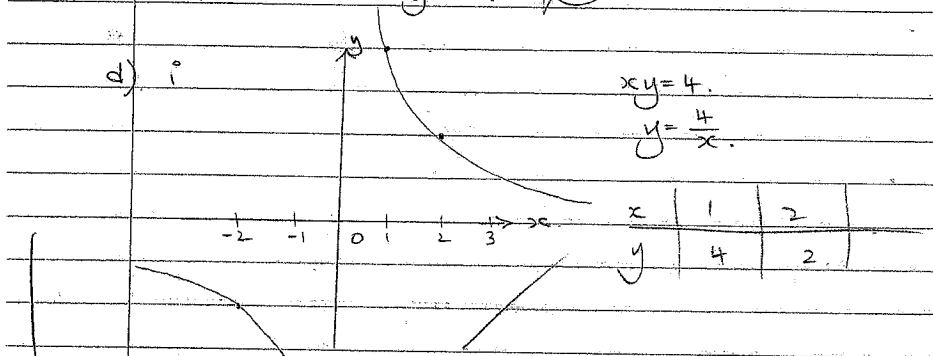
b) i) D. $-\sqrt{3} \leq x \leq \sqrt{3}$, $x \geq 3$

ii) D. $-\sqrt{3} \leq x \leq \sqrt{3}$, $x < 0$

c) D $-2 \leq x \leq 2$ $y \geqslant -4$

R $-2 \leq y \leq 2$

d) i



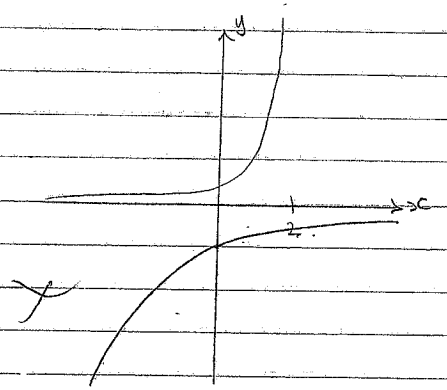
$xy = 4$.

$y = \frac{4}{x}$.

x	1	2
y	4	2

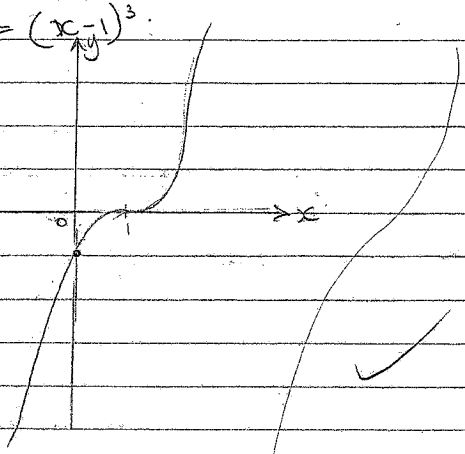
ii) $y = -2^{-x}$.

x	
y	$y = \frac{1}{-2^x}$

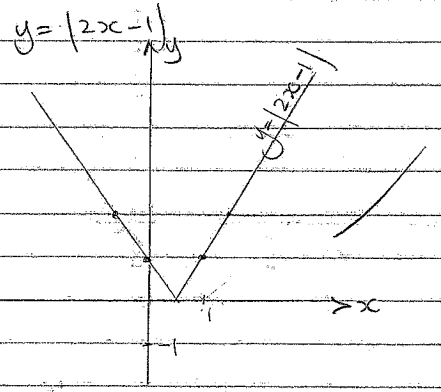


~~iv) y~~

iii) $y = (x-1)^3$.



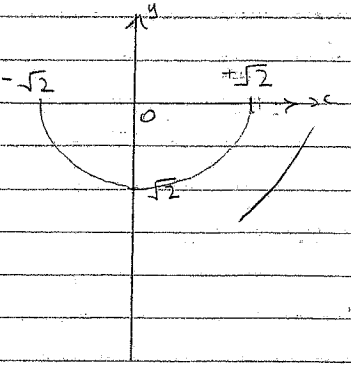
iv) $y = |2x-1|$



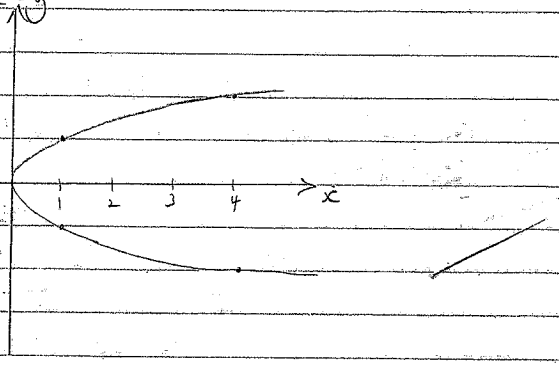
v) $y = -\sqrt{2 - x^2}$

$y^2 = 2 - x^2$.

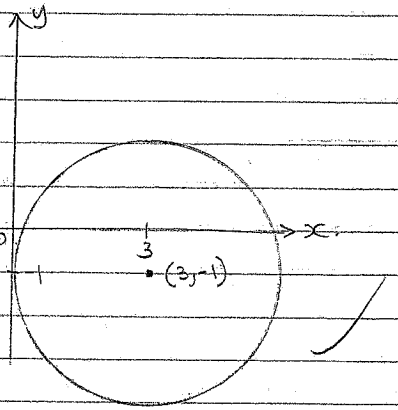
$y^2 + x^2 = 2$



(vi) $x = y^2$.



(vii) $(x-3)^2 + (y+1)^2 = 9$.

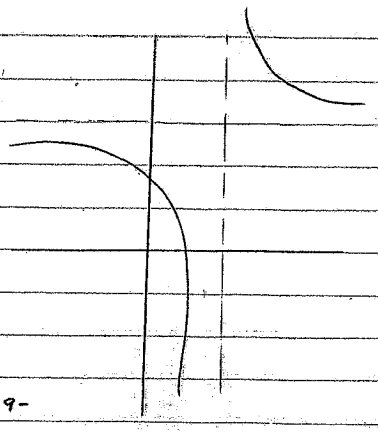


(viii) $y = \frac{1}{x-2} + 9$

=

y asymt $\longrightarrow$ 9.

x asym $\longrightarrow$ 2.



e) odd function

$-f(x) = f(-x)$.

$-(\sin x) = \sin(-x)$

$-\sin x = \sin -x$.

~~eg: $\sin 60°$~~

~~$-(\sin 60°) = \sin -60°$~~

eg: $\sin 30°$

$-(\sin 30°) = \sin -30°$

$-\frac{1}{2} = -\frac{1}{2}$.

$\therefore$ odd function.

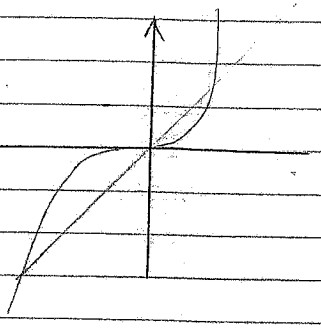
f) $f(-x) = -f(x)$

$f(-3) = 7$.

$f(-3) = -f(3)$

i) -7.

II) $y = f(x)$.



g) any function that is even is symmetrical about the y-axis

$y = x^2$.

HL

Ms Jagger

h) i). A ~~$f(-2) = 3$~~ $-2(-2) + b$.

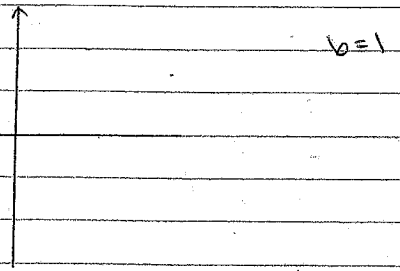
$4 + b$

B $2^0 = 1$

C. 2^{c^2}

2

ii).



0

i) $f(x) = \frac{1}{(x+1)(1-3x)}$

i) $x \neq -1$, ~~$\frac{1}{3}$~~

ii). $\frac{1}{(0+1)(1-3(0))}$

$= \frac{1}{1 \times 1}$

$= 1.$

1