

RANDWICK GIRLS TECHNOLOGY HIGH SCHOOL

YEAR 11 – MATHEMATICS/EXTENSION 1

TERM 1 – 2005 - ASSESSMENT

Time Allowed: 45 minutes

Instructions:

- Attempt all questions.
- All necessary working must be shown.
- Marks will be deducted for careless or badly arranged work.
- Begin each question on a new page.

Q1 — 9
Q2 — 10
Q3 — 10
29.

QUESTION 1: (10 marks)

MARKS

- (i) Simplify $4 - 2(x - 1)$ 1
- (ii) Express $25^{-\frac{1}{2}}$ as a rational number. 1
- (iii) Factorize completely: (a) $x^3 - 1$ 2
(b) $x^4 - 1$
- (iv) Find the values of x and y given $x - y\sqrt{3} = (5 - 2\sqrt{3})(2 - \sqrt{3})$ 2
- (v) Graph the solution set to $|5 - 2x| > 7$ on the real number line. 2
- (vi) Write $0.\dot{6}\dot{9}$ as a rational number. 2

QUESTION 2: (10 marks)

- (i) Solve $3x + 4 = 4x - 3$ 1
- (ii) Solve $2^x = 256$ 1
- (iii) Solve $\frac{2x + 3}{5x - 1} = \frac{2x + 1}{5x - 3}$ 2
- (iv) Evaluate correct to 2 decimal places:
- (a) $\frac{1}{4.7 + 1.3}$ (b) $\sqrt{\frac{0.9 + 1.4}{5.2 - 3.6}}$ 2

- (v) Solve the quadratic equation

$$3x^2 = 5x - 2$$

MARKS

2

- (vi) If $\frac{1}{c+x} + \frac{1}{c+y} = \frac{1}{c}$ prove that $xy = c^2$ 2

QUESTION 3: (10 marks)

- (i) Simplify $\frac{x^2 - x - 20}{x^2 - 25} \div \frac{x + 1}{x^2 + 5x}$ 2
- (ii) Solve $|2x - 5| = |1 - x|$ 2
- (iii) Rationalize the denominator of $\frac{1 + 3\sqrt{3}}{5 - 2\sqrt{3}}$ 2
- (iv) Simplify $2\sqrt{63} - \sqrt{28}$ 1
- (v) Evaluate a^4 given $a = 3\sqrt{2}$ 1
- (vi) Express as a single fraction (in simplest form) $\frac{y}{x^2 - xy} + \frac{1}{x}$ 2

Question 1.

i) $4 - 2(x-1)$
 $= 4 - 2x + 2$
 $= 6 - 2x$ ✓

$\frac{9}{10}$

ii) $25^{-\frac{1}{2}}$
 $= \frac{1}{25^{\frac{1}{2}}}$
 $= \frac{1}{\sqrt{25}}$
 $= \frac{1}{5}$ ✓

iii) a) $x^3 - 1$
 $= (x-1)(x^2 + x + 1)$ ✓

b) $x^4 - 1$
 ~~$(x-1)(x+1)(x+1)(x+1)$~~ $= [(x-1)(x+1)]^2 = (x^2-1)(x^2+1)$
 ~~$(x-1)(x+1)(x^2+2x+1)$~~ $= (x-1)(x+1)(x-1)(x+1)$
 $(x-1)(x+1)(x^2+1)$

iv) $x - y\sqrt{3} = (5 - 2\sqrt{3})(2 - \sqrt{3})$
 $= 10 - 5\sqrt{3} - 4\sqrt{3} + 2 \times 3$
 $= 10 - 9\sqrt{3} + 6$
 $x - y\sqrt{3} = 16 - 9\sqrt{3}$
 $x = 16$
 $y = 9$ ✓

v) $|5 - 2x| > 7$ $|5 - 2x| > 7$
 ~~$5 - 2x > 7$~~ $5 - 2x < -7$
 $5 - 7 > 2x$ $5 + 7 < 2x$
 $-2 > 2x$ $12 < 2x$
 $-\frac{2}{2} > x$ $\frac{12}{2} < x$
 $-1 > x$ $6 < x$
 $x < -1$ ✓ $x > 6$ ✓



Question 1 continued.

(vi) $0.\dot{6}\dot{9} = x$
 $x = 0.\dot{6}\dot{9}$
 $10x = 6.\dot{9}\dot{6}\dot{9}$
 $100x = 69.\dot{6}\dot{9}$
 $99x = 69.\dot{6}\dot{9} - 0.\dot{6}\dot{9}$
 $x = \frac{69}{99}$
 $x = \frac{23}{33}$ ✓

2

2

2

Question 2

(i) $3x + 4 = 4x - 3$
 $3x - 4x = -4 - 3$
 $-x = -7$
 $x = 7$ ✓

10/10

(ii) $2^x = 256$
 $2^x = 2^8$
 $x = 8$ ✓

(iii) $\frac{2x+3}{5x-1} = \frac{2x+1}{5x-3}$

$(2x+3)(5x-3) = (2x+1)(5x-1)$
 $10x^2 - 6x + 15x - 9 = 10x^2 - 2x + 5x - 1$
 $10x^2 - 10x^2 + 9x = 3x - 1 + 9$
 $9x - 3x = 8$
 $6x = 8$
 $x = \frac{8}{6}$
 $= 1\frac{1}{3}$ ✓

(iv) a) $\frac{1}{4-7+13}$
 $= \frac{1}{6}$
 $= 0.17$ ✓

b) $\sqrt{\frac{0.9+1.4}{5.2-3.6}}$
 $= \sqrt{\frac{2.3}{1.6}}$
 $= 1.20$ ✓

(v) $3x^2 = 5x - 2$
 $3x^2 - 5x + 2 = 0$

$\begin{array}{r|l} 3x & -1 \\ \times & -2 \\ \hline 3 & +2 \\ & -7 \end{array}$

Using quadratic formula

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $= \frac{5 \pm \sqrt{5^2 - 4 \times 3 \times 2}}{2 \times 3}$
 $= \frac{5 \pm \sqrt{1}}{6}$

$= 1 \text{ or } \left(\frac{4}{6}\right) = \frac{2}{3}$

Question 2 continued

(vi) $\frac{1}{c+x} + \frac{1}{c+y} = \frac{1}{c}$

$\frac{c+y + c+x}{(c+x)(c+y)} = \frac{1}{c}$

$\frac{2c+x+y}{(c+x)(c+y)} = \frac{1}{c}$

$c(2c+x+y) = (c+x)(c+y)$ ✓

$2c^2 + xc + cy = c^2 + cy + xc + xy$

$2c^2 - c^2 = cy - cy + xc - xc + xy$

$c^2 = xy$

$\therefore xy = c^2$ ✓

Question 3.

(I) $\frac{x^2 - x - 20}{x^2 - 25} = \frac{x+1}{x^2 + 5x}$ 10/10

$\frac{\cancel{(x-5)}(x+4)}{\cancel{(x-5)}(x+5)} \times \frac{x\cancel{(x+5)}}{x+1}$
 $= \frac{x(x+4)}{x+1}$ ✓

II) $|2x-5| = |1-x|$
 $2x-5 = 1-x$
 $2x+x = 1+5$
 $3x = 6$
 $x = 2$ ✓

$-2x+5 = -1+x$
 $-2x-x = -1-5$
 $-3x = -6$
 $x = 2$ ✓

$-2x+5 = 1-x$
 $-2x+x = 1-5$
 $-x = -4$
 $x = 4$ ✓

$2x-5 = -1+x$
 $2x-x = -1+5$
 $x = 4$ ✓

Check:
 $|2(\frac{-4}{3})-5| = |1+\frac{4}{3}|$
 $|\frac{-8}{3}-5| = |\frac{4}{3}|$
 $|\frac{-13}{3}| \neq |\frac{4}{3}|$ X

$|2(2)-5| = |1-2|$
 $|4-5| = |-1|$
 $1 = 1$ ✓

$|2(-4)-5| = |1+4|$
 $|-8-5| = |5|$
 $13 \neq 5$ X

$|2(4)-5| = |1-4|$
 $|8-5| = |-3|$
 $3 = 3$ ✓

Solution = $x = 2$ ✓
 $x = 4$ ✓

Question 3 continued.

III) $\frac{4+3\sqrt{3}}{5-2\sqrt{3}} \times \frac{5+2\sqrt{3}}{5+2\sqrt{3}}$

$\frac{(4+3\sqrt{3})(5+2\sqrt{3})}{25-4 \times 3}$
 $\frac{5+2\sqrt{3}+15\sqrt{3}+6 \times 3}{13}$

$\frac{23+17\sqrt{3}}{13}$ ✓

(IV) $2\sqrt{63} - \sqrt{28}$
 $= 2\sqrt{9 \times 7} - \sqrt{4 \times 7}$
 $= 6\sqrt{7} - 2\sqrt{7}$
 $= 4\sqrt{7}$ ✓

(V) $a^4 = (3\sqrt{2})^4$
 $= (\sqrt{9 \times 2})^4$
 $= (\sqrt{18})^4$
 $= (\sqrt{18})^2 \times (\sqrt{18})^2$
 $= 324$ ✓

(VI) $\frac{y}{x^2-xy} + \frac{1}{x}$

~~$\frac{y}{x^2-xy} + \frac{1}{x}$~~
 $= \frac{y}{x(x-y)} + \frac{1}{x}$
 $= \frac{y+1(x-y)}{x(x-y)}$

$= \frac{y+x-y}{x(x-y)}$
 $= \frac{x}{x(x-y)} = \frac{1}{x-y}$ ✓

$(x^2+1) \left(\frac{x^2-1}{x^2+1} \right)$

$(x^2-1)(x^2+1) = x^4 + x^2 - x^2 - 1 = x^4 - 1$

$(x^2-x+1)(x^2+x+1) = x^4 + 2x^3 + x^2 - x^2 - 2x - 1 = x^4 + 2x^3 - 2x - 1$
 $(x^2-1)(x^2-x+1) = x^4 - x^3 + x^2 - x^2 + x - 1 = x^4 - x^3 + x - 1$

$(x^2-1)(x^2-1) = (x^2-1)(x+1)(x-1)$
 $(x^2-2x+1)(x^2+2x+1) = x^4 + 2x^3 + x^2 - 2x^3 - 4x^2 + 2x + x^2 + 2x + 1 = x^4 - 2x^2 + 4x + 1$

$x^4 - 1 = (x-1)(x+1)(x+1)(x+1)$
 $= (x-1)(x+1)(x^2+x+1)$
 $= (x-1)(x^3+x^2+x+x^2+x+1)$
 $= (x-1)(x^3+2x^2+2x+1)$