RANDWICK GIRLS HIGH SCHOOL

Mathematics Extension 1
Term 3Assessment Task

Year 11 2005

Name ..Hui Hng Lim.......
Class...M1........

Time allowed 48 minutes
Reading Time 2 minutes
Approved calculators may be used
Start a new page for each question
Do not write on the back of the paper
Write neatly and clearly: marks can be deduced
For hard to read or badly organised work
Show all working out

Question 1.........15../16
Question 2........6½/7
Question 3.....12½./17

Total ....34../40

Well done.

Question 1 (Start a new page)

Marks

a. The point P(8, 7) divides the line AB externally in the ratio 4:1. If A has coordinates (–2, 1), find the coordinates of B. **2**

b. Find the acute angle between the lines $2x + y - 5 = 0$ and $2x - y + 5 = 0$ to the nearest minute. **3**

c. Differentiate $\dfrac{(x + 1)}{(2x - 3)}$ giving your answer in simplest form. **3**

d. Differentiate $y = x\sqrt{x + 1}$. giving your answer in simplest form. **4**

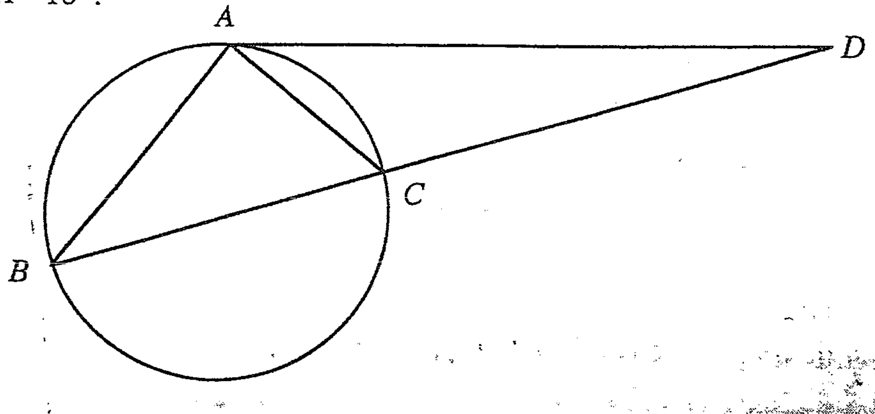
e. i. Sketch the curve $y = \dfrac{1}{x - 1}$, clearly showing the y-intercept, P. **2**

ii. Show that the equation of the tangent at P is given by **2**

$$x + y + 1 = 0$$

Question 2 (Start a new page)

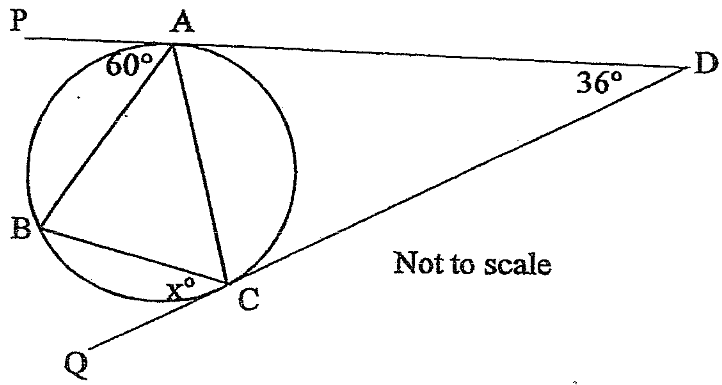
(a) ABC is a triangle inscribed in a circle. **4**
The tangent at A meets BC produced at D.
$D\hat{A}C = 40°$, $C\hat{D}A = 10°$.



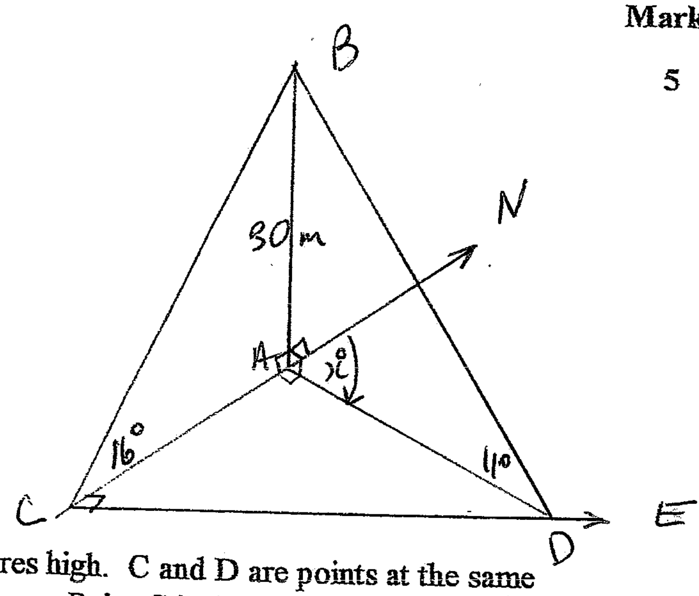
(i) Copy the diagram showing the above information.
(ii) Show that BC is a diameter of the circle.

b. The diagram shows tangents drawn from an external point D to touch the circle at A and C. $\angle PAB = 60°$ and $\angle ADC = 36°$. 3

Copy the diagram into your workbook and find the value of $\angle BCQ$, giving reasons.



Question 3 (Start a new page) | **Marks**



a. AB is a vertical tower 30 metres high. C and D are points at the same level as the base, A, of the tower. Point C is due south of A and point D is due east of C. The angles of elevation from C and D to the top, B, of the tower are 16° and 11° respectively. **5**

i. Draw a diagram to illustrate this information.

ii. Find the bearing of D from the tower to the nearest minute.

iii. Find the distance DC to the nearest metre.

b. i) Express $2\cos\theta + \sin\theta$ in the form $r\cos(\theta - \alpha)$ or where $r > o, 0^\circ \le \alpha \le 90^\circ$. **5**

ii) Hence or otherwise solve, to the nearest minute, the equation

$$2\cos\theta + \sin\theta = \sqrt{5} \text{ for } 0^\circ \le \theta \le 360^\circ$$

c. i. Show that $\dfrac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$ **2**

ii. Hence show that the exact value of $\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$ **2**

d) i) Write $\sin\theta + \cos\theta$ in terms of $\tan\dfrac{\theta}{2}$. **3**

ii) For $0^\circ \le \theta \le 360^\circ$ solve $\sin\theta + \cos\theta = -1$ using "t" substitution.

Hui Ling

Question 1.

15/16.

a.

A $(-2,1)$.

$x = \frac{mx_1 + nx_2}{m+n}$ $\quad$ $y = \frac{my_1 + ny_2}{m+n}$

$8 = \frac{4x_1 + 1(-2)}{5}$ $\quad \frac{4x_1 + -1(-2)}{3}$ $\quad 7 = \frac{4y_1 + 1(1)}{5}$ $\quad \frac{4y_1 + -1(1)}{3}$

Well done.

1

$40 = 4x_1 + -2$ $\quad$ $35 = 4y_1 + 1$

$40 + 2 = 4x_1$ $\quad 5\frac{1}{2} = x$ $\quad$ $\frac{34}{4} = y_1$ $\quad y = 5\frac{1}{2}$

$x_1 = 10.5$ $\quad$ $y_1 = 8.5$.

B $(10\frac{1}{2}, 8\frac{1}{2})$

b. $2x + y - 5 = 0$ $\quad$ $2x - y + 5 = 0$.

$m = \frac{-b}{a}$ $\quad$ $m = \frac{-b}{a}$

$= \frac{-2}{1}$ $\quad$ $= \frac{-2}{-1}$

$= -2$ $\quad$ $= 2$.

3

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{-2 - 2}{1 + 2 \times -2}\right|$

$= \left|\frac{-4}{-3}\right|$

$= 53°8'$

c. $\frac{(x+1)}{(2x-3)}$. $\quad \frac{u}{v}$ $\quad f'(x)\ \frac{vu' - uv'}{v^2}$

3

$f'(x)\ \frac{(2x-3)1 - (x+1)2}{(2x-3)^2}$

$f'(x) = \frac{2x - 3 - 2x - 2}{(2x-3)^2}$

$= \frac{-5}{(2x-3)^2}$

Question 1 continued

d. $y = x\sqrt{x+1}$

$= x(x+1)^{\frac{1}{2}}$.

$u = x$ $\quad$ $v = (x+1)^{\frac{1}{2}}$

$\frac{du}{dx} = 1$ $\quad$ $\frac{dv}{dx} = \frac{1}{2}(x+1)^{-\frac{1}{2}}$.

$= \frac{1}{2\sqrt{x+1}}$

$1(x+1)^{\frac{1}{2}} + \frac{x}{2\sqrt{x+1}}$ $\quad$ $2(x+1)^{\frac{1}{2}} \cdot (x+1)^{\frac{1}{2}} + x$

excellent.

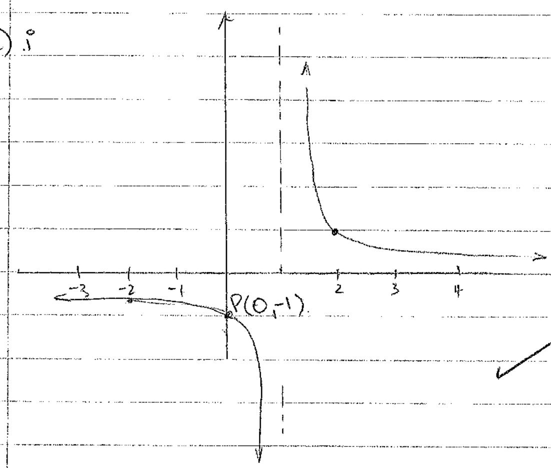
$= \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$

4

$= \frac{2x + 2 + x}{2\sqrt{x+1}}$

$= \frac{3x+2}{2\sqrt{x+1}}$.

e) i



$y = \frac{1}{x-1}$

$\lim_{x \to \infty} = \frac{1}{x-1}$.

$= \frac{\frac{1}{x}}{1 - \frac{1}{x}}$

$=$

2

II). $y = \frac{1}{x-1}$

$= (x-1)^{-1}$

$\frac{dy}{dx} = -1(x-1)^{-2}$

$= \frac{-1}{(x-1)^2}$

$= -1$

$y - y_1 = m(x - x_1)$

$y + 1 = -1(x - 0)$

$y + 1 = -x$

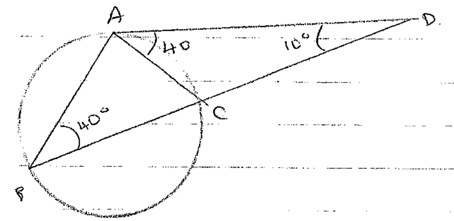
$0 = x + y + 1$

2

Auling.

Question 2. $6\frac{1}{2}/7$.

a).



$3\frac{1}{2}/4$.

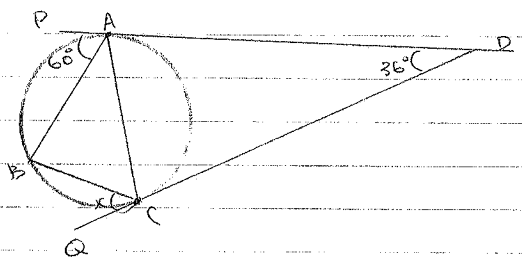
$\angle DAC = \angle ABC$ ($\angle$ between tangent & chord = $\angle$ in alternate segment)
$= 40°$

$\angle ACB = 40+10$ (ext $\angle$ of $\triangle$ = $\angle$ ~~of~~ sum of int. opp $\angle$)
$= 50°$

$\therefore \angle BAC = 180-40-50$ ($\angle$ sum of $\triangle = 180°$)
$= 90°$

$\therefore$ BC is diameter of $\odot$ ($\angle$ in semi circle $= 90°$)

b)



3

$\angle ACB = 60°$ ($\angle$ between tangent & chord = $\angle$ in alternate segment)

$AD = CD$ (tangents meet at ext pt are =)

$\therefore \angle DAC = \angle DCA = \frac{180-36}{2}$

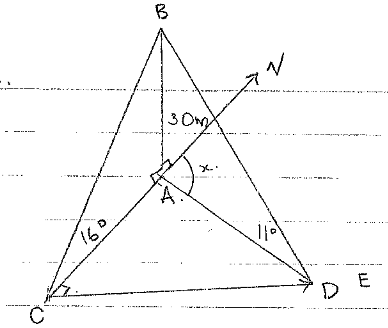
$= 72°$

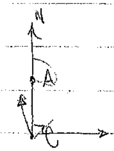
$\therefore \angle BCQ = 180-72-60°$ (supplementary $\angle = 180°$)
$= 48°$

Huiling

Question 3. $12\frac{1}{2}/17$.

a i





1

II. ~~AC~~ $\tan 16° = \frac{30}{AC}$
$= 104.62$ m.

$AD = \tan 11 = \frac{30}{AD}$
$= 154.34$ m.

2

~~$\angle CAD = \cos A = \frac{b^2+c^2-a^2}{}$~~

$\angle CAD = \cos A = \frac{adj}{hyp}$

$= \frac{104.62}{154.34}$

$= 47°19'$.

$180 - 47°19' = 132°41'$ ~~132°4~~ $\underline{132°41'}$

III ~~$\cos\ a^2 = b^2+c^2-2bc\cos A$~~

$\sin 47°19' = \frac{opp}{154.34}$

113.44 m

Opp $= 113°27'$

1½

Not an angle.

b. I $2\cos\theta + \sin\theta$.

$R = \sqrt{a^2+b^2}$
$= \sqrt{2^2+1^2}$
$= \sqrt{5}$

$\tan\alpha = \frac{b}{a}$
$= \frac{1}{2}$
$= 26°34'$

2

$r\cos(\theta-\alpha)$

$\sqrt{5}\cos(\theta - 26°34')$

Q3 continued

3

II. $\sqrt{5} = \sqrt{5}\cos(\theta - 26°34')$

$\cos(\theta - 26°34') = 1$

$\theta - 26°34' = 0°, 360°$

$= 26°34'$

well done

C. I. $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x.$

1/2

using $\cos x = \frac{1-t^2}{1+t^2}$ $\quad t = \tan\frac{\theta}{2}$ $\quad \tan\theta = \frac{2t}{1+t^2}$

$\cos 2x = \tan\frac{\theta}{2}$

$\frac{1-\cos^2 x + \sin^2 x}{1+\cos^2 x - \sin^2 x} = \frac{}{2\cos^2 x}$

Look at LHS only. $\frac{\sin^2 x + \sin^2 x}{\cos^2 x + \cos^2 x}$

$= \tan^2 x.$

$=$

II. $\tan 22\frac{1}{2}° = \sqrt{2} - 1.$

0/2

$\tan\frac{45}{2} = \sqrt{2} - 1$

$\frac{1}{2} = \sqrt{2} - 1.$

$\frac{1-\cos(2\times 22\frac{1}{2})}{1+\cos(2\times 22\frac{1}{2})}$

$= \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}+1} = (\sqrt{2}-1)^2 = \tan^2 22\frac{1}{2}.$

$\tan 22\frac{1}{2} = \sqrt{2} - 1$

d. i. $\sin\theta + \cos\theta.$ $\quad \tan\frac{\theta}{2} =$

1

$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}$

$= \frac{2t+1-t^2}{1+t^2}$

$= \frac{-t^2+2t+1}{1+t^2}.$

II. $-t^2 + 2t + 1 = -1 - t^2$

1

$2t = -2$

$t = -1$

$= \cancel{225},$

$\tan\frac{\theta}{2} = t$

$\tan\frac{\theta}{2} = -1$

$\cancel{\theta = 22\frac{1}{2}}$

$= \cancel{202\frac{1}{2}°, 337\frac{1}{2}°}.$

Q3 cont.

test $180°$

$\sin 180° + \cos 180 = -1$

~~Solution: $\theta = 202\frac{1}{2}°, 337\frac{1}{2}°, 180°$~~

0

$\tan\frac{\theta}{2} = -1$

$\theta = 90°$ X

Solution $\theta = 90°, 180°, 270°.$ X.

$\frac{\theta}{2} = 135°$

$\theta = 270°$

$$\frac{1-\cos 2x}{1+\cos 2x}$$

~~1 - 2cos~~

$$= \frac{1-\cos^2 x + \sin^2 x}{1+\cos^2 x - \sin^2 x}$$

$$= \frac{2}{2\cos^2 x}$$

$=$.

R