NAME :

CLASS:

Randwick Girls High School
Year 12
EXTENSION 1 MATHEMATICS
Assessment 2

MARCH 2011

Time allowed : 50 minutes

Directions to candidates:
Attempt all questions.
Calculators may be used.
Show all working where possible.

Question	Marks
1	/ 10
2	/ 9
3	/ 14
Total	/ 33

Question 1. **(10 marks)**

a) (i) Use Simpson's rule with 5 function values to evaluate **3**

$$\int_0^4 \frac{\sqrt{144-9x^2}}{4}$$

(ii) The formula $A = \frac{\pi ab}{4}$ where $a = 4$ and $b = 3$, gives the exact value of the integral above. Comment on the accuracy of your answer from (i) compared to the exact answer. **1**

b) Integrate

(i) $\int \cos\left(\frac{x}{2}\right)dx$ **1**

(ii) $\int_0^1 \frac{1}{6}e^{3x}dx$ (leave your answer in exact form) **2**

c) Find y if $\frac{dy}{dx} = 2\sin 2x$ and $y = 3$ when $x = \frac{\pi}{4}$ **3**

Question 2. **(9 marks)**

a) Integrate using the substitution $u = 1 + t$ **4**

$$\int_0^1 \frac{t}{\sqrt{1+t}}dt$$

b) Evaluate $\lim_{x\to 0} \frac{\sin\left(\frac{3x}{4}\right)}{2x}$ **1**

c) Differentiate ln (tan3x) **2**

d) Find $\int \cos^2 2x\, dx$ **2**

Question 3. **(14 marks)**

a) For $y = 3\cos(2x - \frac{\pi}{4})$ $0 \le x \le 2\pi$ write down the period **3**

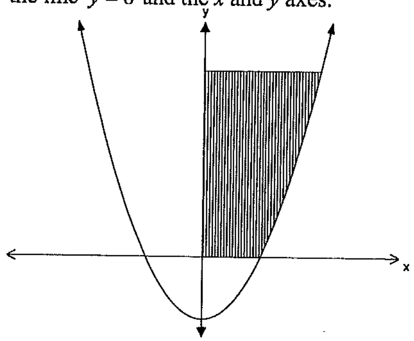
and amplitude then sketch the graph showing all important features.

b) Consider the functions $y = 3 - \frac{x}{2}$ and $y = \frac{1}{2}x^2 - 2x + 1$

Find the area between the curves. **4**

c) **4**

The diagram shows the region bounded by the curve $y = 2x^2 - 2$ the line $y = 6$ and the x and y axes.



Find the volume of the solid of revolution formed when the region is rotated about the y-axis. (answer correct to 3 significant figures)

d) Find the area of the minor segment with a radius of 6.2 cm and subtending an angle of 135 degrees at the centre. (answer to 2 decimal places) **3**

Q1 a) i) $y_0 = 3$

$y_1 = 2.9$ $\quad h = \frac{b-a}{n} = 1$

$y_2 = 2.6$

$y_3 = 2.0$

$y_4 = 0$ $\quad$ 1

$I \doteq \frac{1}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$

$\doteq \frac{1}{3}\left[(3+0) + 4(2.9+2) + 2(2.6)\right]$ 1

$\doteq 9.27$ 1

ii) $A = \frac{\pi(4)(3)}{4}$

$= 9.42$

Formula exact whereas Simpson's Rule is an approximation 1

i) $2\sin\left(\frac{x}{2}\right) + c$ 1

ii) $\frac{1}{6}\left(\frac{1}{3}\right)\left[e^{3x}\right]_0^1$ 1

$= \frac{1}{18}\left(e^3 - 1\right)$ 1 /2

$y' = 2\sin 2x$

$y = -\cos 2x + c$ 1

sub. $3 = -\cos\left(\frac{\pi}{2}\right) + c$

$\therefore c = 3$ 1

$y = 3 - \cos 2x$ 1 /3

Q2 a) $\int_0^1 \frac{t}{\sqrt{1+t}}\,dt$ $\quad du = dt$, $t = u - 1$, $t=0, u=1$, $t=1, u=2$

$= \int_1^2 \frac{u-1}{u^{\frac{1}{2}}}\,du$

$= \int_1^2 \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du$ 1

$= \left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^2$ 1

$= \left(\frac{2}{3}\cdot 2\sqrt{2} - 2\sqrt{2}\right) - \left(\frac{2}{3} - 2\right)$

$= \frac{1}{3}\left(4\sqrt{2} - 6\sqrt{2}\right) - \frac{1}{3}(2-6)$

$= \frac{2}{3}\left(2 - \sqrt{2}\right)$ 1 /4

b) $\lim_{x\to 0} \frac{\sin\left(\frac{3x}{4}\right)}{2x}$ $\quad \times \frac{3}{8}$ / $\times \frac{3}{8}$

$= \frac{3}{8}$ 1

c) $\frac{d}{dx}\ln(u)$ $\quad u = \tan 3x$, $u' = 3\sec^2 3x$ 1

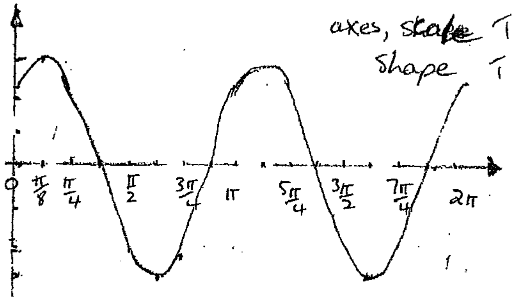
$= \frac{3\sec^2 3x}{\tan 3x}$ 1 /2

d) $= \frac{1}{2}\int(1 + \cos 4x)\,dx$ 1

$\frac{1}{2}x + \frac{1}{8}\sin 4x + c$ 1 /2

3. a) amp $= 3$ $\frac{1}{2}$

Period $= \frac{2\pi}{2} = \pi$ $\frac{1}{2}$ 1.



axes, scale 1

shape 1

$\frac{1}{2}x^2 - 2x + 1 = 3 - \frac{x}{2}$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = -1, 4$ 1

$A = \int_{-1}^{4}\left[\left(3 - \frac{x}{2}\right) - \left(\frac{1}{2}x^2 - 2x + 1\right)\right]dx$ 1

$= \int_{-1}^{4}\left(-\frac{1}{2}x^2 + \frac{3x}{2} + 2\right)dx$

$= \left[-\frac{x^3}{6} + \frac{3x^2}{4} + 2x\right]_{-1}^{4}$ 1

$= \left(-\frac{64}{6} + 12 + 8\right) - \left(\frac{1}{6} + \frac{3}{4} - 2\right)$

$= 10\frac{5}{12}$ units^2 1 /4

) $y = 2x^2 - 2$

$2x^2 = y + 2$

$x^2 = \frac{y}{2} + 1$

$x = \left(\frac{y}{2} + 1\right)^{\frac{1}{2}}$ 1

$V = \pi\int_0^6\left[\left(\frac{y}{2}+1\right)^{\frac{1}{2}}\right]^2 dy$ 1

$= \pi\left[\frac{y^2}{4} + y\right]_0^6$ 1

$= \pi\left[(9+6) - 0\right]$

$= 47.1$ units^3 1 /4

d) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}(6.2)^2\left(\frac{3\pi}{4} - \frac{1}{\sqrt{2}}\right)$ 1

$= 31.70$ units^2 1 /3