NAME : ____________________

CLASS: ____________________

Randwick Girls High School
Year 12
Mathematics

Assessment Task 3
June 2008

General Instructions

- Working Time – 50 minutes
- Write using a black or blue pen
- Approved calculators may be used
- All necessary working should be shown for every question.
- Work down the page, not across!
- START each of the three questions on a NEW page

Question	Mark
1	/ ~~13~~ 15
2	/ 12
3	/ 13
Total	/ ~~38~~ 40.

Question 1: (13 Marks) Marks

a) Differentiate $\left(e^x+e^{-x}\right)^2$ 1

b) Differentiate:

(i) $\log_e(3x+2)$ 1

(ii) $\log_e\dfrac{x^2}{x+1}$ 2

c) Find the area of the region enclosed by the curve $y=\log_e(x+1)$, the y-axis, and the line $y=3$. 3

d) Given $y=e^{\frac{x}{2}}+e^{\frac{-x}{2}}$

(i) Prove that this is an even function. What does this mean geometrically? 2

(ii) Find y', hence, find the coordinates of any turning point. 2

(iii) Find y'' and use it to determine the nature of the turning point. 2

(iv) Discuss the behaviour of the curve as x becomes positively and negatively large, hence, sketch the curve. 2

Question 2: (12 Marks)

a) An arithmetic series has a first term of 1 and a last term of 5. The sum of its terms is 6 times the last term. Find:

(i) the number of terms in the series 2

(ii) the common difference of the series 1

Continued over

Marks

b) One infinite geometric series has a first term of 3 and common ratio of r. Another has a first term of 5 and a common ratio of r^2. If they have the same limiting sum, find:

(i) the value of r, and — 2

(ii) the value of the limiting sum — 1

c) Kelly is going to deposit $205 000 in an account which will pay her interest of 1% each month. Immediately after each interest payment she will withdraw $W.

(i) Show that after she has made her second withdrawal, the balance of her account will be $\$\{205000(1.01)^2 - W(1+1.01)\}$ — 3

(ii) If she wishes to do this for a total period of 10 years, find, to nearest $1, the maximum amount she can withdraw. — 3

Question 3: (13 Marks)

a)

(i) Find the derivative of $x\sqrt{x+3}$ — 1

(ii) Hence find $\int \frac{x+2}{\sqrt{x+3}}dx$ — 2

b)

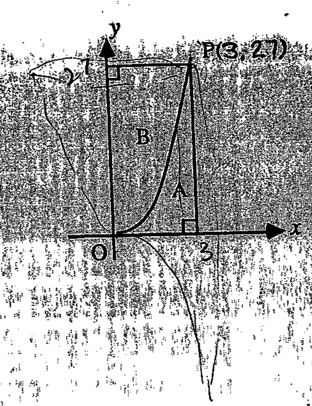
(i) Sketch, on the same number plane, graphs of $y=(x-3)^2$ and $y=9+8x-x^2$ indicating their points of intersection and the points where they meet the co-ordinate axes. — 3

(ii) Calculate the area enclosed between the two curves. — 3

Continued over

Marks

c)



The sketch shows the arc of the curve $y=x^3$ from the origin, O, to the point P(3,27). Calculate the volumes of the solids formed when:

(i) the region A makes a revolution about the x axis. — 2

(ii) the region B makes a revolution about the y axis. — 2

DP – 1
VJ – 2
JA – 3

Question 1 (13 Marks) EXPONENTIAL & LOGARITHMIC FUNCTIONS

$f(x) = (e^{x} + e^{-x})^2$

$f'(x) = 2(e^{x}+e^{-x})\frac{d}{dx}(e^{x}+e^{-x})$

$= \boxed{2(e^{x}+e^{-x})(e^{x}-e^{-x})}$ (1)

ii) a) $f(x) = \ln(3x+2)$

$f'(x) = \frac{1}{3x+2}\cdot\frac{d}{dx}(3x+2)$

$= \boxed{\frac{3}{3x+2}}$ (1)

b) $f(x) = \ln\left(\frac{x^2}{x+1}\right)$

$= \ln x^2 - \ln(x+1)$ (1)

$= 2\ln x - \ln(x+1)$

$\therefore f'(x) = \boxed{\frac{2}{x} - \frac{1}{x+1}}$ (1)

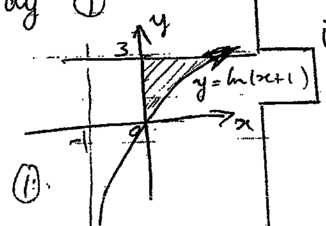
OR $\boxed{\frac{x+2}{x(x+1)}}$

$3^y = 9$
$y = \log_3 9$

c) $y = \ln(x+1)$

$e^{y} = x+1 \Rightarrow x = e^{y} - 1$ (1)

Area $= \int_0^3 (e^{y}-1)\,dy$ (1)



3 marks

$= \left[e^{y} - y\right]_0^3$ (1)

$= \left[e^{3}-3\right] - \left[e^{0}-0\right]$

$= \left[e^{3}-3-1\right]$

$= (e^{3}-4)$ units^2 (1) 3

d) i) $f(x) = e^{x/2} + e^{-x/2}$

$f(-x) = e^{-x/2} + (e^{x/2})$

$f(-x) = e^{x/2} + e^{-x/2}$ (1)

$f(x)$ is symmetrical about the y-axis. (1)

ii) $f'(x) = \frac{1}{2}e^{x/2} - \frac{1}{2}e^{-x/2}$ (1)

$= \frac{1}{2}(e^{x/2} - e^{-x/2})$

For turning pts put $y' = 0$

$\frac{1}{2}(e^{x/2} - e^{-x/2}) = 0$

$e^{x/2} = e^{-x/2}$

$\frac{x}{2} = -\frac{x}{2}$

$\frac{1}{2}(x+x) = 0$

ie $\begin{cases} x = 0 \\ y = 2 \end{cases}$

$\therefore$ (0,2) is a turning pt.

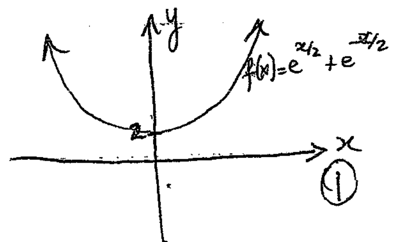
iii) $y'' = \frac{1}{4}e^{x/2} + \frac{1}{4}e^{-x/2}$ (1)

$y''(0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} > 0$ (1)

$\therefore$ turning point is a minimum

iv) As $x \to +\infty$, $y \to +\infty$ (½)

As $x \to -\infty$, $y \to +\infty$ (½)



(1)

13

Question 2: (12 Marks) SERIES

a) i) $T_1 = a = 1$ — (1)

$T_n = a + (n-1)d = 5$ — (2)

$S_n = \frac{n}{2}(a+l) = 6T_n = 30$

ie $\frac{n}{2}(1+5) = 30$ — (3)

From (3) $\boxed{\therefore n = 10}$ (2)

ii) substitute $\begin{cases} n = 10 \text{ in (2)} \\ a = 1 \end{cases}$

$1 + (10-1)d = 5$

$9d = 4$

$\boxed{\therefore d = \frac{4}{9}}$ (1)

b) series 1: $a = 3$, common ratio, r

series 2: $a = 5$, common ratio, r^2

$S_{1\infty} = S_{2\infty}$

$\therefore \frac{3}{1-r} = \frac{5}{1-r^2}$ (1)

$5 - 5r = 3 - 3r^2$

$3r^2 - 5r + 2 = 0$

$(3r-2)(r-1) = 0$ (1)

Either $r = 2/3$ or 1

But for series to have limiting sum, $r \neq 1$. $\boxed{\text{So } r = 2/3}$ (½)

$\therefore S_\infty = \frac{3}{1-2/3}$ or $\frac{5}{1-4/9}$

$\boxed{S_\infty = 9}$ (1) $S_\infty = 9$

c) i) Balance after 1st withdrawal

$B_1 = 205000(1.01) - W$ (1)

Balance after 2nd withdrawal

$B_2 = B_1 \times 1.01 - W$

$= \{205000(1.01) - W\}1.01 - W$

$= 205000(1.01)^2 - W(1.01) - W$

$= \$\{205000(1.01)^2 - W(1+1.01)\}$ (1)

ii) Balance after 120 withdrawals

$= 205000(1.01)^{120} - W(1+1.01+\ldots+1.01^{119})$ (1)

$= 205000(1.01)^{120} - W\left(\frac{1.01^{120}-1}{1.01-1}\right)$

For this to be zero

$\frac{W(1.01^{120}-1)}{0.01} = 205000(1.01)^{120}$ (1)

$\therefore W = \frac{205000(1.01)^{120}(0.01)}{1.01^{120}-1}$

$\boxed{= \$2941}$ (1)

Question 3: (13 Marks) INTEGRATION

a) i) $y = x\sqrt{x+3}$

$y' = 1.\sqrt{x+3} + x\left(\frac{1}{2}(x+3)^{-1/2}\right)$

$= \sqrt{x+3} + \dfrac{x}{2\sqrt{x+3}}$ (1)

i) or ii) $\sqrt{x+3} + \dfrac{x}{2\sqrt{x+3}}$

$= \dfrac{2(x+3)+x}{2\sqrt{x+3}}$

$= \dfrac{2x+6+x}{2\sqrt{x+3}}$

$= \dfrac{3x+6}{2\sqrt{x+3}}$

$= \dfrac{3(x+2)}{2\sqrt{x+3}}$ (1)

ii) Since $\frac{d}{dx}\left(x\sqrt{x+3}\right) = \frac{3}{2}\frac{(x+2)}{\sqrt{x+3}}$

$\therefore \int \dfrac{x+2}{\sqrt{x+3}}\,dx = \boxed{\frac{2}{3}\left[x\sqrt{x+3}\right]+C}$ (1)

OR $\boxed{\dfrac{2x\sqrt{x+3}}{3}+C}$

b) $y = (x-3)^2 = x^2-6x+9$ —(1)

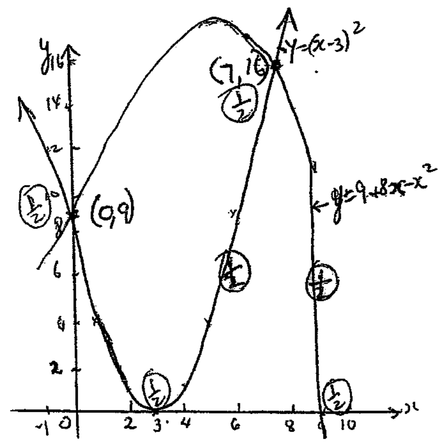
$y = 9+8x-x^2 = (9-x)(1+x)$ (2)

Now, $x^2-6x+9 = 9+8x-x^2$

$2x^2-14x = 0$

$2x(x-7) = 0$

$\begin{cases} x=0 \\ y=9 \end{cases}$, $\begin{cases} x=7 \\ y=16 \end{cases}$



Area $= \int_0^7 \left[(9+8x-x^2)-(x^2-6x+9)\right]dx$ (1)

$= \int_0^7 (-2x^2+14x)\,dx$

$= \left[-2\frac{x^3}{3} + 7x^2\right]_0^7$ (1)

$= \left[-\frac{2}{3}\times 7^3 + 7^3\right] - 0$

$= 114\frac{1}{3}$ units^2 (1)

c) i) $V_A = \pi\int_0^3 \left[x^3\right]^2 dx$ (1)

$= \pi\left[\frac{x^7}{7}\right]_0^3$

$= \frac{2187}{7}\pi$ units^3 (1)

$\doteqdot 982$ units^3

ii) If $y = x^3$

$\therefore x = y^{\frac{1}{3}}$

$V_B = \pi\int_0^{27}\left[y^{1/3}\right]^2 dy$ (1)

$= \left[\frac{3}{5}y^{5/3}\pi\right]_0^{27}$

$= 729\pi$ units^3 (1)