RANDWICK GIRLS HIGH SCHOOL

YEAR 11 MATHEMATICS EXTENSION 1
ASSESSMENT TASK 3

Examiner: D. Posener

Time Allowed: 1 ½ hours

Directions to Candidates:

- All questions may be attempted.
- In every question, show all necessary working.
- Marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Start each question on a new page.

QUESTION	MARK
Question 1	/15
Question 2	/22
Question 3	/31
TOTAL	/68

Question 1

(a) Find the equation of the line through $(4, -3)$ parallel to the join of $(-1, -5)$ and $(-6, 2)$ **3**

(b) Without finding the coordinates of the point of intersection T of the 2 lines $x - 2y - 5 = 0$ and $3x - y + 2 = 0$, find the equation of the straight line through T, and also passing through $(-2, -1)$ **4**

(c) Show that the line $4x + 3y + 18 = 0$ is a tangent to the circle centre $(-1, 2)$ and radius 4 units. **3**

(d) A is the point $(-3, -4)$ and B is the point $(2, -1)$. Find the coordinates of the point P dividing AB externally in the ratio $4 : 7$. **3**

(e) Find the acute angle, to the nearest minute, between the lines $x + y + 1 = 0$ and $2x - y + 4 = 0$ **2**

Question 2

(a) Evaluate the following limits:

(i) $\lim\limits_{x \to 2} \frac{x^2-4}{x-2}$ (ii) $\lim\limits_{x \to \infty} \frac{4x^2+5}{3x^2+8x}$ **2**

(b) Differentiate $f(x) = 2x^2 - 3x + 1$ from first principles. **4**

$4x - 3.$

(c) Differentiate with respect to x: **12**

(i) $y = 2x^3 - 4x^2 - 8x + 2$

(ii) $y = \frac{\sqrt{x}+x}{x^2}$

(iii) $y = \frac{7}{\sqrt[4]{x^3}}$

(iv) $y = (10 - 9x)^8$

(v) $y = 5x^2\sqrt{4x + 9}$

(vi) $y = \frac{x^2}{x+1}$

(d) (i) Find the equation of the tangent to the curve $y = x^2 + \frac{2}{x} + 4$ at the point $P(-1, 3)$ **4**

(ii) State the gradient of the normal at P.

Question 3

(a) Prove $\frac{1 - \sin\theta \cdot \cos\theta}{\cos\theta(\sec\theta - \operatorname{cosec}\theta)} \times \frac{\sin^2\theta - \cos^2\theta}{\sin^4\theta + \cos^3\theta \sin\theta}$ is a constant. **3**

(b) Find the exact value of $\cos 15°$ in its simplest form, with a rational denominator. **2**

(c) Given $\cos\theta = \frac{2}{\sqrt{13}}$ and θ is a reflex angle find **2**

(i) $\sin\theta$ (ii) $\sin 2\theta$

(d) Show that $\cot 2x = \frac{1}{2}\cot x - \frac{1}{2}\tan x$ **3**

(e) Show that $\tan(45° + 2\theta) = \sec 4\theta + \tan 4\theta$ **3**

(f) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ **3**

(g) Find the general solution of $\sin 2\theta = \cos\theta$ **3**

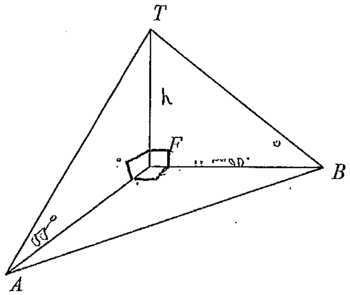
(h) (a) Write $\sin x - \cos x$ in the form $R\sin(x - \alpha)$ where $R > 0$ and α is acute. **5**

(b) Hence state the maximum value of $\sin x - \cos x$ and the smallest positive value of x for which this maximum occurs.

(c) Solve $\sin x - \cos x = 1$ for $0° \leq x \leq 360°$

(i) Given that $\cos\theta = \frac{3}{5}$ find the exact value of $\tan\frac{\theta}{2}$ **3**

(k) A tower of height h metres is standing on level ground. The angles of elevation of the top T of the tower from 2 points A and B on the ground nearby are 55° and 40° respectively. The distance AB is 500 metres and the interval BF is perpendicular to the interval AF, where F is the foot of the tower. **4**



(i) Find AF and BF in terms of h

(ii) Find h correct to the nearest cm.

Question One. SOLUTIONS - RGHS - 2011

$\frac{2+(5)}{-6-(-1)} = \frac{2+5}{-6+1}$

$m_1 = 7/-5$

$m_1 = m_2 = 7/-5$

$y - y_1 = m(x - x_1)$

3 $y+3 = -7/5(x-4)$

$-5y - 15 = 7x - 28$

$\therefore 7x + 5y - 13 = 0$

$(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$

$(-2 + 2(1) - 5) + k(-6 + 1 + 2) = 0$

$= -5 + k(-3) = 0$

$= -6 - 3k = 0$

$= 3k = -6, -5$

$k = -2$ ✗ $k = -5/3$

$(x - 2y - 5) - 2(3x - y + 2) = 0$

$x - 2y - 5 - 6x + 2y - 4 = 0$

$-5x - 9 = 0$

$\therefore 5x + 9 = 0$ ✗

) $\frac{|4(-1) + 3(2) + 18|}{\sqrt{4^2 + 3^2}}$

$= \frac{|-4 + 6 + 18|}{\sqrt{25}}$

$= \frac{20}{5}$

$= 4$

$\therefore$ Perpendicular distance of line is equivalent to 4, and is a tangent to

3 circle of radius 4 units.

) $\frac{kx_2 + lx_1}{k + l}, \frac{ky_2 + ly_1}{k + l}$

$= \frac{4(2) - 7(-3)}{4 - 7} = \frac{4(-1) - 7(-4)}{4 - 7}$

$= \frac{8 + 21}{-3} \quad = \frac{-4 + 28}{-3}$

$= -29/3 \quad = -24/3$

$y = -8$

3 $\therefore P = (-9\tfrac{2}{3}, -8)$

) $x + y + 1 = 0 \quad 2x - y + 4 = 0$

$y = -x - 1 \quad 2x + 4 = y$

$m_1 = -1 \quad m_2 = 2$

$\tan\theta = \left|\frac{2 - (-1)}{1 + 2 \cdot -1}\right|$

$\tan\theta = \left|\frac{3}{-1}\right|$

$\tan\theta = |-3|$

$\tan\theta = 3$

$\theta = 71°34'$

2

11/15

Question Two

) (i) $\lim_{x \to 2} \frac{x^2-4}{x-2}$

$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$\lim_{x \to 2} x+2$

$\lim \to 2 \quad 2+2$

$= 4$

$\therefore 4$

i) $\lim_{x \to \infty} \frac{4x^2+5}{3x^2+8x}$

$\lim_{x \to \infty} \frac{\frac{4x^2}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{8x}{x^2}}$

$\lim_{x \to \infty} \frac{4+0}{3+0}$

$\lim_{x \to \infty} \frac{4}{3}$

$\therefore 4/3$

) $\frac{f(x+h)-f(x)}{h}$ $\quad f(x) = 2x^2 - 3x + 1$

$\frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$

$= \frac{2(x^2+2hx+h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$

$= \frac{\cancel{2x^2} + 4hx + 2h^2 \cancel{-3x} - 3h \cancel{+1} \cancel{-2x^2} + \cancel{3x} \cancel{-1}}{h}$

$\lim_{h \to 0} = \frac{4hx + 2h^2 - 3h}{h}$ $\quad \to \quad 4x - 3$

$\lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}}$ $\quad \therefore 4x - 3$

$\lim_{h \to 0} 4x + 2(0) - 3$

Question Two

(i) $2x^3 - 4x^2 - 8x + 2$

$\frac{dy}{dx} = 6x^2 - 8x - 8$

(ii) $\frac{x^{1/2} + x}{x^2}$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{x^2\left(\left(\frac{1}{2}x^{-1/2}\right)+(1)\right) - (x^{1/2}+x).2x}{x^4}$

$= \frac{\frac{x^2}{2\sqrt{x}} - 2x^{3/2} + 2x^2}{x^4}$

$= \frac{\frac{x}{2} - \frac{4x^{3/2} + 4x^2}{2}}{x^4}$

$= \frac{x - 4\sqrt{x^3} + 4x^2}{2} \times \frac{1}{x^4}$

$= \frac{x - 4\sqrt{x^3} + 4x^2}{2x^4}$

7

iii) $7x^{-(4/3)}$ $7x^{-3/4}$ Be careful!

$\frac{dy}{dx}$ minus

$= -4/3 . 7 (x)^{-7/3}$ correct process

$= -28/3 \, x^{-7/3}$

$= \frac{-28}{3\sqrt[3]{x^7}}$

iv) $\frac{dy}{dx} = 8(10-9x)^7 \cdot -9$

$= -72(10-9x)^7$

v) $5x^2(4x+9)^{1/2}$

$\frac{dy}{dx} = (4x+9)^{1/2} \, 10x + 5x^2(4x+9)^{-1/2} \times 1/2 \times 4$

$= 10(4x+9)^{1/2} + 10x^2(4x+9)^{-1/2}$

$= 10x(4x+9)^{-1/2}[4x+9+x]$

$= 10x(4x+9)^{-1/2}(5x+9)$

$= \frac{10x(5x+9)}{\sqrt{4x+9}}$

6

) $y = \frac{x^2}{x+1}$

$\frac{dy}{dx} = \frac{(x+1).2x - x^2(1)}{(x+1)^2}$

$= \frac{2x^2+2x-x^2}{(x+1)^2}$

$= \frac{x^2+2x}{(x+1)^2}$

(b)

)(i) $y = x^2 + \frac{2}{x} + 4$

$= x^2 + 2x^{-1} + 4$

$\frac{dy}{dx} = 2x - 2x^{-2}$

$= 2x - \frac{2}{x^2}$

At $x = -1$

$2(-1) - \frac{2}{(-1)^2}$

$= -2 - 2$

$m_1 = -4$

$y - y_1 = m(x - x_1)$

$y - 3 = -4(x+1)$

$y - 3 = -4x - 4$

$\therefore 4x + y + 1 = 0$

(ii) $m_1 = -4$

$m_1 \times m_2 = -1$

$-4m_2 = -1$

$m_2 = -1/-4$

at normal $= 1/4$

Question Three

5

) $\frac{1 - \sin\theta\cos\theta}{\cos\theta(\sec\theta - \operatorname{cosec}\theta)} \times \frac{\sin^2\theta - \cos^2\theta}{\sin^4\theta + \cos^3\theta\sin\theta}$

$= \frac{(1-\sin\theta\cos\theta)(\cos\theta - \sin\theta)}{\cos\theta} \times \frac{\sin^2\theta - (1-\sin^2\theta)}{\sin\theta(\sin^3\theta + \cos^3\theta)}$

$= \frac{(1-\sin\theta\cos\theta)(\cos\theta - \sin\theta)}{\cos\theta} \times \frac{\sin^2\theta - 1 + \sin^2\theta}{\sin\theta(\sin^3\theta + \cos^3\theta)}$

$= \frac{\cos\theta - \sin\theta - \sin\theta\cos^2\theta + \sin^2\theta\cos\theta}{\cos\theta} \times \frac{2\sin^2\theta - 1}{\sin\theta(\sin^3\theta + \cos^3\theta)}$

CORRECTIONS :-

LHS $= \frac{1 - \sin\theta\cos\theta}{\cos\theta\left(\frac{1}{\cos\theta} - \frac{1}{\sin\theta}\right)} \times \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta(\sin^3\theta + \cos^3\theta)}$

$\frac{1 - \sin\cos\theta}{\cos\theta\left(\frac{\sin\theta - \cos\theta}{\cos\theta\sin\theta}\right)} \times \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta(\sin\theta + \cos)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}$

$\frac{1 - \sin\cos\theta}{\frac{\sin\theta - \cos\theta}{\sin\theta}} \times \frac{\sin\theta - \cos\theta}{\sin\theta(1 - \sin\theta\cos\theta)}$

$= 1$

) $\cos 15^\circ$

$\cos(45-30)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{2\sqrt{2}}$

$= \frac{\cancel{2}\sqrt{6} + \cancel{2}\sqrt{2}}{\cancel{8}}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$.

7

)3 (i) $(\sqrt{13})^2 - 2^2$

$13 - 4 = 9$

$\sqrt{9} = 3$

$\therefore \sin\theta = \frac{-3}{\sqrt{13}}$

(ii) $2\sin\theta\cos\theta = \sin 2\theta$

$\frac{2}{\sqrt{13}} \times \frac{-3}{\sqrt{13}} \times 2$

$= \frac{-6}{13} \times 2$

$\therefore = \frac{-12}{13}$

) $\cot 2x = \frac{1}{2}\cot x - \frac{1}{2}\tan x$

$RHS = \frac{1}{2}\cot x - \frac{1}{2}\tan x$

$= \frac{1}{2\tan x} - \frac{\tan x}{2}$

$= \frac{\cos x}{2\sin x} - \frac{\sin x}{2\cos x}$

$= \frac{2\cos^2 x - 2\sin^2 x}{4\sin x\cos x}$

$= \frac{\cancel{2}(\cos^2 x - \sin^2 x)}{\cancel{4}\sin x\cos x}$

$= \frac{\cos^2 x - \sin^2 x}{2\sin x\cos x}$

$= \frac{\cos 2x}{\sin 2x}$

$= \cot 2x$

$= LHS$

$\tan(45+2\theta) = \sec 4\theta + \tan 4\theta$

~~$LHS = \tan(45+2\theta)$~~

) $\frac{\tan 45 + \tan 2\theta}{1 - \tan 45 \tan 2\theta}$

$= \frac{1+\tan 2\theta}{1 - \tan 2\theta}$

$= \frac{1 + \frac{2\tan\theta}{1-\tan^2\theta}}{1 - \frac{2\tan\theta}{1-\tan^2\theta}}$

$= \frac{\frac{1-\tan^2\theta + 2\tan\theta}{1-\tan^2\theta}}{\frac{1-\tan^2\theta - 2\tan\theta}{1-\tan^2\theta}}$

$= \frac{1-\tan^2\theta + 2\tan\theta}{1-\tan^2\theta} \times \frac{1-\tan^2\theta}{1-\tan^2\theta - 2\tan\theta}$

$= \frac{1-\tan^2\theta + 2\tan\theta}{1-\tan^2\theta - 2\tan\theta}$

$= \frac{-(\tan^2\theta - 2\tan\theta - 1)}{-(\tan^2\theta + 2\tan\theta - 1)}$

1

$RHS = \sec 4\theta + \tan 4\theta$

$= 2\sec 2\theta + 2\tan 2\theta$

$= \frac{2}{\cos 2\theta} + 2\tan 2\theta$

$= \frac{2}{\cos^2\theta - \sin^2\theta} + 2\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$= \frac{2}{\cos^2\theta - \sin^2\theta} + \frac{4\tan\theta}{1-\tan^2\theta}$

$=$

$RHS = \sec 4\theta + \tan 4\theta$

$\frac{1}{\cos 4\theta} + \frac{\sin 4\theta}{\cos 4\theta}$

$= \frac{1+\sin 4\theta}{\cos 4\theta}$

$= \frac{1 + 2\sin 2\theta}{\cos 4\theta}$

$= \frac{1 + 2\sin 2\theta\cos 2\theta}{(\cos 2\theta + \sin 2\theta)(\cos 2\theta - \sin 2\theta)}$

$= \frac{\cos^2 2\theta + \sin^2 2\theta + 2\sin 2\theta\cos 2\theta}{(\cos 2\theta + \sin 2\theta)(\cos 2\theta - \sin 2\theta)}$

$\frac{(\cos 2\theta + \sin 2\theta)^2}{(\cos 2\theta + \sin 2\theta)(\cos 2\theta - \sin 2\theta)}$

$= \frac{(\cos 2\theta + \sin 2\theta)}{(\cos 2\theta - \sin 2\theta)}$

$= \frac{\frac{\cos 2\theta}{\cos 2\theta} + \frac{\sin 2\theta}{\cos 2\theta}}{\frac{\cos 2\theta}{\cos 2\theta} - \frac{\sin 2\theta}{\cos 2\theta}}$

$= \frac{1 + \tan 2\theta}{1 - \tan 2\theta}$

$= LHS$

f) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\cos(2\theta + \theta)$

$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$

$2\cos^2\theta - 1$

$= \cancel{(1-\sin^2\theta)}\cos\theta - 2\cos\theta\sin^2\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta\sin^2\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$

$=$ RHS

4

g) $\sin 2\theta = \cos\theta$

$\sin 2\theta - \cos\theta = 0$

$2\sin\theta\cos\theta - \cos\theta = 0$

$= \sqrt{\sin 2\theta}$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\therefore \cos\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\dfrac{2\sin\theta\cos\theta}{\cos\theta} = \dfrac{\cos\theta}{\cos\theta}$ ≠ 0

$2\sin\theta = 1$

$\sin\theta = 1/2$ ✓ $\cos\theta = 0$

$\therefore 180n + (-1)^n 30°$ ✓   or $2n(180°) \pm 90°$

h) (a) $\sin x - \cos x$

$R\sin(x - \alpha)$

$R(\sin x\cos\alpha - \cos x\sin\alpha)$

$\sqrt{1^2+1^2} = \sqrt{2}$

$\sqrt{2}(\sin x \cdot 1/\sqrt{2} - \cos x \cdot 1/\sqrt{2})$

$\cos\alpha = 1/\sqrt{2}$, $\sin\alpha = 1/\sqrt{2}$ } $\alpha = 45°$

2

$R\sin(x - 45°)$

$\therefore = \sqrt{2}\sin(x - 45°)$ ✓

(b) $\sqrt{2}\sin(x-45) = 1$

$\sin(x-45) = 1/\sqrt{2}$

1 Maximum $= \sqrt{2}$ ✓

Smallest value $= 45°$ ✗ $135°$

c) $\sqrt{2}\sin(x-45) = 1$

$\sin(x-45) = 1/\sqrt{2}$

1 $x - 45 = 45, 135$ ✓

$x = 90°, 180°$

i) $\cos\theta = 3/5$

$\dfrac{1-t^2}{1+t^2} = \dfrac{3}{5}$ $\cos\theta = 3/5$

$\theta = 53°8'$

1

2

$\dfrac{1 - \tan^2\theta/2}{1 + \tan^2\theta/2} = \dfrac{4}{3}$

$3(1 - \tan^2\theta/2) = 4(1 + \tan^2\theta/2)$

$3 - 3\tan^2\theta/2 = 4 + 4\tan^2\theta/2$

$7\tan^2\theta/2 = -1$

$\tan^2\theta/2 = -1/7$ ✗

$\tan\theta/2 = -\frac{1}{\sqrt{7}}$

(i) $AF = \frac{h}{\tan 55}$ $BF = \frac{h}{\tan 40^\circ}$

$= h \tan 35^\circ$ $= h \tan 50^\circ$

(ii) $500^2 = h^2 \tan^2 35^\circ + h^2 \tan^2 50^\circ$

$500^2 = h^2 (\tan^2 35 + \tan^2 50^\circ)$

$h = \frac{500}{\sqrt{\tan^2 35 + \tan^2 50}}$

$= 361.73m$

4