CEM - Solving difficult rates problems by using Unitary Method

Example 1: Suppose that it takes Sally 3 hours to mow a lawn, and it takes Tom 4 hours to mow the same lawn; Tom's mower is less powerful than Sally's. *Without* using algebra (x or other variables) determine how long it would take Sally and Tom to mow the lawn if they worked together (using both lawn mowers)? (Assume that each works at his/her standard speed and they never get in each others way.)

Method of Unitary Analysis.

Person	Hours worked	Fraction of job done
Sally	3	1
Sally	1	1/3
Tom	4	1
Tom	1	1/4
Sally and Tom	1	1/3 + 1/4 = 7/12
Sally and Tom	12	12(7/12) = 7
Sally and Tom	12/7	7/7 = 1

Answer. It would take Sally and Tom 12/7 hours to mow the lawn if they worked together.

Example 2: It takes Sally 3 hours to walk from her home to Tom's home. It takes Tom 4 hours to walk from his home to Sally's home. They walk the same road. Suppose they both leave their own homes at noon, walking at their standard constant speeds toward each other. At what time do they meet? (The distance between their homes, who lives uphill from the other and their standard constant speeds shall remain unknown.)

Person	Hours walked	Fraction of road walked
Sally	3	1
Sally	1	1/3
Tom	4	1
Tom	1	1/4
Sally and Tom	1	1/3 + 1/4 = 7/12
Sally and Tom	12	12(7/12) = 7
Sally and Tom	12/7	7/7 = 1

Now try the following questions:

(1) Suppose that it takes Tom and Dick 2 hours to do a certain job. But, today, a friend joins them and works at the same rate as Tom and Dick. How long will it take for the three men, together to do the same job?

Ans: 1⅓ hours

(2) 12 men take 20 days to renovate a house. Assuming that they work at the same rate, find

(a) how many days would it take 15 men to renovate a similar house?

Ans: 16 days

(b) how many men would be needed to renovate two such houses in 30 days?

Ans: 16 men

(3) If it takes 16 taps 10 hours to fill 8 pools, how long will it take 12 taps to fill 9 pools?

Ans: 15 hours

(4) Machine A can produce 80 bolts per minute while Machine B can produce only 50 bolts per minute. Only Machine A is used initially to make the bolts but it breaks down after eight and a half minutes. Machine B is then used to complete the job. If the job requires 1000 bolts, how long does the whole operation take?

Ans: 14.9 mins

(5) Jane takes 5 hours to sew buttons on a batch of uniforms. Oprah will take 7 hours to sew buttons on a similar batch of uniforms. How long will it take both of them, working together to complete sewing the buttons on the similar batch of uniforms? Give your answer in hours and minutes.

Ans: 2 hr 55 mins