18 Surds

Question 1 Simplify the following:

(a) $-\sqrt{8}$

(b) $\sqrt{200}$

(c) $\sqrt{98}$

(d) $\sqrt{288}$

(e) $\sqrt{63}$

(f) $\sqrt{7x^2}$

(g) $\sqrt{5n^6}$

(h) $\sqrt{2m^4n^2}$

(i) $\sqrt{8x^2}$

(j) $6\sqrt{18}$

(k) $4\sqrt{56}$

(l) $5\sqrt{68}$

Question 2 Simplify:

(a) $7\sqrt{7} - 3\sqrt{7} - \sqrt{7}$

(b) $10\sqrt{2} - \sqrt{2} + 2\sqrt{2}$

(c) $5\sqrt{3} + \sqrt{3} - 3\sqrt{2}$

(d) $\sqrt{p} + 3\sqrt{p}$

(e) $6\sqrt{x} - \sqrt{x} + 2\sqrt{x}$

(f) $7\sqrt{y} - 2\sqrt{y}$

(g) $3m\sqrt{n} - m\sqrt{n}$

(h) $a\sqrt{x} + b\sqrt{x}$

(i) $2a\sqrt{x} + a\sqrt{x} - b\sqrt{x}$

Question 3 Simplify by collecting like terms:

(a) $3\sqrt{2} + 2\sqrt{3} + 5\sqrt{2}$

(b) $3\sqrt{3} + 5\sqrt{2} + \sqrt{3} - 2\sqrt{2}$

(c) $7\sqrt{5} - \sqrt{3} - 2\sqrt{5} + 4\sqrt{3}$

(d) $7\sqrt{3} + 8\sqrt{5} - 3\sqrt{5} - 9\sqrt{3}$

(e) $3\sqrt{m} - \sqrt{n} + 2\sqrt{m} + 5\sqrt{n}$

(f) $p\sqrt{q} - a\sqrt{b} + q\sqrt{q} - b\sqrt{b}$

Question 4 Simplify completely:

(a) $\sqrt{50} + 3\sqrt{2}$

(b) $\sqrt{72} + \sqrt{18}$

(c) $\sqrt{98} + \sqrt{50}$

(d) $2\sqrt{32} + \sqrt{98}$

(e) $3\sqrt{11} - \sqrt{44}$

(f) $\sqrt{121y} - 2\sqrt{9y}$

Question 5 Simplify completely:

(a) $3\sqrt{8} - \sqrt{18} + 5\sqrt{2}$

(b) $\sqrt{54} + \sqrt{75} + \sqrt{24} + \sqrt{48}$

(c) $6\sqrt{20} - 3\sqrt{128} - 2\sqrt{45} + \sqrt{512}$

(d) $\sqrt{200} + \sqrt{40} - \sqrt{72} + 2\sqrt{90}$

(e) $2\sqrt{9m} + \sqrt{16n} - \sqrt{36m} + 3\sqrt{4n}$

(f) $\sqrt{x^3} + m\sqrt{4y} - \sqrt{9x} + \sqrt{y^3}$

Question 6 Simplify completely:

(a) $3\sqrt{2} \times 5\sqrt{3}$

(b) $6\sqrt{7} \times \sqrt{14}$

(c) $\sqrt{10} \div \sqrt{2}$

(d) $\sqrt{48} \div \sqrt{3}$

(e) $\sqrt{45} \div \sqrt{15}$

(f) $\sqrt{18} \div \sqrt{2}$

(g) $3\sqrt{8} \div 2\sqrt{2}$

(h) $3\sqrt{50} \div \sqrt{10}$

(i) $3\sqrt{6} \times 2\sqrt{27}$

(j) $\sqrt{45} \div \sqrt{3}$

(k) $\sqrt{x} \times 2\sqrt{x}$

(l) $\sqrt{8x} \div \sqrt{2}$

(m) $12\sqrt{18} \div 3\sqrt{6}$

(n) $2\sqrt{x} \times \sqrt{x^3}$

(o) $\sqrt{8x} \times \sqrt{16y}$

Question 7 Expand and simplify:

(a) $\sqrt{a}\left(2 + \sqrt{a}\right)$

(b) $\sqrt{2}\left(\sqrt{8} - \sqrt{2}\right)$

(c) $2\sqrt{3}\left(\sqrt{3} + \sqrt{27}\right)$

(d) $2\sqrt{7}\left(\sqrt{7} - 2\sqrt{2}\right)$

(e) $4\sqrt{2}\left(\sqrt{3} + \sqrt{2}\right)$

(f) $4\sqrt{3}\left(\sqrt{2} - \sqrt{5}\right)$

(g) $\frac{1}{\sqrt{3}}\left(\sqrt{6} - \sqrt{3}\right)$

(h) $3\sqrt{x}\left(2\sqrt{x} + 3\right)$

(i) $2\sqrt{xy}\left(5\sqrt{x} + 3\sqrt{y}\right)$

Question 8 Simplify completely:

(a) $\dfrac{2\sqrt{3}\times 8\sqrt{6}}{4}$

(b) $\dfrac{3\sqrt{5}\times 2\sqrt{15}}{10}$

(c) $\dfrac{\sqrt{21}\times\sqrt{7}}{7\sqrt{3}}$

(d) $\dfrac{4\sqrt{6}\times 5\sqrt{5}}{10\sqrt{15}}$

(e) $\dfrac{6\sqrt{2}\times\sqrt{6}}{3\sqrt{3}}$

(f) $\dfrac{\sqrt{24}\times\sqrt{72}}{\sqrt{8}\times 3\sqrt{6}}$

Question 9 Factorise the following:

(a) $\sqrt{3}+\sqrt{6}$

(b) $2\sqrt{5}+\sqrt{10}$

(c) $2\sqrt{3}+\sqrt{8}$

(d) $\sqrt{a^3}+3\sqrt{a}$

(e) $\sqrt{24}-\sqrt{18}$

(f) $15\sqrt{10}+5\sqrt{15}$

Question 10 Simplify the following by first factorising:

(a) $\dfrac{\sqrt{3}+\sqrt{6}}{\sqrt{3}}$

(b) $\dfrac{2\sqrt{5}-\sqrt{10}}{\sqrt{5}}$

(c) $\dfrac{3\sqrt{2}+\sqrt{8}}{2\sqrt{2}}$

(d) $\dfrac{6\sqrt{10}+8\sqrt{15}}{2\sqrt{5}}$

(e) $\dfrac{12-\sqrt{18}}{3}$

(f) $\dfrac{\sqrt{x^3}+3\sqrt{x}}{\sqrt{x}}$

Question 11 Expand and simplify:

(a) $\left(2\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$

(b) $\left(\sqrt{7}+2\right)\left(2\sqrt{7}-5\right)$

(c) $\left(5\sqrt{3}+1\right)\left(5\sqrt{3}-2\right)$

(d) $\left(4\sqrt{3}+\sqrt{2}\right)\left(3\sqrt{3}-2\sqrt{2}\right)$

(e) $\left(\sqrt{2}+1\right)\left(\sqrt{3}-2\right)$

(f) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{c}+\sqrt{d}\right)$

(g) $\left(2\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}+2\sqrt{2}\right)$

(h) $\left(2\sqrt{x}+\sqrt{y}\right)\left(3\sqrt{x}-2\sqrt{y}\right)$

(i) $\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)$

(j) $\left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)$

(k) $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$

(l) $\left(3\sqrt{6}-\sqrt{5}\right)\left(3\sqrt{6}+\sqrt{5}\right)$

(m) $\left(3\sqrt{7}+2\sqrt{5}\right)\left(3\sqrt{7}-2\sqrt{5}\right)$

(n) $\left(\sqrt{3}+\sqrt{2}\right)^2$

(o) $\left(3-\sqrt{5}\right)^2$

(p) $\left(3\sqrt{7}+\sqrt{10}\right)^2$

(q) $\left(\sqrt{11}-\sqrt{5}\right)\left(\sqrt{11}+\sqrt{5}\right)$

(r) $\left(3\sqrt{2}-2\sqrt{3}\right)^2$

(s) $\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)$

(t) $\left(3\sqrt{m}+2\sqrt{n}\right)^2$

(u) $\left(2\sqrt{a}+5\sqrt{b}\right)\left(2\sqrt{a}-5\sqrt{b}\right)$

Question 12 **Rationalise the denominator for each of the following:**

(a) $\dfrac{3}{\sqrt{2}}$

(b) $\dfrac{2}{\sqrt{7}}$

(c) $\dfrac{7}{2\sqrt{5}}$

(d) $\sqrt{\dfrac{7}{12}}$

(e) $\dfrac{\sqrt{5}}{\sqrt{18}}$

(f) $\dfrac{3+\sqrt{5}}{3\sqrt{2}}$

(g) $\dfrac{\sqrt{10}-\sqrt{5}}{2\sqrt{10}}$

(h) $\dfrac{1}{\sqrt{2}-1}$

(i) $\dfrac{3}{\sqrt{5}+2}$

(j) $\dfrac{20}{\sqrt{5}-\sqrt{2}}$

(k) $\dfrac{1}{2\sqrt{5}-4}$

(l) $\dfrac{1}{3\sqrt{2}+2\sqrt{3}}$

(m) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

(n) $\dfrac{5+\sqrt{2}}{2\sqrt{2}+\sqrt{3}}$

(o) $\dfrac{3\sqrt{5}+2\sqrt{7}}{3\sqrt{5}-2\sqrt{7}}$

Question 13 Express the following as single fractions by first rationalising each denominator:

(a) $\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}$

(b) $\dfrac{1}{\sqrt{7}}-\dfrac{1}{\sqrt{5}}$

(c) $\dfrac{3}{\sqrt{10}}+\dfrac{1}{\sqrt{5}}$

(d) $\dfrac{\sqrt{2}}{\sqrt{5}}-\dfrac{\sqrt{7}}{\sqrt{2}}$

(e) $\dfrac{1}{5-\sqrt{2}}+\dfrac{1}{5+\sqrt{2}}$

(f) $\dfrac{1}{\sqrt{5}+\sqrt{3}}-\dfrac{1}{\sqrt{7}+\sqrt{5}}$

Question 14

(a) The square root of 3 less than a certain number is 11. Find the number.

(b) Seven times the square root of a number is equal to the sum of twice the square root of the number and 30. Find the number.

(c) The square root of the sum of two consecutive numbers is 7.

Find the odd number of the pair.

(d) When Leonardo da Vinci dropped a stone from a window in the leaning tower of Pisa, he found it took $3\frac{1}{4}$ seconds to reach the ground. If the time (t seconds) to fall h metres is given by $t = \frac{13}{8\sqrt{2}}\sqrt{h}$,

what was the height of the window?

SOLUTIONS

18 Surds

1. (a) $-2\sqrt{2}$ (b) $10\sqrt{2}$ (c) $7\sqrt{2}$
   (d) $12\sqrt{2}$ (e) $3\sqrt{7}$ (f) $\sqrt{7}x$
   (g) $\sqrt{5}n^3$ (h) $\sqrt{2}m^2n$ (i) $2\sqrt{2}\,x$
   (j) $18\sqrt{2}$ (k) $8\sqrt{14}$ (l) $10\sqrt{17}$

2. (a) $3\sqrt{7}$ (b) $11\sqrt{2}$ (c) $6\sqrt{3}-3\sqrt{2}$
   (d) $4\sqrt{p}$ (e) $7\sqrt{x}$ (f) $5\sqrt{y}$
   (g) $2m\sqrt{n}$ (h) $(a+b)\sqrt{x}$ (i) $(3a-b)\sqrt{x}$

3. (a) $8\sqrt{2}+2\sqrt{3}$ (b) $4\sqrt{3}-3\sqrt{2}$ (c) $5\sqrt{5}+3\sqrt{2}$
   (d) $5\sqrt{5}-2\sqrt{3}$ (e) $5\sqrt{m}+4\sqrt{n}$
   (f) $(p+q)\sqrt{q}-(a+b)\sqrt{b}$

4. (a) $8\sqrt{2}+2\sqrt{3}$ (b) $9\sqrt{2}$ (c) $12\sqrt{2}$
   (d) $15\sqrt{2}$ (e) $\sqrt{11}$ (f) $5\sqrt{y}$

5. (a) $8\sqrt{2}$ (b) $5\sqrt{6}+9\sqrt{3}$
   (c) $6\sqrt{5}-8\sqrt{2}$ (d) $4\sqrt{2}+8\sqrt{10}$
   (e) $10\sqrt{n}$ (f) $(x-3)\sqrt{x}+(2m+y)\sqrt{y}$

6. (a) $15\sqrt{6}$ (b) $42\sqrt{2}$ (c) $\sqrt{5}$
   (d) 4 (e) $\sqrt{3}$ (f) 3
   (g) 3 (h) $3\sqrt{5}$ (i) $54\sqrt{2}$
   (j) $\sqrt{15}$ (k) $2x$ (l) $2\sqrt{x}$
   (m) $4\sqrt{3}$ (n) $2x^2$ (o) $8\sqrt{2xy}$

7. (a) $2\sqrt{a}+a$ (b) 2 (c) 24
   (d) $14-4\sqrt{14}$ (e) $4\sqrt{6}+8$
   (f) $4\sqrt{6}-4\sqrt{15}$ (g) $\sqrt{2}-1$
   (h) $6x+9\sqrt{x}$ (i) $10x\sqrt{y}+6y\sqrt{x}$

8 (a) $12\sqrt{2}$ (b) $3\sqrt{3}$ (c) 1 (d) $2\sqrt{2}$ (e) 4 (f) 2

9 (a) $\sqrt{3}\left(1+\sqrt{2}\right)$ (b) $\sqrt{5}\left(2+\sqrt{2}\right)$
   (c) $2\left(\sqrt{3}+\sqrt{2}\right)$ (d) $\sqrt{a}(a+3)$
   (e) $\sqrt{2}\left(2\sqrt{3}-3\right)$ (f) $5\sqrt{5}\left(3\sqrt{2}+\sqrt{3}\right)$

10 (a) $1+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $2\frac{1}{2}$
   (d) $3\sqrt{2}+4\sqrt{3}$ (e) $4-\sqrt{2}$ (f) $x+3$

11 (a) $4+\sqrt{6}$ (b) $4-\sqrt{7}$ (c) $73-5\sqrt{3}$
   (d) $32-5\sqrt{6}$ (e) $\sqrt{6}-2\sqrt{2}+\sqrt{3}-2$
   (f) $\sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$ (g) $10+5\sqrt{6}$
   (h) $6x-\sqrt{xy}-2y$ (i) 4 (j) 2
   (k) 1 (l) 49 (m) 43
   (n) $5+2\sqrt{6}$ (o) $14-6\sqrt{5}$ (p) $73+6\sqrt{70}$
   (q) 6 (r) $30-12\sqrt{6}$ (s) 6
   (t) $9m+12\sqrt{mn}+4n$ (u) $4a-25b$

12 (a) $\frac{3\sqrt{2}}{2}$ (b) $\frac{2\sqrt{7}}{7}$ (c) $\frac{7\sqrt{5}}{10}$
   (d) $\frac{\sqrt{21}}{6}$ (e) $\frac{\sqrt{10}}{6}$ (f) $\frac{3\sqrt{2}+\sqrt{10}}{6}$
   (g) $\frac{2-\sqrt{2}}{4}$ (h) $\sqrt{2}+1$ (i) $3(\sqrt{5}-2)$
   (j) $20(\sqrt{5}+\sqrt{2})$ (k) $\frac{\sqrt{5}+2}{2}$
   (l) $\frac{3\sqrt{2}-2\sqrt{3}}{6}$ (m) $\frac{5+2\sqrt{3}}{2}$
   (n) $\frac{10\sqrt{2}-5\sqrt{3}+4-\sqrt{6}}{5}$ (o) $\frac{73+12\sqrt{35}}{17}$

13 (a) $\frac{3\sqrt{2}+2\sqrt{3}}{6}$ (b) $\frac{5\sqrt{7}-7\sqrt{5}}{35}$ (c) $\frac{3\sqrt{10}+2\sqrt{5}}{10}$
   (d) $\frac{2\sqrt{10}-5\sqrt{14}}{10}$ (e) $\frac{10}{23}$ (f) $\frac{2\sqrt{5}-\sqrt{3}-\sqrt{7}}{2}$

14 (a) $11+\sqrt{3}$ (b) 36 (c) 25 (d) 8 m