

NAME : \_\_\_\_\_

CLASS : \_\_\_\_\_

## SIMPLIFYING SURDS

A surd is expressed in its simplest form when the number under the radical sign is as small as possible.

*Example:*

$$\begin{aligned}\sqrt{20} &= \sqrt{4} \times \sqrt{5} \\ &= 2\sqrt{5}\end{aligned}$$

### EXERCISE 2.15

Write the following surds in their simplest form.

- |                         |                            |                           |                           |                            |
|-------------------------|----------------------------|---------------------------|---------------------------|----------------------------|
| 1. $\sqrt{8}$           | 7. $\sqrt{28}$             | 13. $\sqrt{80}$           | 19. $\sqrt{40}$           | 25. $\sqrt{160}$           |
| 2. $\sqrt{20}$          | 8. $\sqrt{18}$             | 14. $\sqrt{500}$          | 20. $\sqrt{162}$          | 26. $\sqrt{192}$           |
| 3. $\sqrt{12}$          | 9. $\sqrt{45}$             | 15. $\sqrt{125}$          | 21. $\sqrt{108}$          | 27. $\sqrt{450}$           |
| 4. $\sqrt{24}$          | 10. $\sqrt{63}$            | 16. $\sqrt{54}$           | 22. $\sqrt{72}$           | 28. $\sqrt{96}$            |
| 5. $\sqrt{32}$          | 11. $\sqrt{48}$            | 17. $\sqrt{150}$          | 23. $\sqrt{245}$          | 29. $\sqrt{242}$           |
| 6. $\sqrt{\frac{2}{9}}$ | 12. $\sqrt{\frac{10}{25}}$ | 18. $\sqrt{\frac{3}{16}}$ | 24. $\sqrt{\frac{27}{4}}$ | 30. $\sqrt{\frac{72}{25}}$ |

## ADDITION AND SUBTRACTION OF SURDS

Like surds can be added or subtracted.

*Example:*

$$5\sqrt{3} + \sqrt{3} - 2\sqrt{3} = 4\sqrt{3}$$

In general before attempting to add or subtract surds each surd should be reduced to its simplest form.

*Example:*

$$\begin{aligned}\sqrt{27} + \sqrt{108} - \sqrt{12} &= 3\sqrt{3} + 6\sqrt{3} - 2\sqrt{3} \\ &= 7\sqrt{3}\end{aligned}$$

### EXERCISE 2.16

Simplify:

- |  |  |  |
|--|--|--|
| 1. $4\sqrt{3} - 2\sqrt{3} + 5\sqrt{3}$   | 11. $\sqrt{128} - \sqrt{98} + \sqrt{72}$ | 21. $\sqrt{80} + 2\sqrt{245}$                |
| 2. $7\sqrt{2} - 9\sqrt{2} + 5\sqrt{2}$   | 12. $\sqrt{75} + \sqrt{32} - \sqrt{27}$  | 22. $\sqrt{128} + 3\sqrt{18} - \sqrt{162}$   |
| 3. $\sqrt{8} + \sqrt{32}$                | 13. $\sqrt{50} + \sqrt{63} - \sqrt{28}$  | 23. $4\sqrt{45} - 3\sqrt{54} - 6\sqrt{5}$    |
| 4. $\sqrt{12} + \sqrt{27}$               | 14. $\sqrt{180} + \sqrt{28} - 2\sqrt{5}$ | 24. $3\sqrt{8} + 5\sqrt{18} - 9\sqrt{2}$     |
| 5. $\sqrt{18} + \sqrt{50}$               | 15. $4\sqrt{48} - 5\sqrt{27}$            | 25. $7\sqrt{24} - 4\sqrt{5} + 2\sqrt{6}$     |
| 6. $\sqrt{75} - \sqrt{48}$               | 16. $2\sqrt{54} + 3\sqrt{150}$           | 26. $3\sqrt{32} + 2\sqrt{50} - 8\sqrt{18}$   |
| 7. $\sqrt{20} + \sqrt{125} - \sqrt{45}$  | 17. $3\sqrt{24} - 5\sqrt{54}$            | 27. $\sqrt{128} + 4\sqrt{75} - \sqrt{162}$   |
| 8. $\sqrt{27} + \sqrt{108} - \sqrt{12}$  | 18. $\sqrt{12} + 2\sqrt{48}$             | 28. $2\sqrt{363} + 5\sqrt{243} + \sqrt{192}$ |
| 9. $\sqrt{125} + \sqrt{5} - \sqrt{80}$   | 19. $\sqrt{147} - 4\sqrt{3}$             | 29. $\sqrt{a^3} + 2\sqrt{a}$                 |
| 10. $\sqrt{63} - \sqrt{28} - \sqrt{112}$ | 20. $3\sqrt{125} - 2\sqrt{80}$           | 30. $\sqrt{x^3} + \sqrt{x} - \sqrt{4x}$      |

## PRODUCTS INVOLVING SURDS

Examples:

$$(i) \sqrt{3}(\sqrt{3} + 5\sqrt{2}) = 3 + 5\sqrt{6}$$

$$(ii) (\sqrt{5} - \sqrt{2})(\sqrt{3} + \sqrt{2}) = \sqrt{5}(\sqrt{3} + \sqrt{2}) - \sqrt{2}(\sqrt{3} + \sqrt{2}) \\ = \sqrt{15} + \sqrt{10} - \sqrt{6} - 2$$

$$(iii) (\sqrt{3} + \sqrt{5})^2 = 3 + 2\sqrt{15} + 5 \\ = 8 + 2\sqrt{15}$$

### EXERCISE 2.17

Write the following products in their simplest form.

- |  |  |  |
|--|--|--|
| 1. $\sqrt{3} \times \sqrt{3}$                    | 13. $\sqrt{2}(\sqrt{8} - \sqrt{2})$              | 25. $(\sqrt{3} + \sqrt{2})^2$                        |
| 2. $(\sqrt{5})^2$                                | 14. $\sqrt{3}(\sqrt{3} + 2\sqrt{3})$             | 26. $(\sqrt{5} + 3)^2$                               |
| 3. $3\sqrt{3} \times 5\sqrt{3}$                  | 15. $\sqrt{7}(3\sqrt{7} - \sqrt{2})$             | 27. $(\sqrt{7} - \sqrt{2})^2$                        |
| 4. $\sqrt{5} \times \sqrt{6}$                    | 16. $2\sqrt{3}(4\sqrt{2} - 5\sqrt{3})$           | 28. $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$     |
| 5. $\sqrt{7} \times \sqrt{3}$                    | 17. $2\sqrt{7}(\sqrt{3} + 6\sqrt{2})$            | 29. $(2\sqrt{3} + \sqrt{2})(3\sqrt{3} - \sqrt{2})$   |
| 6. $\sqrt{2} \times 3\sqrt{6}$                   | 18. $\sqrt{2}(\sqrt{8} + \sqrt{32})$             | 30. $(2\sqrt{7} + 1)^2$                              |
| 7. $4\sqrt{5} \times 2\sqrt{2}$                  | 19. $(3\sqrt{2})^2$                              | 31. $(3\sqrt{5} - 2\sqrt{3})^2$                      |
| 8. $10\sqrt{3} \times 5\sqrt{11}$                | 20. $3\sqrt{3}(\sqrt{8} + 2\sqrt{2})$            | 32. $(\sqrt{7} + 1)(\sqrt{7} - 1)$                   |
| 9. $2\sqrt{3} \times 3\sqrt{2} \times \sqrt{3}$  | 21. $2\sqrt{6}(3\sqrt{3} - 5\sqrt{2})$           | 33. $\sqrt{2}(\sqrt{18} + \sqrt{8} - 4\sqrt{2})$     |
| 10. $2\sqrt{7} \times \sqrt{63}$                 | 22. $(\sqrt{3} + 4)(\sqrt{3} - 2)$               | 34. $(2\sqrt{5} - 3)(2\sqrt{5} + 3)$                 |
| 11. $\sqrt{27} \times \frac{1}{3}\sqrt{3}$       | 23. $(\sqrt{2} + \sqrt{3})(\sqrt{5} + \sqrt{3})$ | 35. $(\sqrt{7} - 2\sqrt{3})(\sqrt{5} - 3\sqrt{2})$   |
| 12. $\sqrt{8} \times \sqrt{5} \times \sqrt{125}$ | 24. $(\sqrt{7} - \sqrt{2})(\sqrt{3} + \sqrt{5})$ | 36. $(3\sqrt{5} - 2\sqrt{2})(2\sqrt{5} + 3\sqrt{2})$ |

## THE PRODUCT OF TWO CONJUGATE SURDS

Consider the product

$$(5 - \sqrt{2})(5 + \sqrt{2}) = 5^2 - (\sqrt{2})^2 \\ = 23.$$

The factors  $(5 - \sqrt{2})$  and  $(5 + \sqrt{2})$  multiply together to give a *rational* number 23.

Because of this  $(5 - \sqrt{2})$  and  $(5 + \sqrt{2})$  are called conjugate surds. Each is the conjugate of the other.

Here are two other examples of pairs of conjugate surds multiplying to give a rational product:

$$(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}) = (\sqrt{7})^2 - (\sqrt{3})^2 = 7 - 3 = 4 \\ (2\sqrt{3} - 1)(2\sqrt{3} + 1) = (2\sqrt{3})^2 - 1^2 = 12 - 1 = 11$$

### EXERCISE 2.18

1. Find the product of each pair of conjugate surds.

- |  |  |  |
|--|--|--|
| (a) $(3 + \sqrt{2})(3 - \sqrt{2})$               | (h) $(\sqrt{7} - \sqrt{2})(\sqrt{7} + \sqrt{2})$   | (o) $(7 + 2\sqrt{3})(7 - 2\sqrt{3})$                 |
| (b) $(\sqrt{5} - 1)(\sqrt{5} + 1)$               | (i) $(\sqrt{11} + \sqrt{5})(\sqrt{11} - \sqrt{5})$ | (p) $(5\sqrt{3} + 3\sqrt{2})(5\sqrt{3} - 3\sqrt{2})$ |
| (c) $(7 - \sqrt{3})(7 + \sqrt{3})$               | (j) $(3\sqrt{2} - 1)(3\sqrt{2} + 1)$               | (q) $(4\sqrt{11} - 5)(4\sqrt{11} + 5)$               |
| (d) $(\sqrt{6} - 2)(\sqrt{6} + 2)$               | (k) $(2\sqrt{7} - 3)(2\sqrt{7} + 3)$               | (r) $(2\sqrt{7} - \sqrt{13})(2\sqrt{7} + \sqrt{13})$ |
| (e) $(4 + \sqrt{5})(4 - \sqrt{5})$               | (l) $(6 - 2\sqrt{2})(6 + 2\sqrt{2})$               | (s) $(9\sqrt{2} + 5\sqrt{3})(9\sqrt{2} - 5\sqrt{3})$ |
| (f) $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$ | (m) $(2\sqrt{5} + 2)(2\sqrt{5} - 2)$               | (t) $(6\sqrt{7} + \sqrt{3})(6\sqrt{7} - \sqrt{3})$   |
| (g) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$ | (n) $(\sqrt{12} - 4)(\sqrt{12} + 4)$               |  |

2. Multiply each expression by its conjugate and check that the product is rational.

- (a)  $\sqrt{2} - 1$       (d)  $\sqrt{8} - 2$       (g)  $2\sqrt{3} + 5$       (j)  $3\sqrt{7} - \sqrt{5}$       (m)  $8\sqrt{2} - 2\sqrt{3}$   
 (b)  $5 - \sqrt{3}$       (e)  $6 + \sqrt{3}$       (h)  $6\sqrt{2} - 1$       (k)  $2\sqrt{5} + \sqrt{3}$       (n)  $4\sqrt{5} + 5\sqrt{2}$   
 (c)  $\sqrt{7} + 2$       (f)  $\sqrt{11} - \sqrt{5}$       (i)  $5 - 2\sqrt{2}$       (l)  $5\sqrt{8} - \sqrt{3}$       (o)  $2\sqrt{11} - 3\sqrt{5}$

## FRACTIONS WITH SURDS IN THE DENOMINATOR

To simplify a fraction such as  $\frac{8}{\sqrt{2}}$  the denominator is changed into a rational number. This is called rationalising the denominator.

Example (i):

$$\begin{aligned} \frac{8}{\sqrt{2}} &= \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} && \text{(What number law is this?)} \\ &= \frac{8\sqrt{2}}{2} \\ &= 4\sqrt{2} \end{aligned}$$

Example (ii):

$$\begin{aligned} \frac{6}{5\sqrt{3}} &= \frac{6}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{6\sqrt{3}}{15} \\ &= \frac{2\sqrt{3}}{5} \end{aligned}$$

### EXERCISE 2.19

Simplify the following by rationalising the denominator in each case.

- |                          |                                  |                                    |   |
|--------------------------|----------------------------------|------------------------------------|---|
| 1. $\frac{3}{\sqrt{2}}$  | 6. $\frac{\sqrt{3}}{\sqrt{6}}$   | 11. $\frac{1}{4\sqrt{5}}$          | 16. $\frac{\sqrt{2} + 1}{\sqrt{3}}$           |
| 2. $\frac{4}{\sqrt{5}}$  | 7. $\frac{4}{3\sqrt{7}}$         | 12. $\frac{\sqrt{12}}{5\sqrt{3}}$  | 17. $\frac{\sqrt{3} - 1}{\sqrt{3}}$           |
| 3. $\frac{1}{\sqrt{3}}$  | 8. $\frac{\sqrt{7}}{\sqrt{14}}$  | 13. $\frac{2\sqrt{5}}{3\sqrt{10}}$ | 18. $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{6}}$    |
| 4. $\frac{7}{\sqrt{7}}$  | 9. $\frac{\sqrt{10}}{2\sqrt{5}}$ | 14. $\frac{7\sqrt{3}}{2\sqrt{6}}$  | 19. $\frac{3\sqrt{2} + \sqrt{5}}{2\sqrt{5}}$  |
| 5. $\frac{4}{2\sqrt{2}}$ | 10. $\frac{3\sqrt{2}}{\sqrt{8}}$ | 15. $\frac{4\sqrt{2}}{3\sqrt{8}}$  | 20. $\frac{4\sqrt{3} - 2\sqrt{2}}{3\sqrt{2}}$ |

Example:

$$\begin{aligned}\frac{3}{\sqrt{5}-2} &= \frac{3}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} \\ &= \frac{3(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} \\ &= \frac{3\sqrt{5}+6}{1} \\ &= 3\sqrt{5}+6\end{aligned}$$

(Why is the value of the original fraction unchanged?)

## EXERCISE 2.20

Simplify the following fractions.

1.  $\frac{1}{\sqrt{2}+1}$

6.  $\frac{6}{\sqrt{3}-1}$

11.  $\frac{\sqrt{7}}{\sqrt{7}+1}$

16.  $\frac{3-\sqrt{2}}{3+\sqrt{2}}$

2.  $\frac{1}{2+\sqrt{3}}$

7.  $\frac{5}{\sqrt{5}+\sqrt{7}}$

12.  $\frac{\sqrt{6}}{3\sqrt{6}-\sqrt{2}}$

17.  $\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}+\sqrt{2}}$

3.  $\frac{1}{\sqrt{3}-\sqrt{2}}$

8.  $\frac{1}{2\sqrt{5}-1}$

13.  $\frac{1}{4\sqrt{5}+3\sqrt{7}}$

18.  $\frac{3\sqrt{2}-1}{3\sqrt{2}+1}$

4.  $\frac{3}{\sqrt{5}+\sqrt{3}}$

9.  $\frac{5}{5+2\sqrt{6}}$

14.  $\frac{2\sqrt{5}}{3\sqrt{5}-2\sqrt{2}}$

19.  $\frac{3+2\sqrt{5}}{2\sqrt{5}-1}$

5.  $\frac{2}{2\sqrt{2}-1}$

10.  $\frac{\sqrt{3}}{4\sqrt{3}+5}$

15.  $\frac{\sqrt{2}}{3\sqrt{8}-\sqrt{12}}$

20.  $\frac{4\sqrt{7}+2}{2\sqrt{7}+1}$

## DECIMAL APPROXIMATIONS

For many practical purposes a decimal approximation is required rather than an answer in the exact surd form.

## EXERCISE 2.21

Use your calculator to find decimal approximations for the following. Give each answer correct to 3 decimal places.

1.  $\sqrt{108}$

6.  $\frac{\sqrt{7}-\sqrt{3}}{2}$

11.  $\frac{8}{\sqrt{3}}$

16.  $\frac{1}{\sqrt{3}+\sqrt{2}}$

2.  $\sqrt{96}$

7.  $5\sqrt{3}+8\sqrt{2}$

12.  $\frac{6}{4\sqrt{7}}$

17.  $\frac{6}{7-2\sqrt{5}}$

3.  $\sqrt{450}$

8.  $(2+\sqrt{3})^2$

13.  $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{6}}$

18.  $\frac{\sqrt{5}}{\sqrt{5}-1}$

4.  $5+\sqrt{3}$

9.  $\sqrt{7}(\sqrt{5}+\sqrt{3})$

14.  $\frac{4\sqrt{2}-2\sqrt{3}}{3\sqrt{2}}$

19.  $\frac{5+2\sqrt{3}}{2\sqrt{3}-1}$

5.  $\sqrt{\frac{34}{27}}$

10.  $(3+\sqrt{6})(5-\sqrt{3})$

15.  $\frac{11-3\sqrt{7}}{6\sqrt{2}}$

20.  $\frac{5\sqrt{2}-1}{5\sqrt{2}+1}$

EXERCISE 2.15 (Page 27)

- |                  |                  |                           |
|------------------|------------------|---------------------------|
| 1. $2\sqrt{2}$   | 2. $2\sqrt{5}$   | 3. $2\sqrt{3}$            |
| 4. $2\sqrt{6}$   | 5. $4\sqrt{2}$   | 6. $\frac{\sqrt{2}}{3}$   |
| 7. $2\sqrt{7}$   | 8. $3\sqrt{2}$   | 9. $3\sqrt{5}$            |
| 10. $3\sqrt{6}$  | 11. $4\sqrt{3}$  | 12. $\frac{\sqrt{10}}{5}$ |
| 13. $4\sqrt{5}$  | 14. $10\sqrt{5}$ | 15. $5\sqrt{5}$           |
| 16. $3\sqrt{6}$  | 17. $5\sqrt{6}$  | 18. $\frac{\sqrt{3}}{4}$  |
| 19. $2\sqrt{10}$ | 20. $9\sqrt{2}$  | 21. $6\sqrt{3}$           |
| 22. $6\sqrt{2}$  | 23. $7\sqrt{5}$  | 24. $\frac{3\sqrt{3}}{2}$ |
| 25. $4\sqrt{10}$ | 26. $8\sqrt{3}$  | 27. $15\sqrt{2}$          |
| 28. $4\sqrt{6}$  | 29. $11\sqrt{2}$ | 30. $\frac{6\sqrt{2}}{5}$ |

EXERCISE 2.16 (Page 27)

- |                              |                             |                             |
|------------------------------|-----------------------------|-----------------------------|
| 1. $7\sqrt{3}$               | 2. $3\sqrt{2}$              | 3. $6\sqrt{2}$              |
| 4. $5\sqrt{3}$               | 5. $8\sqrt{2}$              | 6. $\sqrt{3}$               |
| 7. $4\sqrt{5}$               | 8. $7\sqrt{3}$              | 9. $2\sqrt{5}$              |
| 10. $-3\sqrt{7}$             | 11. $7\sqrt{2}$             | 12. $2\sqrt{3} + 4\sqrt{2}$ |
| 13. $5\sqrt{2} + \sqrt{7}$   | 14. $4\sqrt{3} + 2\sqrt{7}$ | 15. $\sqrt{3}$              |
| 16. $21\sqrt{6}$             | 17. $-9\sqrt{6}$            | 18. $10\sqrt{3}$            |
| 19. $3\sqrt{3}$              | 20. $7\sqrt{5}$             | 21. $18\sqrt{5}$            |
| 22. $8\sqrt{2}$              | 23. $6\sqrt{5} - 9\sqrt{6}$ | 24. $12\sqrt{2}$            |
| 25. $16\sqrt{6} - 4\sqrt{5}$ | 26. $-2\sqrt{2}$            | 27. $20\sqrt{3} - \sqrt{2}$ |
| 28. $75\sqrt{3}$             | 29. $(a+2)\sqrt{a}$         | 30. $(x-1)\sqrt{x}$         |

EXERCISE 2.17 (Page 28)

- |   |  |                  |
|---|--|------------------|
| 1. 3  | 2. 5   | 3. 45            |
| 4. $\sqrt{30}$  | 5. $\sqrt{21}$                                     | 6. $6\sqrt{3}$   |
| 7. $8\sqrt{10}$                                       | 8. $50\sqrt{33}$                                   | 9. $18\sqrt{2}$  |
| 10. 42  | 11. 3  | 12. $50\sqrt{2}$ |
| 13. 2   | 14. 9  |                  |
| 15. $21 - \sqrt{14}$                                  | 16. $8\sqrt{6} - 30$                               |                  |
| 17. $2\sqrt{21} + 12\sqrt{14}$                        | 18. 12   |                  |
| 19. 18  | 20. $12\sqrt{6}$                                   |                  |
| 21. $18\sqrt{2} - 20\sqrt{3}$                         | 22. $2\sqrt{3} - 5$                                |                  |
| 23. $\sqrt{10} + \sqrt{6} + \sqrt{15} + 3$            | 24. $\sqrt{21} + \sqrt{35} - \sqrt{6} - \sqrt{10}$ |                  |
| 25. $5 + 2\sqrt{6}$                                   | 26. $14 + 6\sqrt{5}$                               |                  |
| 27. $9 - 2\sqrt{14}$                                  | 28. 3  |                  |
| 29. $16 + \sqrt{6}$                                   | 30. $29 + 4\sqrt{7}$                               |                  |
| 31. $57 - 12\sqrt{15}$                                | 32. 6  |                  |
| 33. 2   | 34. 11   |                  |
| 35. $\sqrt{35} - 3\sqrt{14} - 2\sqrt{15} + 6\sqrt{6}$ |  |                  |
| 36. $18 + 5\sqrt{10}$                                 |  |                  |

EXERCISE 2.18 (Page 28)

- |          |         |         |
|----------|---------|---------|
| 1. (a) 7 | (b) 4   | (c) 46  |
| (d) 2    | (e) 11  | (f) 1   |
| (g) 2    | (h) 5   | (i) 6   |
| (j) 17   | (k) 19  | (l) 28  |
| (m) 16   | (n) -4  | (o) 37  |
| (p) 57   | (q) 151 | (r) 15  |
| (s) 87   | (t) 249 |         |
| 2. (a) 1 | (b) 22  | (c) 47  |
| (d) 4    | (e) 33  | (f) 6   |
| (g) -13  | (h) 71  | (i) 17  |
| (j) 58   | (k) 17  | (l) 197 |
| (m) 116  | (n) 30  | (o) -1  |

EXERCISE 2.19 (Page 29)

- |                                     |                               |                                       |
|-------------------------------------|-------------------------------|---------------------------------------|
| 1. $\frac{3\sqrt{2}}{2}$            | 2. $\frac{4\sqrt{5}}{5}$      | 3. $\frac{\sqrt{3}}{3}$               |
| 4. $\sqrt{7}$                       | 5. $\sqrt{2}$                 | 6. $\frac{\sqrt{2}}{2}$               |
| 7. $\frac{4\sqrt{7}}{21}$           | 8. $\frac{\sqrt{2}}{2}$       | 9. $\frac{\sqrt{2}}{2}$               |
| 10. $1\frac{1}{2}$                  | 11. $\frac{\sqrt{5}}{20}$     | 12. $\frac{2}{3}$                     |
| 13. $\frac{\sqrt{2}}{3}$            | 14. $\frac{7\sqrt{2}}{4}$     | 15. $\frac{2}{3}$                     |
| 16. $\frac{\sqrt{6} + \sqrt{3}}{3}$ | 17. $\frac{3 - \sqrt{3}}{3}$  | 18. $\frac{3\sqrt{2} - 2\sqrt{3}}{6}$ |
| 19. $\frac{3\sqrt{10} + 5}{10}$     | 20. $\frac{2\sqrt{6} - 2}{3}$ |                                       |

EXERCISE 2.20

- |  |                                  |                                 |
|--|----------------------------------|---------------------------------|
| 1. $\sqrt{2} - 1$                      | 2. $2 - \sqrt{3}$                | 3. $\sqrt{3} + \sqrt{2}$        |
| 4. $\frac{3\sqrt{5} - 3\sqrt{3}}{2}$   | 5. $\frac{4\sqrt{2} + 2}{7}$     | 6. $3\sqrt{3} + 3$              |
| 7. $\frac{5\sqrt{7} - 5\sqrt{5}}{2}$   | 8. $\frac{2\sqrt{5} + 1}{19}$    | 9. $25 - 10\sqrt{6}$            |
| 10. $\frac{12 - 5\sqrt{3}}{23}$        | 11. $\frac{7 - \sqrt{7}}{6}$     | 12. $\frac{9 + \sqrt{3}}{26}$   |
| 13. $\frac{4\sqrt{5} - 3\sqrt{7}}{17}$ | 14. $\frac{30 + 4\sqrt{10}}{37}$ | 15. $\frac{6 + \sqrt{6}}{30}$   |
| 16. $\frac{11 - 6\sqrt{2}}{7}$         | 17. $\frac{9 - 2\sqrt{14}}{5}$   | 18. $\frac{19 - 6\sqrt{2}}{17}$ |
| 19. $\frac{23 + 8\sqrt{5}}{19}$        | 20. 2                            |                                 |

EXERCISE 2.21

- |           |            |           |           |
|-----------|------------|-----------|-----------|
| 1. 10-392 | 2. 9-798   | 3. 21-213 | 4. 6-732  |
| 5. 1-414  | 6. 0-457   | 7. 19-974 | 8. 13-928 |
| 9. 10-499 | 10. 17-809 | 11. 4-619 | 12. 0-567 |
| 13. 0-336 | 14. 0-517  | 15. 0-361 | 16. 0-318 |
| 17. 2-374 | 18. 1-809  | 19. 3-435 | 20. 0-752 |