St Catherine's School

Year: 10
Subject: Advanced Mathematics

Time allowed: 55 minutes
Assessment Task No: 1
Date: March 3, 2005

Student Name: _____

Directions to candidates:

- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work
- All necessary **working** must be shown in every question.
- The answers to Section I are to be circled on the question paper
- The answers to Section II are to be written on the question paper in the space provided.
- Approved calculators and rulers are required.

TEACHERS' USE ONLY

	Section I	
Multiple Choice	Q1-10	/10
Page 2	Q11-13	/19
Page 3	Q14-19	/16
Page 4	Q20	/3
Bonus	Q21	/5
	TOTAL	/48

SECTION I – Circle the correct answer 10 × 1M

1) Which of the following numbers are rational?

$\sqrt{36}$, 3, $\sqrt{45}$, π, -2, $\frac{3}{11}$

(A) $\sqrt{36}$, 3, -2 (B) 3, π, -2, $\frac{3}{11}$ (C) 3, -2, $\frac{3}{11}$ (D) $\sqrt{36}$, 3, -2, $\frac{3}{11}$

2) $\sqrt{24} =$

(A) $2\sqrt{6}$ (B) $4\sqrt{3}$ (C) $4\sqrt{6}$ (D) $6\sqrt{2}$

3) Fully simplify $2\sqrt{3}-7\sqrt{3}+\sqrt{3}$

(A) $-4\sqrt{3}$ (B) $6\sqrt{3}$ (C) $\sqrt{6}-\sqrt{21}+\sqrt{3}$ (D) $-5\sqrt{3}+\sqrt{3}$

4) Fully simplify $3\sqrt{2}+\sqrt{8}$

(A) $5\sqrt{2}$ (B) $7\sqrt{2}$ (C) $3\sqrt{10}$ (D) $4\sqrt{10}$

5) Consider the following statements for $a > 0$ and $b > 0$:

I. $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$

II. $\sqrt{a\times b}=\sqrt{a}\times\sqrt{b}$

Which statement is always true?

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

6) If $5\sqrt{2}=\sqrt{x}$, find x.

(A) $x = 10$ (B) $x = 50$

(C) $x = 20$ (D) $x = 200$

7) Arrange in ascending order: $2\sqrt{5}, \sqrt{16}, \sqrt{19}$.

(A) $2\sqrt{5}, \sqrt{16}, \sqrt{19}$ (B) $\sqrt{16}, \sqrt{19}, 2\sqrt{5}$

(C) $2\sqrt{5}, \sqrt{19}, \sqrt{16}$ (D) $\sqrt{19}, \sqrt{16}, 2\sqrt{5}$

8) Rationalise $\frac{1}{\sqrt{6}-1}$ and express your answer in its simplest form.

(A) $\frac{1}{\sqrt{6}+1}$ (B) $\frac{\sqrt{6}}{5}$

(C) $\frac{\sqrt{6}+1}{5}$ (D) $\frac{(\sqrt{6}+1)}{(\sqrt{6}-1)(\sqrt{6}+1)}$

9) If $(a+\sqrt{2})^2 = m+6\sqrt{2}$, where a and m are integers, then which of the following is true?

(A) $a=3, m=9$ (B) $a=3, m=11$

(C) $a=6, m=36$ (D) $a=6, m=38$

10) Use your calculator to evaluate $\frac{\sqrt{3}-1}{2\sqrt{2}}$ correct to 2 decimal places.

(A) 0·26 (B) 0·52 (C) 1·02 (D) 1·38

SECTION II – Show your working on the space provided

11) Simplify

i. $\sqrt{50}+\sqrt{75}$ 2M

ii. $2\sqrt{28}+3\sqrt{7}-5\sqrt{63}$ 2M

iii. $2\sqrt{3}\times 5\sqrt{2}\times\sqrt{3}$ 2M

iv. $\frac{\sqrt{8}\times\sqrt{5}}{20}$ 2M

v. $(2+3\sqrt{5})(3-2\sqrt{3})$ 2M

vi. $(6+\sqrt{5})(6-\sqrt{5})$ 2M

vii. $(4\sqrt{2}-3)^2$ 2M

viii. $(\sqrt{5}-1)^2$ 2M

12) Express $3\sqrt{3}$ as an entire surd. 1M

13) Find b if $\sqrt{12}+\sqrt{3}=\sqrt{b}$ 2M

14) If $a = 3\sqrt{2}$ find a^3 in exact form. 2M

15) Rationalise the denominator and simplify where necessary:

i. $\frac{2}{\sqrt{10}} =$ ii. $\frac{5}{2\sqrt{5}} =$

16) Simplify leaving your answer with a rational denominator: $\frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} =$

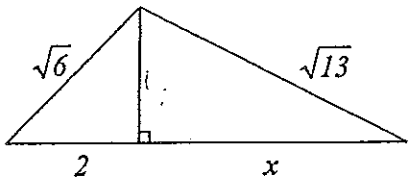
3M

17) Rationalise the denominator of $\frac{1}{\sqrt{7}-2}$ and simplify. 2M

18) Simplify $(\sqrt{5}-1)^2(\sqrt{5}+1)^2 =$ 3M

19) Show that $\frac{1}{2\sqrt{11}-1} - \frac{1}{2\sqrt{11}+1}$ is a rational number. 2M

20)



NOT TO SCALE

Consider the diagram above. What is the value of x? 3M

Hint: You may need Pythagoras' Theorem to find the height of the triangle.

BONUS QUESTION

21) i. Show $(x+\frac{1}{x})^2 = x^2+2+\frac{1}{x^2}$ 1M

ii. If $x=\sqrt{3}-\sqrt{2}$ find:

a. $x+\frac{1}{x}$ 2M

Note: Ensure your answer has a rational denominator if required.

b. $x^2+\frac{1}{x^2}$

YR 10 Surds

SOLUTIONS

1) $\sqrt{36} = 6, 3, -2,$ 3/11 (D)

2) $\sqrt{24} = \sqrt{4} \times \sqrt{6}$
$= 2\sqrt{6}$ (A)

3) $2\sqrt{3} - 7\sqrt{3} + \sqrt{3}$
$= -4\sqrt{3}$ (A)

4) $3\sqrt{2} + \sqrt{8}$
$= 3\sqrt{2} + \sqrt{4} \times \sqrt{2}$
$= 3\sqrt{2} + 2\sqrt{2}$
$= 5\sqrt{2}$ (A)

5) II (B)

6) $5\sqrt{2} = \sqrt{x}$
$25 \times 2 = x$
$x = 50$ (B)

7) $2\sqrt{5}, \sqrt{16}, \sqrt{19}$
$\sqrt{4 \times 5}, \sqrt{16}, \sqrt{19}$
$\sqrt{20}, \sqrt{16}, \sqrt{19}$
$\therefore \sqrt{16}, \sqrt{19}, 2\sqrt{5}$ (B)

8) $\frac{1}{\sqrt{6}-1} \times \frac{\sqrt{6}+1}{\sqrt{6}+1}$
$= \frac{\sqrt{6}+1}{6-1}$
$= \frac{\sqrt{6}+1}{5}$ (C)

9) $(a + \sqrt{2})^2 = a^2 + 2\sqrt{2}a + 2$
$m + 6\sqrt{2} = a^2 + 2\sqrt{2}a + 2$
$6\sqrt{2} = 2\sqrt{2}a$
$\therefore a = 3$
$m = a^2 + 2$
$m = 9 + 2$
$= 11$ (D)

10) $\frac{\sqrt{3}-1}{(2\sqrt{2})} \doteq 0.2588$ (A)

(iii) $\sqrt{50} + \sqrt{75} = \sqrt{25} \times \sqrt{2} + \sqrt{25} \times \sqrt{3}$
$= 5\sqrt{2} + 5\sqrt{3}$

(ii) $2\sqrt{28} + 3\sqrt{7} - 5\sqrt{63}$
$= 2 \times \sqrt{4} \times \sqrt{7} + 3\sqrt{7} - 5\sqrt{9} \times \sqrt{7}$
$= 2 \times 2\sqrt{7} + 3\sqrt{7} - 5 \times 3 \times \sqrt{7}$
$= 4\sqrt{7} + 3\sqrt{7} - 15\sqrt{7}$
$= -8\sqrt{7}$ ✓

(iii) $2\sqrt{3} \times 5\sqrt{2} \times \sqrt{3}$
$= 2 \times 5 \times 1 \times \sqrt{3 \times 3 \times 2}$
$= 10 \times 3\sqrt{2}$
$= 30\sqrt{2}$ ✓

(iv) $\frac{\sqrt{8} \times \sqrt{5}}{20} = \frac{\sqrt{40}}{20}$ ✗
$= \frac{\sqrt{4} \times \sqrt{10}}{20}$ ✗
$= \frac{2\sqrt{10}}{20_{10}}$ ✗
$= \frac{\sqrt{10}}{10}$ ✗

(v) $(2 + 3\sqrt{5})(3 - 2\sqrt{3})$ ✗
$= 6 - 4\sqrt{3} + 9\sqrt{5} - 6\sqrt{15}$
-1M for subsequent work.

(vi) $(6+\sqrt{5})(6-\sqrt{5})$
$= 36 - 5$ ✓
$= 31$ ✓

(vii) $(4\sqrt{2} - 3)^2 = 16 \times 2 - 2 \times 3 \times 4\sqrt{2} + 9$ ✗ ✗ ✗
$= 32 - 24\sqrt{2} + 9$
$= 41 - 24\sqrt{2}$ ✗

(viii) $(\sqrt{5} - 1)^2 = 5 - 2\sqrt{5} + 1$ ✗ ✗ ✗

(i) $\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$ ✓
$= \frac{\sqrt{7}+2}{7-4}$ ✗

(12) $3\sqrt{3} = \sqrt{9} \times \sqrt{3}$
$= \sqrt{27}$

(13) $\sqrt{12} + \sqrt{3}$
$= \sqrt{4} \times \sqrt{3} + \sqrt{3}$
$= 2\sqrt{3} + \sqrt{3}$
$= 3\sqrt{3}$ ✓

(14) $a = 3\sqrt{2}$
$a^3 = 3^3 \times (\sqrt{2})^3$
$= 27 \times 2\sqrt{2}$
$= 54\sqrt{2}$

(15i) $\frac{2}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{2\sqrt{10}}{10}$
$= \frac{\sqrt{10}}{5}$ ✓

(ii) $\frac{5}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{2 \times 5}$
$= \frac{\sqrt{5}}{2}$ ✓

(16) $\frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} = \frac{2 \times 2}{2\sqrt{2}} + \frac{\sqrt{2} \times \sqrt{2}}{2\sqrt{2}}$
OR
$= \frac{4 + 2}{2\sqrt{2}}$
$= \frac{3}{2} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{3\sqrt{2}}{2 \times 2}$
$= \frac{3\sqrt{2}}{2}$

$\frac{2\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$\frac{3\sqrt{2}}{2}$

$= \frac{\sqrt{7} + 2}{3}$

(18) $(\sqrt{5} - 1)^2 (\sqrt{5} + 1)^2$
$= \left[(\sqrt{5} - 1)(\sqrt{5} + 1) \right]^2$ ✓
$= (5 - 1)^2$ ✓
$= 4^2$
$= 16$ ✓

(19) $\frac{1}{2\sqrt{11} - 1} - \frac{1}{2\sqrt{11} + 1}$
$= \frac{2\sqrt{11} + 1 - (2\sqrt{11} - 1)}{(2\sqrt{11} - 1)(2\sqrt{11} + 1)}$
$= \frac{2\sqrt{11} + 1 - 2\sqrt{11} + 1}{4 \times 11 - 1}$
$= \frac{2}{43}$ ∴ a rational no.

(20) Let "h" be the height of the triangle.
$h^2 = (\sqrt{6})^2 - 2^2$ (By Pythag)
$= 6 - 4$
$= 2$
$x^2 = (\sqrt{13})^2 - h^2$ (By Pythag)
$= 13 - 2$
$= 11$
$\therefore x = \sqrt{11}$

(21i) $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2}$
$= x^2 + 2 + \frac{1}{x^2}$

(iia) $\sqrt{3} - \sqrt{2} + \frac{1}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$
$= \sqrt{3} - \sqrt{2} + \frac{\sqrt{3} + \sqrt{2}}{3 - 2}$
$= \sqrt{3} - \sqrt{2} + \frac{\sqrt{3} + \sqrt{2}}{1}$
$= \sqrt{3} - \sqrt{2} + \sqrt{3} + \sqrt{2}$
$= 2\sqrt{3}$

(b) $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$ ✓
$\left(x + \frac{1}{x}\right)^2 - 2 = 4 \times 3 - 2$
$= 10$