1. Simplify these surds:

   (a) $\sqrt{28}$,

   (b) $\sqrt{72}$,

   (c) $\sqrt{50} - \sqrt{18}$,

   (d) $3\sqrt{2} + 5\sqrt{8}$,

   (e) $3\sqrt{15} \times 5\sqrt{6}$,

   (f) $18\sqrt{150} \div 3\sqrt{3}$,

   (g) $3\sqrt{x} + \sqrt{9x}$,

   (h) $5a\sqrt{a} - \sqrt{a^3}$.

2. Expand and simplify:

   (a) $\left(5 + 2\sqrt{2}\right)\left(3 - 4\sqrt{10}\right)$,

   (b) $\left(2 + \sqrt{3}\right)^2$,

   (c) $\left(3 - 2\sqrt{5}\right)^2$,

   (d) $\left(4 - 7\sqrt{2}\right)\left(4 + 7\sqrt{2}\right)$

   (e) $\left(1 + \sqrt{m}\right)^2$,

   (f) $\left(3\sqrt{p} - 2\right)\left(3\sqrt{p} + 2\right)$.

3. Rationalize the denominator of:

   (a) $\dfrac{5}{\sqrt{5}}$,

   (b) $\dfrac{3\sqrt{2}}{5\sqrt{10}}$,

   (c) $\dfrac{1}{\sqrt{6} - \sqrt{5}}$,

   (d) $\dfrac{5 + \sqrt{2}}{3 - 2\sqrt{2}}$.

4. Simplify these expressions, leaving your answer as a single fraction without any surds in the denominator:

   (a) $2 + 5\sqrt{3} + \dfrac{1}{2 + 5\sqrt{3}}$,

   (b) $\dfrac{\sqrt{x} + \sqrt{2 + x}}{\sqrt{x} - \sqrt{2 + x}} + \dfrac{\sqrt{x} - \sqrt{2 + x}}{\sqrt{x} + \sqrt{2 + x}}$.

5. Given $x = 2\sqrt{3} - 1$, find $\dfrac{x}{x + \sqrt{2}}$ expressing your answer as a fraction with a rational denominator.

QUESTION 1

(a) $\sqrt{28} = 2\sqrt{7}$

(b) $\sqrt{72} = 6\sqrt{2}$

(c) $\sqrt{50} - \sqrt{18} = 5\sqrt{2} - 3\sqrt{2}$
$$= 2\sqrt{2}$$

(d) $3\sqrt{2} + 5\sqrt{8} = 3\sqrt{2} + 10\sqrt{2}$
$$= 13\sqrt{2}$$

(e) $3\sqrt{15} \times 5\sqrt{6} = 15\sqrt{90}$
$$= 15 \times 3\sqrt{10}$$
$$= 45\sqrt{10}$$

(f) $18\sqrt{150} \div 3\sqrt{3} = 6\sqrt{50}$
$$= 6 \times 5\sqrt{2}$$
$$= 30\sqrt{2}$$

(g) $3\sqrt{x} + \sqrt{9x} = 3\sqrt{x} + 3\sqrt{x}$
$$= 6\sqrt{x}$$

(h) $5a\sqrt{a} - \sqrt{a^3} = 5a\sqrt{a} - a\sqrt{a}$
$$= 4a\sqrt{a}$$

QUESTION 2

(a) $(5 + 2\sqrt{2})(3 - 4\sqrt{10}) = 15 - 20\sqrt{10} + 6\sqrt{2} - 8\sqrt{20}$
$$= 15 - 20\sqrt{10} + 6\sqrt{2} - 16\sqrt{5}$$

(b) $(2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3$
$$= 7 + 4\sqrt{3}$$

(c) $(3 - 2\sqrt{5})^2 = 9 - 12\sqrt{5} + 20$
$$= 29 - 12\sqrt{5}$$

(d) $(4 - 7\sqrt{2})(4 + 7\sqrt{2}) = 16 - 98$
$$= -82$$

(e) $(1 + \sqrt{m})^2 = 1 + 2\sqrt{m} + m$

(f) $(3\sqrt{p} - 2)(3\sqrt{p} + 2) = 9p - 4$

QUESTION 3

(a) $\dfrac{5}{\sqrt{5}} = \dfrac{5}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$
$$= \dfrac{5\sqrt{5}}{5}$$
$$= \sqrt{5}$$

(b) $\dfrac{3\sqrt{2}}{5\sqrt{10}} = \dfrac{3\sqrt{2}}{5\sqrt{10}} \times \dfrac{\sqrt{10}}{\sqrt{10}}$
$$= \dfrac{3\sqrt{20}}{50}$$
$$= \dfrac{6\sqrt{5}}{50}$$
$$= \dfrac{3\sqrt{5}}{25}$$

(c) $\dfrac{1}{\sqrt{6} - \sqrt{5}} = \dfrac{1}{\sqrt{6} - \sqrt{5}} \times \dfrac{\sqrt{6} + \sqrt{5}}{\sqrt{6} + \sqrt{5}}$
$$= \dfrac{\sqrt{6} + \sqrt{5}}{1}$$
$$= \sqrt{6} + \sqrt{5}$$

(d) $\dfrac{5 + \sqrt{2}}{3 - 2\sqrt{2}} = \dfrac{5 + \sqrt{2}}{3 - 2\sqrt{2}} \times \dfrac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}}$
$$= \dfrac{15 + 10\sqrt{2} + 3\sqrt{2} + 4}{9 - 8}$$
$$= 19 + 13\sqrt{2}$$

QUESTION 4

(a) $2 + 5\sqrt{3} + \dfrac{1}{2 + 5\sqrt{3}} = 2 + 5\sqrt{3} + \dfrac{1}{2 + 5\sqrt{3}} \times \dfrac{2 - 5\sqrt{3}}{2 - 5\sqrt{3}}$
$$= 2 + 5\sqrt{3} + \dfrac{2 - 5\sqrt{3}}{4 - 75}$$
$$= \dfrac{2 + 5\sqrt{3}}{1} + \dfrac{2 - 5\sqrt{3}}{-71}$$
$$= \dfrac{71(2 + 5\sqrt{3}) - (2 - 5\sqrt{3})}{71}$$
$$= \dfrac{142 + 355\sqrt{3} - 2 + 5\sqrt{3}}{71}$$
$$= \dfrac{140 + 360\sqrt{3}}{71}$$

(b) $\dfrac{\sqrt{x} + \sqrt{2+x}}{\sqrt{x} - \sqrt{2+x}} + \dfrac{\sqrt{x} - \sqrt{2+x}}{\sqrt{x} + \sqrt{2+x}}$
$$= \dfrac{(\sqrt{x} + \sqrt{2+x})^2 + (\sqrt{x} - \sqrt{2+x})^2}{x - (2+x)}$$
$$= \dfrac{x + 2\sqrt{x(2+x)} + 2+x + x - 2\sqrt{x(2+x)} + 2+x}{-2}$$
$$= \dfrac{4x + 4}{-2}$$
$$= -2x - 2$$

QUESTION 5

$\dfrac{x}{x + \sqrt{2}} = \dfrac{2\sqrt{3} - 1}{2\sqrt{3} - 1 + \sqrt{2}}$
$$= \dfrac{2\sqrt{3} - 1}{(2\sqrt{3} - 1) + \sqrt{2}} \times \dfrac{(2\sqrt{3} - 1) - \sqrt{2}}{(2\sqrt{3} - 1) - \sqrt{2}}$$
$$= \dfrac{(2\sqrt{3} - 1)^2 - \sqrt{2}(2\sqrt{3} - 1)}{(2\sqrt{3} - 1)^2 - 2}$$
$$= \dfrac{13 - 4\sqrt{3} - 2\sqrt{6} + \sqrt{2}}{13 - 4\sqrt{3} - 2}$$
$$= \dfrac{13 - 4\sqrt{3} - 2\sqrt{6} + \sqrt{2}}{11 - 4\sqrt{3}} \times \dfrac{11 + 4\sqrt{3}}{11 + 4\sqrt{3}}$$
$$= \dfrac{143 - 44\sqrt{3} - 22\sqrt{6} + 11\sqrt{2} + 52\sqrt{3} - 48 - 8\sqrt{18} + 4\sqrt{6}}{121 - 48}$$
$$= \dfrac{95 + 8\sqrt{3} - 18\sqrt{6} + 11\sqrt{2} - 24\sqrt{2}}{73}$$
$$= \dfrac{95 + 8\sqrt{3} - 18\sqrt{6} - 13\sqrt{2}}{73}$$