



FORM III

MATHEMATICS

Examination date

Wednesday 18th May 2005

Time allowed

1 hour 30 minutes

Instructions

- All eight questions may be attempted.
- All eight questions are of equal value.
- All necessary working must be shown.
- Marks may not be awarded for careless or badly arranged work.
- Calculators are not to be used.

Collection

- Write your name, class and master clearly on the front.
- Hand in all the writing paper in a single well-stapled bundle.
- Keep the printed examination paper and bring it to your next Mathematics lesson.

3A: MLS	3B: GJ	3C: FMW/DS
3D: KWM/JNC	3E: JCM/BJC	3F: JMR
3G: PKH	3H: DNW	3I: WMP

Checklist

- Writing paper required.
- Candidature: 185 boys.

Examiner

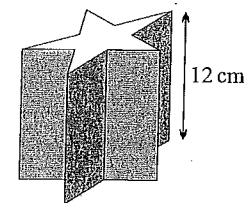
DNW

QUESTION ONE Start a new page.

- (a) Evaluate $10\frac{3}{4} - 2\frac{1}{4}$.
- (b) Round 2.71828 correct to two decimal places.
- (c) What is the perimeter of a square if the length of one side is 5 cm?
- (d) Find the area of a triangle with base 7 mm and perpendicular height 4 mm.
- (e) Simplify $8x - y - 5x$.
- (f) Evaluate $6 - 2c$ when $c = -3$.
- (g) Use the index laws to simplify:
 - (i) $a^6 \div a^4$
 - (ii) $(b^3)^5$
- (h) Simplify:
 - (i) $5\sqrt{3} - 2\sqrt{3}$
 - (ii) $\sqrt{20}$
- (i) The diameter of the moon Titan is 202,752,000 inches. Write this number in scientific notation, correct to four significant figures.

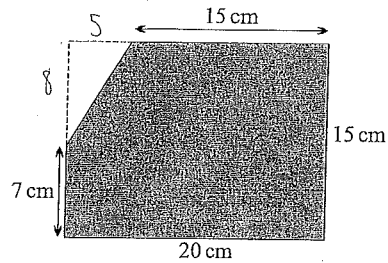
QUESTION TWO Start a new page.

(a)



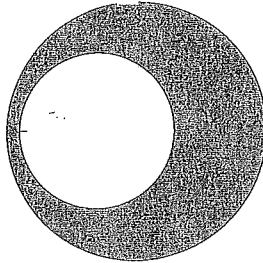
The prism above has a star-shaped base of area 5 cm^2 . What is the volume of the prism?

(b)



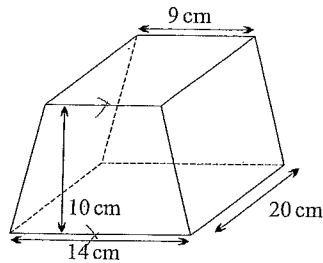
The shaded figure above is the result of cutting a corner off a rectangle. Find the shaded area.

(c)



In the diagram above, the inner circle has radius 3 cm and the outer circle has radius 5 cm. Find the area of the shaded region between the two circles, leaving your answer in terms of π .

(d)



A special container has dimensions as shown above. The shape of the front end is a trapezium.

- (i) What is the area of the trapezium?
- (ii) Find the volume of the container in millilitres.
(Remember that $1 \text{ cm}^3 = 1 \text{ mL}$.)
- (iii) How much will it cost to fill the container with an expensive chemical, which costs 45 cents per mL?

Exam continues overleaf ...

QUESTION THREE Start a new page.

(a) Evaluate:

(i) $27^{\frac{1}{3}}$

(ii) $16^{\frac{3}{2}}$

(b) Simplify:

(i) $\sqrt{50} - \sqrt{8}$

(ii) $(3\sqrt{5})^2$

(c) Rationalise the denominator and simplify $\frac{4}{\sqrt{6}}$.

(d) Simplify:

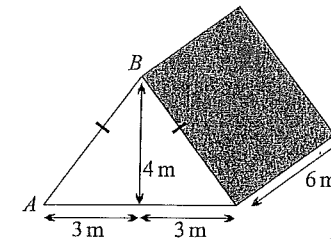
(i) $\frac{3}{2q^2} \times \frac{q^3}{6}$

(ii) $\frac{15a}{4b} \div \frac{3ab}{8}$

(iii) $\frac{5}{2x} + \frac{4}{3x}$

QUESTION FOUR Start a new page.

(a)



The diagram above shows an isosceles triangular prism.

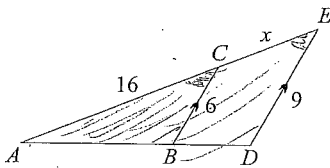
- (i) Find the length of the side marked AB .
 - (ii) Hence find the surface area of the prism.
- (b) Solve the following equations:
- (i) $3x + 2 = -7$
 - (ii) $4(x - 3) = 3x - 4$
 - (iii) $\frac{2x}{7} - \frac{x}{2} = 1$

Exam continues next page ...

- (c) (i) Solve the inequation $2 - 3x \leq 17$.
 (ii) Graph the solution on the number line.

QUESTION FIVE Start a new page.

- (a) Consider the formula $s = ut - 5t^2$.
 (i) Find s when $u = 22$ and $t = 3$.
 (ii) Make u the subject of the equation.
- (b)



In the diagram above, $BC \parallel DE$, $BC = 6$, $DE = 9$ and $AC = 16$.

Let the length of CE be x .

- (i) Prove that $\triangle ABC \parallel \triangle ADE$, giving all reasons.
 (ii) Form an equation and solve it to find the value of x .
- (c) Simplify:
 (i) $(4x^6y^2)^{\frac{5}{2}}$
 (ii) $\left(\frac{a^2b^{-3}}{a^{-1}b^4}\right)^{-1}$

QUESTION SIX Start a new page.

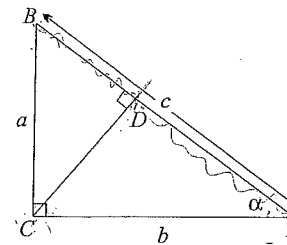
- (a) Expand and simplify $(x + 3)^2 - (x + 5)(x - 5)$.
 (b) Expand and simplify $(\sqrt{2} + 2\sqrt{3})(3\sqrt{2} + 4\sqrt{3})$.
 (c) Express $\frac{2}{\sqrt{11} - 1}$ with a rational denominator and in simplest form.
 (d) Solve $(x + 1)(x - 1) = x^2 + 2x + 3$.

QUESTION SEVEN Start a new page.

- (a) Solve $\frac{1}{x} + \frac{1}{x-1} = 0$.
 (b) Solve the inequation $\frac{3x-4}{3} - \frac{x+2}{2} < 4 - x$.
 (c) (i) Simplify $\frac{p^9q^{-3}}{p(p^2q)^3}$.
 (ii) Hence express $\sqrt{\frac{p^9q^{-3}}{p(p^2q)^3}}$ in simplest form, and without fractions.
- (d) In a certain cylinder, the height h is equal to the diameter. Let the radius of the cylinder be r .
 (i) Draw a diagram of the situation and find h in terms of r .
 (ii) Show that the ratio of the volume to the surface area is $\frac{r}{3}$.

QUESTION EIGHT Start a new page.

(a)



In the diagram above, let $BC = a$, $CA = b$ and $AB = c$.
 Let $\angle CAB = \alpha$.

- (i) Show that $\triangle ABC \parallel \triangle ACD$.
 (ii) Hence show that $AD = \frac{b^2}{c}$.
 (iii) Show that $BD = \frac{a^2}{c}$.
 (iv) Hence complete a proof of Pythagoras' theorem, that is
 $c^2 = a^2 + b^2$.

(b) The value of $\sqrt{5}$ is approximately 2.236.

Consider the problem of finding the approximate value of $\frac{1}{\sqrt{5}}$.

- (i) You are not allowed to use calculators in this exam. Why will it be easier to rationalise the denominator first?
- (ii) Hence approximate $\frac{1}{\sqrt{5}}$ correct to three decimal places.

(c) Let the number x be the difference between $\sqrt{5}$ and the approximation 2.236.

That is,

$$x = \sqrt{5} - 2.236.$$

Let

$$J = \frac{1}{\sqrt{5}} - \frac{\sqrt{5} - x}{5}$$

and let

$$M = \frac{1}{\sqrt{5} - x} - \frac{1}{\sqrt{5}}.$$

(i) Show that

$$M - J = \frac{x^2(\sqrt{5} + x)}{5(5 - x^2)}.$$

(ii) Which of $\frac{\sqrt{5} - x}{5}$ and $\frac{1}{\sqrt{5} - x}$ is the better approximation to $\frac{1}{\sqrt{5}}$?

Explain your reasons carefully.

END OF EXAMINATION



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Examiner

DNW

V.K. Chau

QUESTION ONE Start a new page.

- (a) $10\frac{3}{4} - 2\frac{1}{4} = 8\frac{1}{2}$
- (b) $2.718\ 28 \div 2.72$ (correct to two decimal places)
- (c) perimeter = 4×5
= 20 cm
- (d) Area = $\frac{1}{2} \times 7 \times 4$
= 14 mm^2
- (e) $8x - y - 5x = 3x - y$
- (f) $6 - 2c = 6 - 2 \times (-3)$
= 12
- (g) (i) $a^6 \div a^4 = a^2$
(ii) $(b^3)^5 = b^{15}$
- (h) (i) $5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$
(ii) $\sqrt{20} = 2\sqrt{5}$
- (i) $202\ 752\ 000 = 2.028 \times 10^8$
scientific notation
- rounding*

Total for Question 1: 12 Marks

QUESTION TWO Start a new page.

- (a) Volume = 5×12
= 60 cm^3
answer
- (b) $\Delta = \frac{1}{2} \times 5 \times 8$
= 20 cm^2
rect = 20×15
= 300 cm^2
Total Area = $300 - 20$
= 280 cm^2
units

Exam continues next page ...

(c) Area = $\pi \times 5^2 - \pi \times 3^2$
 = $16\pi \text{ cm}^2$

(d) (i) Trap = $\frac{1}{2}(9 + 14) \times 10$
 = 115 cm^2

(ii) Capacity = 115×20
 = 2300 cm^3
 = 2300 mL

(iii) Cost = $2300 \times 45 \text{ cents}$
 = $\$1035$

Total for Question 2: 12 Marks

QUESTION THREE Start a new page.

(a) (i) $27^{\frac{1}{3}} = 3$

(ii) $16^{\frac{3}{2}} = 4^3$
 = 64

(b) (i) $\sqrt{50} - \sqrt{8} = 5\sqrt{2} - 2\sqrt{2}$
 = $3\sqrt{2}$

(ii) $(3\sqrt{5})^2 = 9 \times 5$
 = 45

(c) $\frac{4}{\sqrt{6}} = \frac{4}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$
 = $\frac{4\sqrt{6}}{6}$
 = $\frac{2\sqrt{6}}{3}$

(d) (i) $\frac{3}{2q^2} \times \frac{q^3}{6} = \frac{q}{4}$

(ii) $\frac{15a}{4b} \div \frac{3ab}{8} = \frac{15a}{4b} \times \frac{8}{3ab}$
 = $\frac{10}{b^2}$

Exam continues overleaf ...

(iii) $\frac{5}{2x} + \frac{4}{3x} = \frac{15}{6x} + \frac{8}{6x}$
 = $\frac{23}{6x}$

Total for Question 3: 12 Marks

QUESTION FOUR Start a new page.

(a) (i) $(AB)^2 = 3^2 + 4^2$ (by Pythagoras)
 = 25

so $AB = 5$

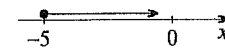
(ii) Surface Area = $2 \times \text{front} + 2 \times \text{side} + \text{bottom}$
 = $2 \times \frac{1}{2} \times 6 \times 4 + 2 \times 5 \times 6 + 6 \times 6$
 = 120 m^2

(b) (i) $3x + 2 = -7$
 $\boxed{-2}$ $3x = -9$
 $\boxed{\div 3}$ $x = -3$

(ii) $4(x - 3) = 3x - 4$
 expand the brackets to get
 $4x - 12 = 3x - 4$
 $\boxed{+12}$ $4x = 3x + 8$
 $\boxed{-3x}$ $x = 8$

(iii) $\frac{2x}{7} - \frac{x}{2} = 1$
 $\boxed{\times 14}$ $4x - 7x = 14$
 so $-3x = 14$
 $\boxed{\div (-3)}$ $x = -\frac{14}{3}$

(c) (i) $2 - 3x \leq 17$
 $\boxed{-2}$ $-3x \leq 15$
 $\boxed{\div (-3)}$ $x \geq -5$



Total for Question 4: 12 Marks

Exam continues next page ...

QUESTION FIVE Start a new page.

(a) (i) $S = 22 \times 3 - 5 \times 3^2$
 $= 21$ ✓

(ii) $ut - 5t^2 S$
 $\boxed{+5t^2}$ $ut = S + 5t^2$ ✓
 $\boxed{\div t}$ $u = \frac{S + 5t^2}{t}$ (or $\frac{S}{t} + 5t$) ✓

(b) (i) In $\triangle ABC$ and $\triangle ADE$
 $\angle BAC \angle DAE$ (common) ✓
 $\angle ABC \angle ADE$ (corresponding angles on $BC \parallel DE$) ✓
 hence $\triangle ABC \parallel \triangle ADE$ (AA) ✓

(ii) $\frac{AE}{AC} = \frac{DE}{BC}$ (ratio of matching sides of similar triangles)
 thus $\frac{16 + x}{16} = \frac{3}{2}$ ✓
 $\boxed{\times 16}$ $16 + x = 24$
 $\boxed{-16}$ $x = 8$ ✓

(c) (i) $(4x^6 y^2)^{\frac{5}{2}} = (2x^3 y)^5$ ✓
 $= 32x^{15} y^5$ ✓

(ii) $\left(\frac{a^2 b^{-3}}{a^{-1} b^4}\right)^{-1} = \left(\frac{a^3}{b^7}\right)^{-1}$ ✓
 $= \frac{b^7}{a^3}$ (or $b^7 a^{-3}$) ✓

Total for Question 5: 12 Marks

QUESTION SIX Start a new page.

(a) $(x + 3)^2 - (x + 5)(x - 5) = x^2 + 6x + 9 - (x^2 - 25)$ ✓✓
 $= 6x + 34$ ✓

(b) $(\sqrt{2} + 2\sqrt{3})(3\sqrt{2} + 4\sqrt{3}) = 6 + 4\sqrt{6} + 6\sqrt{6} + 24$ ✓✓
 $= 30 + 10\sqrt{6}$ ✓

Exam continues overleaf ...

(c) $\frac{2}{\sqrt{11} - 1} = \frac{2}{\sqrt{11} - 1} \times \frac{\sqrt{11} + 1}{\sqrt{11} + 1}$ ✓
 $= \frac{2(\sqrt{11} + 1)}{10}$ ✓
 $= \frac{\sqrt{11} + 1}{5}$ ✓

(d) $(x + 1)(x - 1) = x^2 + 2x + 3$
 expand the brackets to get
 $x^2 - 1 = x^2 + 2x + 3$ ✓
 the x^2 terms cancel to leave
 $2x + 3 = -1$ ✓
 $\boxed{-3}$ $2x = -4$
 $\boxed{\div 2}$ $x = -2$ ✓

Total for Question 6: 12 Marks

QUESTION SEVEN Start a new page.

(a) $\frac{1}{x} + \frac{1}{x - 1} = 0$
 $\boxed{\times x(x - 1)}$ $x - 1 + x = 0$ ✓
 $\boxed{+1}$ $2x = 1$
 $\boxed{\div 2}$ $x = \frac{1}{2}$ ✓

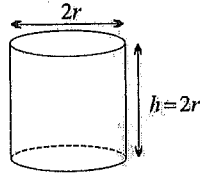
(b) $\frac{3x - 4}{3} - \frac{x + 2}{2} < 4 - x$
 $\boxed{\times 6}$ $2(3x - 4) - 3(x + 2) < 24 - 6x$ ✓
 expand to get
 $3x - 14 < 24 - 6x$ ✓
 $\boxed{+6x - 14}$ $9x < 38$
 $\boxed{\div 9}$ $x < \frac{38}{9}$ ✓

(c) (i) $\frac{p^9 q^{-3}}{p(p^2 q)^3} = \frac{p^9 q^{-3}}{p^7 q^3}$ ✓
 $= \frac{p^2}{q^6}$ (or $p^2 q^{-6}$) ✓

(ii) $\sqrt{\frac{p^9 q^{-3}}{p(p^2 q)^3}} = \sqrt{p^2 q^{-6}}$ (from part (i))
 $= pq^{-3}$ ✓

Exam continues next page ...

(d) (i)



(ii) Volume = $\pi r^2 \times 2r$
 $= 2\pi r^3$
 Surface Area = $2\pi r^2 + 2\pi r \times 2r$
 $= 6\pi r^2$
 hence $\frac{\text{Vol}}{\text{SA}} = \frac{2\pi r^3}{6\pi r^2}$
 $= \frac{r}{3}$

Total for Question 7: 12 Marks

QUESTION EIGHT Start a new page.

(a) (i) In $\triangle ABC$ and $\triangle ACD$
 $\angle BAC = \angle CAD$ (common)
 $\angle ACB = \angle ADC$ (given)
 hence $\triangle ABC \parallel \triangle ACD$ (AA)

(ii) In $\triangle ABC$ and $\triangle ACD$
 $\frac{AD}{b} = \frac{b}{c}$ (matching sides of similar triangles)
 hence $AD = \frac{b^2}{c}$

(iii) In $\triangle ABC$ and $\triangle CBD$
 $\angle ABC = \angle CBD$ (common)
 $\angle ACB = \angle CDB$ (given)
 hence $\triangle ABC \parallel \triangle CBD$ (AA)

so $\frac{BD}{a} = \frac{a}{c}$ (matching sides of similar triangles)
 thus $BD = \frac{a^2}{c}$

(iv) $AB = AD + DB$
 so $c = \frac{b^2}{c} + \frac{a^2}{c}$ (from parts (ii) and (iii))
 thus $c^2 = a^2 + b^2$

(b) (i) It is easier to evaluate $\frac{2,236}{5}$ than $\frac{1}{2,236}$.

(ii) $\frac{1}{\sqrt{5}} \approx \frac{2,236}{5}$
 ≈ 0.447

(c) (i) $M - J = \frac{1}{\sqrt{5-x}} - \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{5}} + \frac{\sqrt{5-x}}{5}$
 $= \frac{1}{\sqrt{5-x}} + \frac{\sqrt{5-x}}{5} - \frac{2}{\sqrt{5}}$
 $= \frac{\sqrt{5+x}}{5-x^2} + \frac{\sqrt{5-x}}{5} - \frac{2\sqrt{5}}{5}$
 $= \frac{\sqrt{5+x}}{5-x^2} - \frac{\sqrt{5-x}}{5}$
 $= \frac{5(\sqrt{5+x}) - (5-x^2)(\sqrt{5-x})}{5(5-x^2)}$
 $= \frac{5\sqrt{5} + 5x - 5\sqrt{5} + x^2\sqrt{5} - 5x + x^3}{5(5-x^2)}$
 $= \frac{x^2\sqrt{5} + x^3}{5(5-x^2)}$
 $= \frac{x^2(\sqrt{5} + x)}{5(5-x^2)}$

(ii) J represents the error in the approximation of $\frac{1}{\sqrt{5}}$ by $\frac{\sqrt{5-x}}{5}$.
 M represents the error in the approximation of $\frac{1}{\sqrt{5}}$ by $\frac{1}{\sqrt{5-x}}$.

Clearly $0 < x^2 < 1$ so $5 - x^2 > 0$. Thus $M - J$ is positive.
 That is, the error M is bigger than the error J .

Hence $\frac{\sqrt{5-x}}{5}$ is the better approximation to $\frac{1}{\sqrt{5}}$.

Total for Question 8: 12 Marks

END OF EXAMINATION