HIGHER SCHOOL CERTIFICATE EXAMINATION TRIAL PAPER

2012

MATHEMATICS

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
  Black is preferred
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in Questions 11 – 16

Total marks – 100

Section I: Multiple Choice

Questions 1 – 10 10 marks

- Attempt all questions
- Answer on the Answer Sheet provided
- Allow about 15 minutes for this section

Section II: Extended Response

Questions 11 – 16 90 marks

- Attempt all questions
- Allow about 2 hours 45 minutes for this section

Section I

Questions 1 – 10 (1mark for each question)

Read each question and choose an answer A, B, C or D.
Record your answer on the Answer Sheet provided.
Allow about 15 minutes for this section

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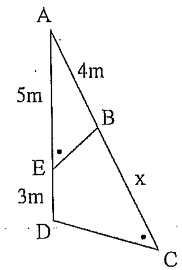
1. $(2-\sqrt{a})^2 = b - 4\sqrt{3}$

   What is the value of a and b?

   A) $a = 3,\ b = 1$ B) $a = -3,\ b = 7$ C) $a = 3,\ b = 4$ D) $a = 3,\ b = 7$

2. $\frac{d}{dx}(3\cos 2x) =$

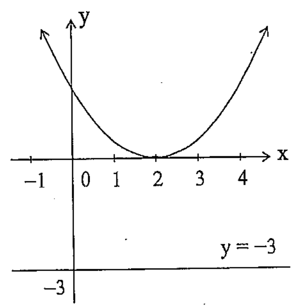
   A) $6\cos 2x$ B) $6\sin 2x$ D) $-3\sin 2x$ D) $-6\sin 2x$

3. 

   In the diagram $\angle AEB = \angle ACD$.

   The value of x is:

   A) 2.4m B) 4m
   C) 6m D) 6.4m

4. 

   The equation of the directrix of this parabola is $y = -3$.

   Which of these is the equation of this parabola?

   A) $(x-2)^2 = 6y$
   B) $(x-2)^2 = 12y$
   C) $(y-2)^2 = 6x$
   D) $(x-2)^2 = -12y$

5. $\int (e^x+1)^2 dx =$

A) $\frac{1}{2}e^{2x}+2e^x+x+c$

B) $\frac{1}{3}(e^x+1)^3+c$

C) $\frac{1}{2}e^{2x}+x+c$

D) $\frac{1}{2}e^{2x}+e^x+x+c$

6. $y=\ln\left(\frac{2}{x}\right)$ then $\frac{dy}{dx}=?$

A) $\frac{2}{x}$

B) $-\frac{2}{x^3}$

C) $-\frac{1}{x}$

D) $-\frac{1}{2x}$

7. A piece of wood 4m long is cut into 16 pieces.
The lengths of these 12 pieces form an arithmetic sequence.
The first piece is 15cm.

What is the length of the 10th piece?

A) $25\frac{2}{3}$cm

B) 27cm

C) $28\frac{1}{3}$cm

D) 35cm

8. Given that $\int_0^2 f(x)\,dx = 8$.

What is the value of $\int_0^2 (f(x)+2)\,dx$?

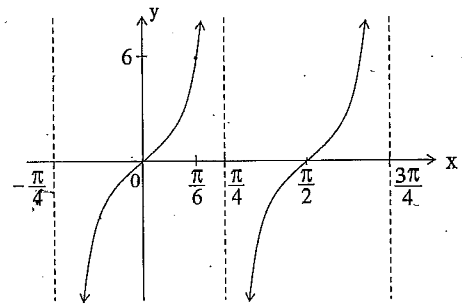
A) 12

B) 10

C) 8

D) 6

9. The diagram shows the graph of a function for $-\frac{\pi}{4}<x<\frac{3\pi}{4}$.



Which of these is the equation of this curve?

A) $y=\sqrt{3}\tan 2x$

B) $y=3\tan x$

C) $y=6\tan\frac{x}{2}$

D) $y=2\sqrt{3}\tan 2x$

10. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

The 8 cards are placed in a bag. Two of them are to be selected at random and placed beside each other to form a two digit number.

What is the probability that the number formed is an odd number greater than 50?

A) $\frac{1}{4}$

B) $\frac{2}{7}$

C) $\frac{7}{28}$

D) $\frac{7}{32}$

Section II

Question 11 – 16 (15 marks each)

Allow about 2 hours 45 minutes for this section

Question 11 — MARKS

a) Simplify $\dfrac{x^2 - 2x}{x^2 - 5x + 6}$ — 2

b) Solve $e^{2x+1} = \sqrt{e}$ — 2

c) Evaluate $\displaystyle\int_1^e \frac{2}{x}\,dx$ — 2

d) Vanessa has 792 flowers in her shop. — 2
The probability to select a rose is $\frac{5}{8}$ and the probability to select a tulip is $\frac{1}{12}$.
How many flowers in her shop are neither roses nor tulip?

e) The roots of the quadratic equation $2x^2 - 7x + 2 = 0$ are α and β.

i) Find $\alpha + \beta$. — 1

ii) Find $\alpha^2\beta + \alpha\beta^2$. — 2

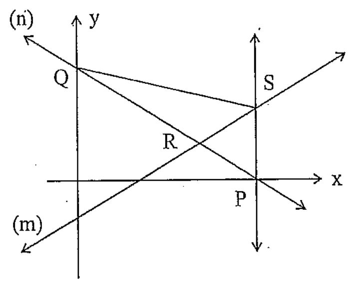
f) Find the exact values of x such that $\sqrt{2}\cos x + 1 = 0$, where $0 \le x \le 2\pi$. — 2

g) Find the equation of the normal to the curve $y = \ln(2x + 3)$ at the point where $x = -1$. — 2

Question 12 — MARKS

a) Differentiate $\dfrac{x^2}{\ln x}$ with respect to x. — 2

b) The diagram shows a line (n) with equation $2x + 3y - 12 = 0$ which intersect the x axis at P and the y axis at Q.
A second line (m) with equation $3x - 5y = 8$ intersect line (n) at R and the vertical line through P at S.



NOT TO SCALE

i) Show that the coordinates of S are (6, 2). — 2

ii) Show that the perpendicular distance from the point S to the line (n) is $\dfrac{6}{\sqrt{13}}$ — 2

iii) Given that the area of triangle QRS is $\dfrac{84}{19}$, find the length of interval QR. — 2

c) i) Differentiate $y = \sqrt{5\cos 2x}$ with respect to x. — 2

ii) Hence, or otherwise, find $\displaystyle\int \frac{4\sin 2x}{\sqrt{5\cos 2x}}\,dx$. — 2

d) A circus was run for 20 weeks. In the first week, 825 people attended.
in each of the following week25 people less than the previous week attended.

i) How many people attended in the 20^{th} week? — 1

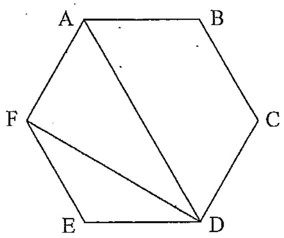
ii) If each person attended the circus paid $12, what was the total amount of money made over the 20 weeks? — 2

Question 13

MARKS

a) Use Simpson's Rule, with 3 function values, to find the area between the curve $y = \frac{1}{\ln x}$, the x axis, and the lines $x = 2$ and $x = 4$. Give your answer correct to 3 decimal places **3**

b) The diagram shows a regular hexagon ABCDEF. Diagonals DA and DF are drawn.



i) Find the size of angle DEF. **1**

ii) Show that triangle AFD is right angled. **2**

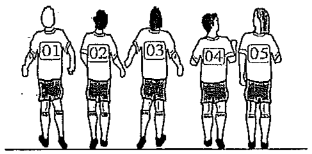
c) The mass of papers recycled by a new factory is doubling every week. 20 kg were recycled on week 1 and 40 kg were recycled on week 2.

i) How many kg were recycled on week 10? **1**

ii) On which week was the mass recycled just over 300 tonnes? **2**

iii) The company earns \$5 for every 4 kg recycled. How much money did the company earned in the first 10 weeks? **2**

d) These soccer players are to come to the field one at the time, what is the probability that:



i) ~~the~~ player 2 comes to the field first? **1**

ii) they come to the field in their correct ascending number order? **1**

iii) player 1 and player 2 come in the correct order and none of the remaining 3 players comes in his correct order? **2**

Question 14

MARKS

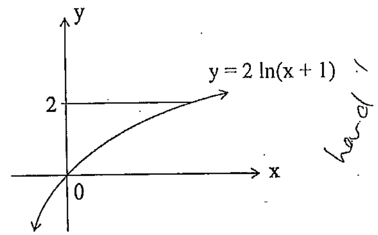
a) Consider the curve $y = x^3 - 6x^2 + 12$

i) Find the coordinates of the turning points of the curve and determine their nature. **2**

ii) Find the coordinates of the point of inflexion. **2**

iii) Sketch the curve showing the turning points and the points of inflexion and the y intercept. **2**

b) The region shaded in the diagram is bounded by $y = 2\ln(x + 1)$, the y axis and $y = 2$. **3**



Find the volume of the solid form when the shaded region is rotated about the y axis.

c) The velocity of a particle moving along the x axis is given by

$$V = 2 + \frac{5}{2t+1}$$

where t is the time in seconds and x is the displacement in metres.

i) Find its initial velocity. **1**

ii) Find the acceleration of the particle. **1**

iii) Explain why the particle is always moving in the positive direction. **2**

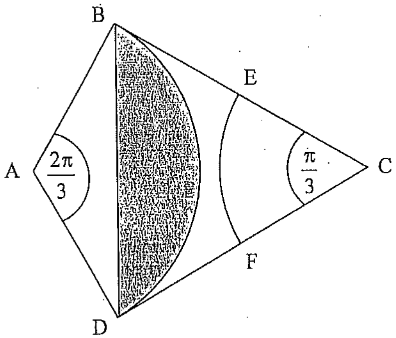
iv) Sketch the graph of the velocity of the particle as a function of time. **2**

Question 15 **MARKS**

a) The diagram ABCD is a kite with $\angle BAD = \frac{2\pi}{3}$ and $\angle BCD = \frac{\pi}{3}$.

The points E and F are the midpoints of sides BC and CD.

The area of the segment with centre A and radius AB is $(4\pi - 3\sqrt{3})\text{cm}^2$.



i) Show that the radius AB is $2\sqrt{3}$ cm. **2**

ii) Prove that ΔECF is similar to ΔBCD. **2**

iii) Find the area of the sector ECF. **2**

b) A particle moves along the x axis. At a time t seconds, its distance x distance from the origin O is given by $x = 2 - \cos 3t$.

i) Sketch the graph of x as a function of t for $0 \le t \le \pi$. **2**

ii) The particle is initially at rest, find the next time it is at rest. **1**

iii) Find the total distance travelled by the particle for $0 \le t \le \pi$. **1**

c) Kevin decided to set up a trust fund for his daughter Tania.
He deposited \$250 at her first birthday and continued to deposit the same amount on each of her birthdays until and including her 17th birthday.

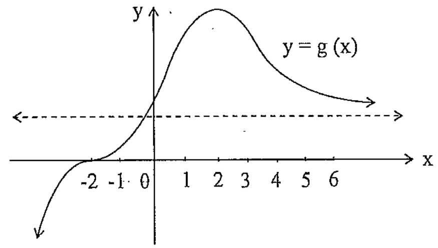
The money is invested at 7.2% per annum compounded monthly and she will be able to withdraw on her 18th birthday.

i) What will be the value of his first deposit on her 18th birthday? **2**

ii) What will be the total value of the fund on Tania's 18th birthday? **3**

Question 16 **MARKS**

a) The diagram shows the graph of $y = g(x)$.



Sketch the graph $y = g'(x)$ **2**

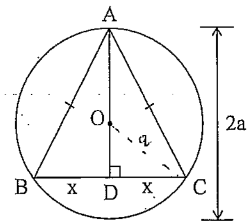
b) The number N of members of a social website can be modelled using the formula $N = Ae^{kt}$ where t is the time in months after its start.

After p months, the number of members reached 16 000, and 4 months after that the number became 64 000.

i) Find the value of k. **3**

ii) Given that after $p + T$ months, the number of members of the website becomes 4 096 000, what is the value of T? **2**

c) The diagram shows an isosceles triangle ABC inscribed in a circle with centre O and diameter 2a.
The side BC has length 2x. The perpendicular from A bisects BC at D.



i) Show that the area of triangle ABC is **2**

$A = ax + x\sqrt{a^2 - x^2}$

ii) Show that the area of triangle is maximum at $x = \frac{a\sqrt{3}}{2}$. **4**

You may use $\frac{d^2A}{dx^2} = \frac{x(2x^2 - 3a^2)}{(a^2 - x^2)\sqrt{a^2 - x^2}}$ (Do Not Prove it)

iii) Show that the triangle ABC has a maximum area when it is equilateral. **2**

2012 Mathematics Solutions

1. D

$(2-\sqrt{a})^2 = 4 - 4\sqrt{a} + a$

so $4 + a - 4\sqrt{a} = b - 4\sqrt{3}$

Hence, $4 + a = b$ and $a = 3$

$4 + 3 = b$

$7 = b$

2. D

$$\frac{d}{dx}(3\cos 2x) = -6\sin 2x$$

3. C

ΔAEB is similar to ΔACD

Hence, $\frac{AE}{AC} = \frac{AB}{AD}$

$\frac{5}{4+x} = \frac{4}{8}$

$16 + 4x = 40$

$x = 6$

4. B

Distance from the vertex (2, 0)
to the directrix $y = -3$ is
3units.
So, the focal length is $a = 3$.
Hence, the equation of the parabola is

$(x-2)^2 = 4 \times 3\,(y-0)$

$(x-2)^2 = 12y$

5. A

$$\int (e^x + 1)^2\,dx = \int (e^{2x} + 2e^x + 1)\,dx$$

$$= \frac{1}{2}e^{2x} + 2e^x + x + c$$

6. C

$y = \ln\left(\frac{2}{x}\right) = \ln 2 - \ln x$

$\therefore \frac{dy}{dx} = -\frac{1}{x}$

7. B

The sum of the 16 pieces is $S_{16} = 4m = 400cm$.
So $400 = \frac{16}{2}(2a + 15d)$ and as a= 15 cm then

$400 = \frac{16}{2}(2 \times 15 + 15d)$

$400 = 8(30 + 15d)$

$50 = 30 + 15d$

$d = \frac{4}{3}$

Now, the length of the 10^{th} piece is

$t_{10} = a + 9d$

$= 15 + 9 \times \frac{4}{3} = 27cm$

8. A

$$\int_0^2 \left(f(x) + 2\right) dx = \int_0^2 f(x)\,dx + \int_0^2 2\,dx$$

$= 8 + [\,2x\,]_0^2 = 8 + (4 - 0) = 12$

9. D

The period of the curve $y = a\tan bx$ is $\frac{\pi}{b}$.
From the diagram, it can be seen that this
period is $\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$. So $\frac{\pi}{b} = \frac{\pi}{2}$ then $b = 2$.
Now, by substituting $\left(\frac{\pi}{6}, 6\right)$ into $y = a\tan 2x$
we get $6 = a\tan\frac{\pi}{3}$ hence $6 = a\sqrt{3}$
that is $a = 2\sqrt{3}$
Hence, the equation is $y = 2\sqrt{3}\tan 2x$

10. A

There are 8 possible ways to select for the first digit and 7 possible ways for the second digit.
So, there are $8 \times 7 = 56$ possible numbers that could be formed.
As the number must be greater than 50, then it must start with 5, 6, 7 or 8. If it starts with either 5 or 7 it should end with one of the 3 odd numbers left
and if it starts with 6 or 8 it should end with one of the 4 odd numbers left.
This means there are a total of 14 possible numbers out of 56, a probability of $\frac{1}{4}$.

Question 11

a) $\dfrac{x^2-2x}{x^2-5x+6}=\dfrac{x(x-2)}{(x-2)(x-3)}$

$=\dfrac{x}{x-3}$

b) If $e^{2x+1}=e^{\frac{1}{2}}$, then

$2x+1=\frac{1}{2}$

$2x=-\frac{1}{2}$

$x=-\frac{1}{4}$

c) $\displaystyle\int_1^e \frac{2}{x}\,dx = 2\int_1^e \frac{1}{x}\,dx$

$=2\,[\ln x]_1^e$

$=2(\ln e-\ln 1)$

$=2(1-0)$

$=2$

d) $\frac{5}{8}+\frac{1}{12}=\frac{34}{48}$

$1-\frac{34}{48}=\frac{14}{48}$

Hence, there are neither roses nor tulip

$\frac{14}{48}\times 792=231$

e) i) $\alpha+\beta=-\frac{b}{a}=\frac{7}{2}$

ii) $\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)$

$=1\times\frac{7}{2}$

$=\frac{7}{2}$

f) $\sqrt{2}\cos x+1=0,\ 0\le x\le 2\pi$

$\sqrt{2}\cos x=-1$

$\cos x=-\frac{1}{\sqrt{2}}$

$\cos x=\cos\frac{3\pi}{4}$

Hence, $x=\frac{3\pi}{4}$ or $\frac{5\pi}{4}$

g) $y=\ln(2x+3)$

$\frac{dy}{dx}=\frac{2}{2x+3}$

when $x=-1$, m tangent $=2$

So, the gradient of the Normal is $-\frac{1}{2}$

and $y=\ln 1=0$.

Hence, the equation of normal is

$y-0=-\frac{1}{2}(x+1)$

$2y=-x-1$

$x+2y+1=0$

Question 12

a) Use the Quotient Rule

Let $u=x^2$ then $u'=2x$

Let $v=\ln x$ then $v'=\frac{1}{x}$

$\frac{dy}{dx}=\frac{vu'-uv'}{v^2}$

$\frac{dy}{dx}=\dfrac{2x\ln x-x^2\times\frac{1}{x}}{(\ln x)^2}$

$\frac{dy}{dx}=\dfrac{2x\ln x-x}{(\ln x)^2}$

b) i) To find the x coordinate of P, we let $y=0$ in the equation of n, we get:

$2x+0-12=0$ so $x=6$.

To find the coordinates of S, we let $x=6$ in the equation of m, we get:

$3\times 6-5y=8$, then $-5y=-10$ so $y=2$

Hence, the coordinates S are (6, 2)

ii) $d=\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$d=\dfrac{|2\times 6+3\times 2-12|}{\sqrt{2^2+3^2}}$

$d=\frac{6}{\sqrt{13}}$ units

iii) Area of triangle QRS $=\frac{1}{2}\times QR\times d$

$\frac{84}{19}=\frac{1}{2}\times QR\times\frac{6}{\sqrt{13}}$

$QR=\frac{84}{19}\times\frac{\sqrt{13}}{3}$

$\therefore QR=\frac{28\sqrt{13}}{19}$ units

c) i) $y=(5\cos 2x)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}(-10\sin 2x)(5\cos 2x)^{-\frac{1}{2}}$

$=\dfrac{-5\sin 2x}{\sqrt{5\cos 2x}}$

ii) $\displaystyle\int\frac{4\sin 2x}{\sqrt{5\cos 2x}}\,dx=-\frac{4}{5}\int\frac{-5\sin 2x}{\sqrt{5\cos 2x}}\,dx$

$=-\frac{4}{5}\sqrt{5\cos 2x}+c$

d) i) The attendance pattern forms an Arithmetic sequence, with $a=825$ and $d=-25$.

$T_{20}=a+19d$

$T_{20}=825-19\times 25=350$

Hence, 350 people attended in the 20th week.

ii) The total number attending was

$S_{20}=\frac{20}{2}(825+350)$

$S_{20}=11750$

Money made $=11750\times\$12$

$=\$141000$

Question 13

a) $h=\frac{4-2}{2}=1$

x	2	3	4
y	$\frac{1}{\ln 2}$	$\frac{1}{\ln 3}$	$\frac{1}{\ln 4}$
	y_0	y_1	y_2

$A\approx\frac{h}{3}[y_0+4y_1+y_2]$

$A\approx\frac{1}{3}\left(\frac{1}{\ln 2}+\frac{4}{\ln 3}+\frac{1}{\ln 4}\right)$

≈ 1.935 units²

b) i) Angle sum of a hexagon

$=180°(6-2)$

$=180°\times 4=720°$

$\therefore\angle DEF=720°\div 6=120°$ (All angles of a regular hexagon are equal)

ii) ED = EF (equal sides of a regular hexagon)

$\therefore\triangle EDF$ is isosceles

$\therefore\angle EFD=\angle EDF$ (base angles of isosceles triangle are equal)

Now, $\angle EFD+\angle EDF+120°=180°$ (angle sum of triangle DEF)

$\therefore 2\angle EFD=60°\ \therefore\angle EFD=30°$

As $\angle AFE=\angle DEF=120°$ (proven in part i)

then $\angle AFD=120°-30°=90°$

c) i) The amounts recycled each week form a Geometric Sequence with a = 20kg and r = 2

As $t_{10}=ar^9$ so $t_{10}=20\times 2^9$

Hence, 10240 kg were recycled in week 10.

ii) The mass recycled was over 300 tonnes if $t_n > 300\,000$ i.e. $20 \times 2^{n-1} > 300\,000$

$2^{n-1} > 15000$

$(n-1) \times \ln 2 > \ln 15000$

$n-1 > \ln 15000 \div \ln 2$

$n > 14.87267$

Hence, the first time the mass was over 300 tonnes was week 15.

iii) The total mass recycled over the 10 weeks was $S_{10} = \dfrac{20(2^{10}-1)}{2-1} = 20460$

Money earned = $(20460 \div 4) \times \$5$
= \$25 575

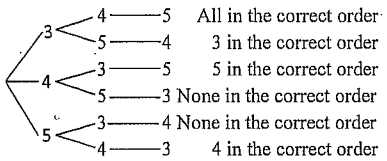
d) i) Probability of player 2 being first = $\dfrac{1}{5}$

ii) The total number of ways they could come to the field is $5 \times 4 \times 3 \times 2 \times 1 = 120$.

They could come in their correct order from smallest to largest in just one way.

Hence, the probability that they come in the correct order is $\dfrac{1}{120}$.

iii) As player 1 and 2 come to the field in their correct order, so, we are concern only with the remaining 3 players.
The diagram shows all the possible ways for these 3 players to come to the field.



Hence, the probability that none of the remaining 3 players will be in his correct position is $\dfrac{2}{6}$ or $\dfrac{1}{3}$.

Question 14

a) i) $y = x^3 - 6x^2 + 12$

$\dfrac{dy}{dx} = 3x^2 - 12x$ (gradient function)

Let $\dfrac{dy}{dx} = 0$ to find the possible stationary turning points.

$\therefore 3x^2 - 12x = 0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0, x = 4$

for $x = 0$, $y = 12$

for $x = 4$, $y = -20$

x		0		4	
$\frac{dy}{dx}$	+	0	−	0	+
y	↗	— max (0, 12)	↘	— min (4, −20)	↗

$\therefore$ the curve has a maximum at (0, 12) and minimum at (4, −20).

ii) $\dfrac{d^2y}{dx^2} = 6x - 12$

Let $\dfrac{d^2y}{dx^2} = 0$ to find the possible points of inflexion.

$\therefore 6x - 12 = 0$

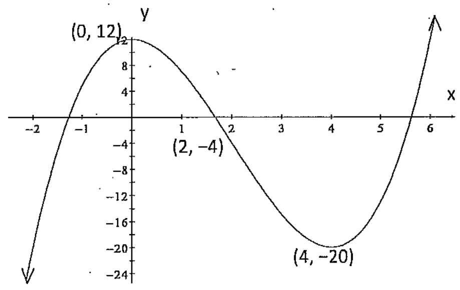
$x = 2$

$y = -4$

x		2	
$\frac{d^2y}{dx^2}$	−	0	+
y	concave down	−4	concave up

$\therefore (2, -4)$ is a point of inflexion.

iii) The curve cuts y axis when x = 0, ie. (0, 12)



b) $y = 2\ln(x+1)$

$\dfrac{y}{2} = \log_e(x+1)$

$e^{\frac{y}{2}} = x + 1$

$x = e^{\frac{y}{2}} - 1$

$\therefore x^2 = (e^{\frac{y}{2}} - 1)^2$

$= e^y - 2e^{\frac{y}{2}} + 1$

$V = \pi \int_0^2 x^2\, dy$

$= \pi \int_0^2 (e^y - 2e^{\frac{y}{2}} + 1)\, dy$

$= \pi \left[e^y - 4e^{\frac{y}{2}} + y \right]_0^2$

$= \pi\left[(e^2 - 4e + 2) - (e^0 - 4e^0 + 0)\right]$

$= \pi(e^2 - 4e + 2 + 3)$

$= \pi(e^2 - 4e + 5)$ units^3

c) i) Initial velocity occur when t = 0, that is v = 2 + 5 = 7m/s

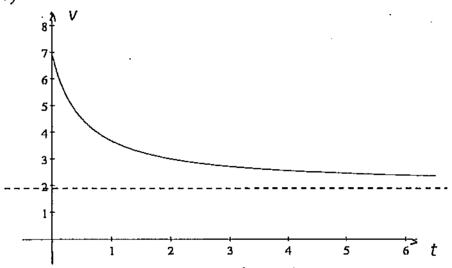
ii) $v = 2 + 5(2t+1)^{-1}$

$\therefore a = \dfrac{dv}{dt} = -5(2t+1)^{-2} \times 2 = -10$

$= \dfrac{-10}{(2t+1)^2}$

iii) The initial velocity of the particle is positive. This indicates that the particle starts moving in the positive direction.
As the velocity is the sum of two positive terms which means it is always positive for all values of t, this indicates that the particle will always move in the positive direction as the particle needs to stop to be able to change direction.

iv)



Question 15

a) i) Area of a segment = $\dfrac{1}{2}r^2(\theta - \sin\theta)$

For this segment AB = r and

$\angle BAD = \theta = \dfrac{2\pi}{3}$

$\therefore 4\pi - 3\sqrt{3} = \dfrac{1}{2}r^2\left(\dfrac{2\pi}{3} - \sin\dfrac{2\pi}{3}\right)$

$8\pi - 6\sqrt{3} = r^2\left(\dfrac{2\pi}{3} - \dfrac{\sqrt{3}}{2}\right)$

$8\pi - 6\sqrt{3} = \dfrac{2\pi r^2}{3} - \dfrac{\sqrt{3}r^2}{2}$

$48\pi - 36\sqrt{3} = 4\pi r^2 - 3\sqrt{3}r^2$

$12(4\pi - 3\sqrt{3}) = r^2(4\pi - 3\sqrt{3})$

$\therefore r^2 = 12$ Hence, $r = 2\sqrt{3}$ cm

ii) In ΔCEF and ΔCBD

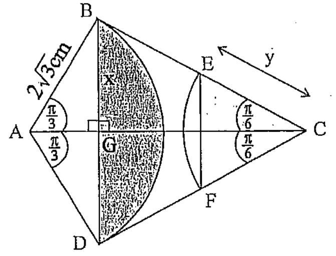
CB:CE = 2:1 (given E midpoint of BC)

CD:CF = 2:1 (given F midpoint of DC)

$\angle BCD$ is common.

Hence ΔECF is similar to ΔBCD (2 pairs of sides in the same ratio and the included angles are equal)

iii)



Construct the diagonal AC in the kite ABCD to meet BD at G.

This diagonal bisects $\angle BAD$ and $\angle BCD$.

Also $\angle BGA = \angle BGC = \frac{\pi}{2}$

In ΔABG

$\sin\frac{\pi}{3} = \frac{x}{2\sqrt{3}}$

$\frac{\sqrt{3}}{2} = \frac{x}{2\sqrt{3}}$ $\quad \therefore$ x = 3cm

From part (ii) EF = ½ BD

$\therefore$ EF = BG = 3, but EH = ½ EF

$\therefore$ EH = ½ × 3 = 1.5 cm

Now, In ΔEHC

$\sin\frac{\pi}{6} = \frac{1.5}{y}$

$\frac{1}{2} = \frac{1.5}{y}$

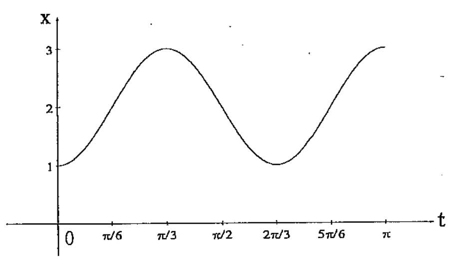
y = 3 cm

$\therefore$ the radius of sector ECF = 3cm

Hence, the area of sector ECF $= \frac{1}{2}\times 3^2 \times \frac{\pi}{3}$

$= \frac{3\pi}{2} cm^2$

b) i)



ii) From the graph, the particle is next at rest at the first turning point after t = 0 which is at

$t = \frac{\pi}{3}$ seconds.

Alternatively,

$v = \frac{dx}{dt} = 3\sin 3t$

Particle at rest when v = 0

$0 = 3\sin 3t$

$3t = 0, \pi, 2\pi, \ldots$

$t = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \ldots$

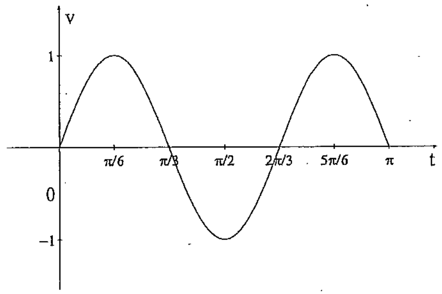
The second time the particle is at rest is when $t = \frac{\pi}{3}$ seconds.

iii) From the displacement graph in part i), it can be seen that the particle moves 2 units in the first $\frac{\pi}{3}$ sec.

It repeats this motion every $\frac{\pi}{3}$ seconds.

Hence, it will move 6 units in π seconds.

Alternatively, the total distant travelled is equal to the area between the velocity curve and the horizontal axis.



The required area between the curve and the t axis, from 0 to π is equivalent to

3 × the area from 0 to $\frac{\pi}{3}$.

$A = 3\int_0^{\frac{\pi}{3}} 3\sin 3t \, dt$

$= 3\left[-\cos 3t\right]_0^{\frac{\pi}{3}}$

= 6 units

c) i) The first \$250 will compound at

r = 7.2% ÷ 12 = 0.6% = 0.006 per month

After 17 years, this amount will be

A_1 = \$250 × 1.006^{204} = \$847.09

ii) The 2nd amount will be

A_2 = \$250 × 1.006^{192}

The 3rd amount will be

A_3 = \$250 × 1.006^{180}

⋮ ⋮ ⋮ ⋮

The 16th amount will be

A_{16} = \$250 × 1.006^{24}

The 17th amount will be

A_{17} = \$250 × 1.006^{12}

The total amount Tania will receive on her 18th birthday will be

S = \$250 × 1.006^{12} + \$250 × 1.006^{24} + …
+ \$250 × 1.006^{192} + \$250 × 1.006^{204}

= \$250 × 1.006^{12} (1 + 1.006^{12} + … + 1.006^{192})

This is a geometric series with a = 1 and r = 1.006^{12}

S = \$250 × $1.006^{12} \times \frac{(1.006^{12})^{17}-1}{1.006^{12}-1}$

S = \$250 × $1.006^{12} \times \frac{1.006^{204}-1}{1.006^{12}-1}$

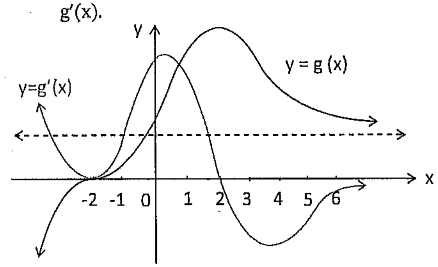
$\therefore$ S = \$8619.85

Question 16

a) Note:

To differentiate g(x) graphically, we should consider the following:

- The x coordinate of the turning points of g(x) becomes x intercepts of g′(x)
- The x coordinate of the points of inflexion of g(x) becomes the x of the turning points of g′(x).



b) i) $N = Ae^{kt}$, where t is the time in months after the start of the website.

When t = p, N = 16000

$\therefore 16000 = Ae^{pk}$ (1)

When t = p + 4, N = 64000

$\therefore 64000 = Ae^{(p+4)k}$ (2)

Dividing (2) by (1), we get:

$4 = e^{4k}$

ln4 = 4k

4k = 2ln2 $\quad \therefore$ k = ½ ln2

ii) When t = p + T, N = 4096000

$\therefore 4096000 = Ae^{(p+T)k}$ (3)

Dividing (3) by (1), we get:

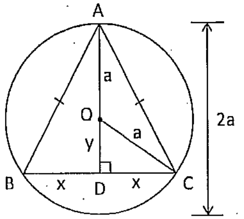
$256 = e^{kT}$

ln256 = kT

$T = \frac{\ln 256}{\frac{1}{2}\ln 2}$

$\therefore$ T = 16 months

c)



i) Using Pythagoras theorem in ΔOCD, we get:

$OD^2 = a^2 - x^2$ that is

$OD = \sqrt{a^2 - x^2}$

$\therefore AD = a + \sqrt{a^2 - x^2}$

Hence, the area of ΔABC is

$$A = \frac{1}{2} \times 2x \times \left(a + \sqrt{a^2 - x^2} \right)$$

$$= x\left(a + \sqrt{a^2 - x^2} \right)$$

$$= ax + x\sqrt{a^2 - x^2}$$

ii) Using the product rule

$u = x \qquad v = \sqrt{a^2 - x^2}$

$u' = 1 \qquad v' = \dfrac{-x}{\sqrt{a^2 - x^2}}$

$$\frac{dA}{dx} = a + \sqrt{a^2 - x^2} - \frac{x^2}{\sqrt{a^2 - x^2}}$$

$$= a + \frac{a^2 - x^2 - x^2}{\sqrt{a^2 - x^2}}$$

$$= a + \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}}$$

Let $\dfrac{dA}{dx} = 0$ to find the possible stationary turning point.

$$\therefore a + \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}} = 0$$

$$\frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}} = -a$$

$$\frac{2x^2 - a^2}{\sqrt{a^2 - x^2}} = a$$

By squaring both sides, we get

$$\frac{(2x^2 - a^2)^2}{a^2 - x^2} = a^2$$

$4x^4 - 4x^2a^2 + a^4 = a^4 - x^2a^2$

$4x^4 - 3x^2a^2 = 0$

$x^2(4x^2 - 3a^2) = 0$

$\therefore x^2 = 0 \quad , \quad 4x^2 - 3a^2 = 0$

$x = 0 \quad \text{or} \quad x = \pm\dfrac{\sqrt{3}\,a}{2}$

As x represents the length of DC it should be positive.

$\therefore x = \dfrac{\sqrt{3}\,a}{2}$

$$f''\left(\frac{a\sqrt{3}}{2}\right) = \frac{\frac{a\sqrt{3}}{2}\left(2 \times \frac{3a^2}{4} - 3a^2\right)}{\left(a^2 - \frac{3a^2}{4}\right)\sqrt{a^2 - \frac{3a^2}{4}}}$$

$$= \frac{\frac{a\sqrt{3}}{2}\left(\frac{-3a^2}{2}\right)}{\frac{a^2}{4} \times \sqrt{\frac{a^2}{4}}}$$

$$= \frac{\frac{-3a^3\sqrt{3}}{4}}{\frac{a^2}{4} \times \frac{a}{2}}$$

$$= \frac{-3a^3\sqrt{3}}{4} \div \frac{a^3}{8}$$

$$= \frac{-3a^3\sqrt{3}}{4} \times \frac{8}{a^3}$$

$$= -6\sqrt{3} < 0$$

As $f''\left(\dfrac{a\sqrt{3}}{2}\right) < 0$,

then the area maximum is at $x = \dfrac{a\sqrt{3}}{2}$

iii) When $x = \dfrac{a\sqrt{3}}{2}$

$$AD = a + \sqrt{a^2 - \frac{3a^2}{4}}$$

$$= a + \sqrt{\frac{a^2}{4}}$$

$$= a + \frac{a}{2}$$

$$= \frac{3a}{2}$$

and $DC = \dfrac{a\sqrt{3}}{2}$

Let $\angle ACB = \alpha$

In ΔADC

$$\tan\alpha = \frac{AD}{DC}$$

$$= \frac{3a}{2} \div \frac{a\sqrt{3}}{2}$$

$$= \frac{3a}{2} \times \frac{2}{a\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \sqrt{3}$$

$\therefore \alpha = 60°$

$\therefore \angle ABC = 60°$ (base angles of isosceles triangle ABC are equal)

$\therefore \angle BAC = 60°$ (remaining angle)

Hence, the triangle ABC is equilateral (all angles are equal) to 60°)

$\therefore$ the area of ΔABC is maximum when ΔABC is equilateral.