St George Girls High School

Year 11

Common Test – 2

2015

Mathematics

Instructions

1. Time – 70 minutes
2. Total Marks: 64
3. Write using blue or black pen.
4. Start each question in a new booklet.
5. Calculators may be used.
6. Attempt all Questions 1 – 8.

Question 1 (8 Marks) – Start on a new sheet — Marks

a) If $a > b > 0$ which of the following is ALWAYS NEGATIVE: — 1

(A) $|-ab|$

(B) $\frac{b}{a}$

(C) $b - a$

(D) a^{-b}

b) Sketch the region defined by $y < \sqrt{2 - x}$. — 3

c) By showing suitable working state whether $f(x) = \frac{3x}{x^2 - 3}$ is odd, even or neither. — 2

d) Draw a neat sketch of $y = |x - 2|$ showing all relevant features. — 2

Question 2 (8 Marks) – Start on a new sheet — Marks

a) The domain of $y = \sqrt{4 - x^2}$ is: — 1

(A) $x \geq 2$

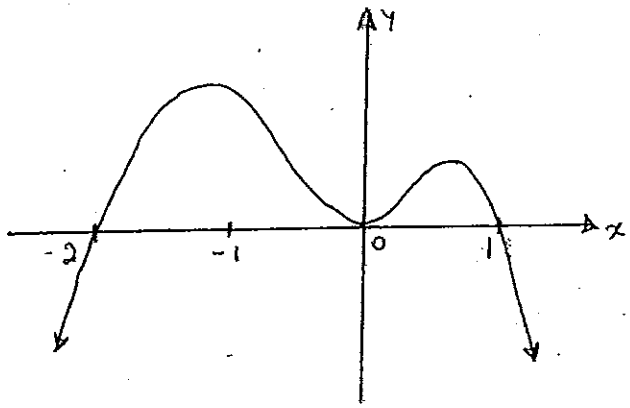
(B) $x \leq -2$

(C) $-2 \leq x \leq 2$

(D) $0 < x \leq 2$

b) Solve $|2x - 1| = 7$. — 3

c) — 2



This is a graph of $y = x^2(x + 2)(x - 1)$. Use it to solve $x^2(x + 2)(x - 1) \leq 0$.

d) Draw a neat sketch to show the solution of $(x + 3)(2 - x) > 0$.

Question 3 (8 Marks) – Start on a new sheet — Marks

a) Draw a neat sketch of $y = x^2 - 2$ and $y = |x|$ on the same axes and use your graph to solve $x^2 - 2 < |x|$. — 3

b) On the sheet at the back of this test complete the graph of $y = f(x)$ given $y = f(x)$ is an even function. — 2

c) Sketch the ODD function that has: — 3

- $f(0) = 0$
- asymptotes at $x = \pm 2$
- when $x < -2$, $f(x) < 0$
- when $-2 < x < 0$, $f(x) > 0$
- as $x \to -\infty$, $f(x) \to 0$

Question 4 (8 Marks) – Start on a new sheet Marks

a) The exact value of $\sin(-225°)$ is: 1

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{-\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{2}}$

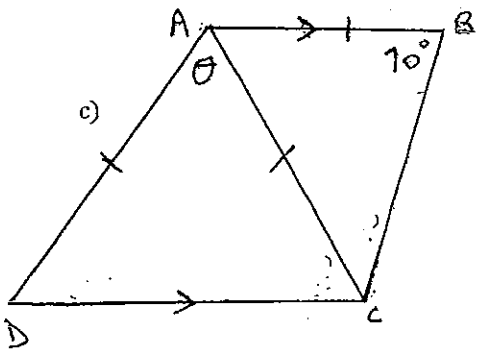
(D) $\frac{-1}{\sqrt{2}}$

b) If $0° < \theta < 90°$ and $\sin\theta = \frac{2}{3}$ write down the EXACT value of: 2

(i) $\cos\theta$.

(ii) $\cot\theta$.

c)



Find the size of θ, giving reason if: 2

$AB = AC = AD$
$AB \parallel DC$
$\angle ABC = 70°$

d) Solve $\sin(x - 280°) = \frac{1}{2}$ for $0° \le x \le 360°$. 3

Question 5 (8 Marks) – Start on a new sheet Marks

a) The area of ΔABC can be found using the formula: 1

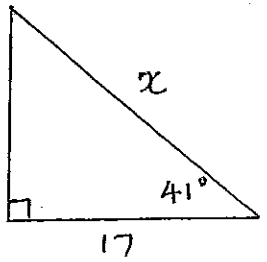
(A) $A = \frac{1}{2}ab\sin C$

(B) $A = \frac{1}{2}ab\cos C$

(C) $ab\sin C$

(D) $ab\cos C$

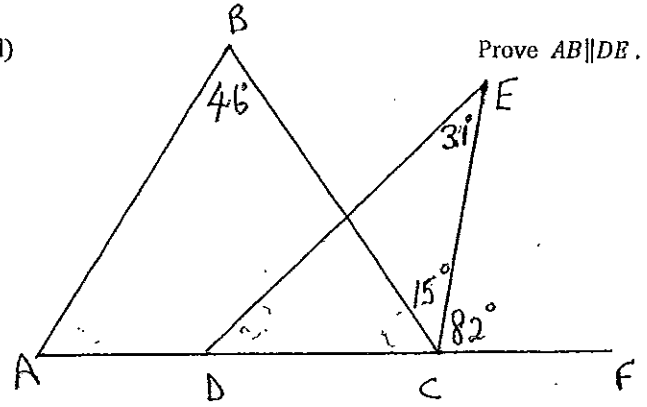
b)



Find x correct to 1 decimal place. 2

c) If $\text{cosec}\,\theta = 1.7$ and θ is obtuse find θ to the nearest degree. 2

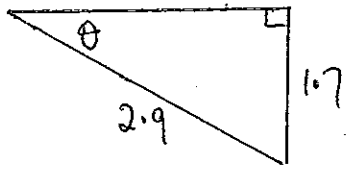
d)



Prove $AB \parallel DE$. 3

Question 6 (8 Marks) – Start on a new sheet — Marks

a) Find θ to the nearest minute. 2

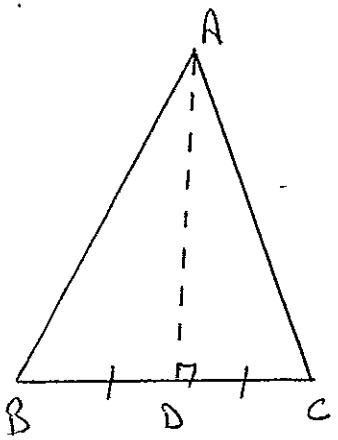


b) A car travels 90km from Town A to Town B on a bearing of $074°T$ and then travels 120km to Town C on a veering of $140°T$.

(i) Calculate the distance AC. 2

(ii) Find the bearing of Town A from Town C. 1

c)

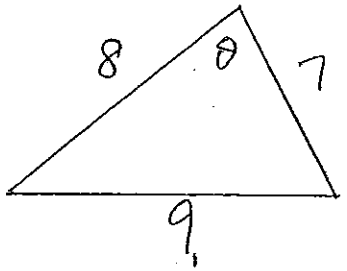


In ΔABC, D is the midpoint of BC and $AD \perp BC$, as shown in the diagram. Prove ΔABC is isosceles by showing $AB = AC$. 3

Question 7 (8 Marks) – Start on a new sheet — Marks

a) Simplify $\dfrac{\cos(90-\theta)}{\sin\theta}$. 1

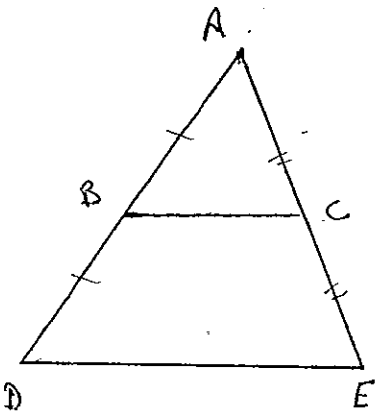
b)



Find θ to the nearest degree. 2

c) If $0° \le \theta \le 720°$ and $\sin\theta = 0$ write down all possible values of θ. 2

d)



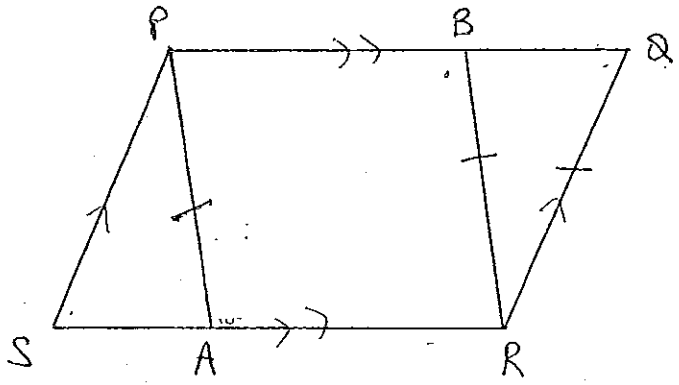
B and C are the midpoints of AD and AE respectively. 3

Prove $BC = \frac{1}{2}DE$.

Question 8 (8 Marks) – Start on a new sheet Marks

a) Prove $(1-\tan x)^2+(1+\tan x)^2=2\sec^2 x$. 3

b)



$PQRS$ is a parallelogram. The point A is on RS, the point B is on PQ and $AP=BR=RQ$ as shown on the diagram.

(i) Why does $\angle PSA=\angle BQR$. 1

(ii) Show $PS=AP$. 1

(iii) Show $\Delta PAS\equiv\Delta RBQ$. 2

(iv) Hence prove $PBRA$ is a parallelogram. 1

End of Paper

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

YEAR 11 2U CT2 2015

QUESTION 1

a) A $|-ab| > 0$

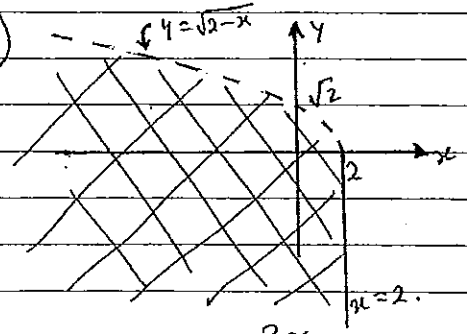
B $\frac{b}{a} > 0$

C. $b - a < 0$ ∴ C

D. $a^{-b} = \frac{1}{a^b}$

> 0

b) $y = \sqrt{2-x}$



c) $f(x) = \frac{3x}{x^2-3}$

$f(a) = \frac{3a}{a^2-3}$ $f(-a) = \frac{3(-a)}{(-a)^2-3}$

$= \frac{-3a}{a^2-3}$

$= -f(a)$

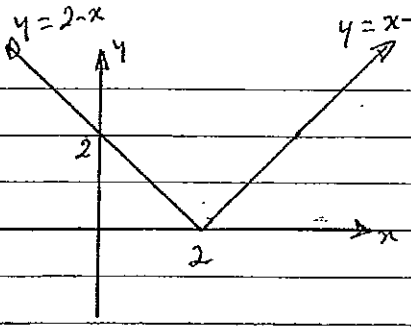
∴ ODD AS $f(a) = -f(-a)$

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

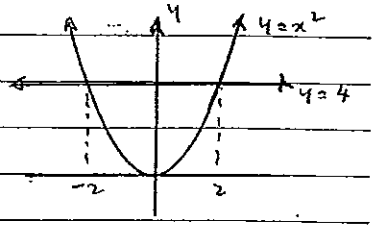
d)



QUESTION 2

a) D: $4 - x^2 \geq 0$

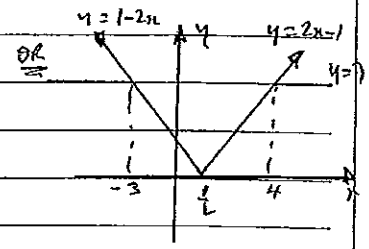
$x^2 \leq 4$



∴ $-2 \leq x \leq 2$

∴ C

b) $|2x-1| = 7$



$2x - 1 = 7$ or $2x - 1 = -7$

$2x = 8$ $2x = -6$

$x = 4$ or $x = -3$

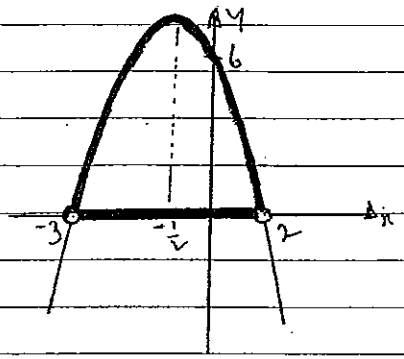
c) $x \leq -2$, $x = 0$, $x \geq 1$

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

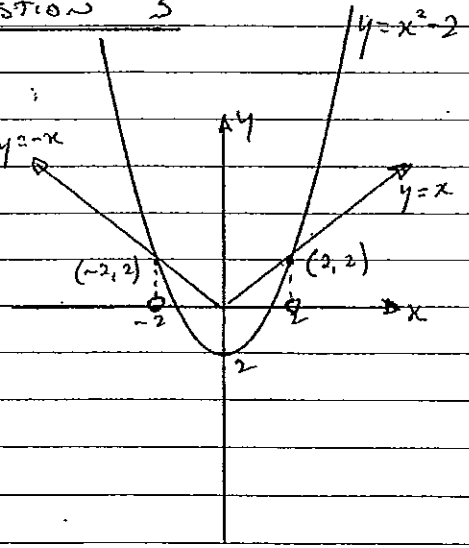
TEACHER: _____

d)



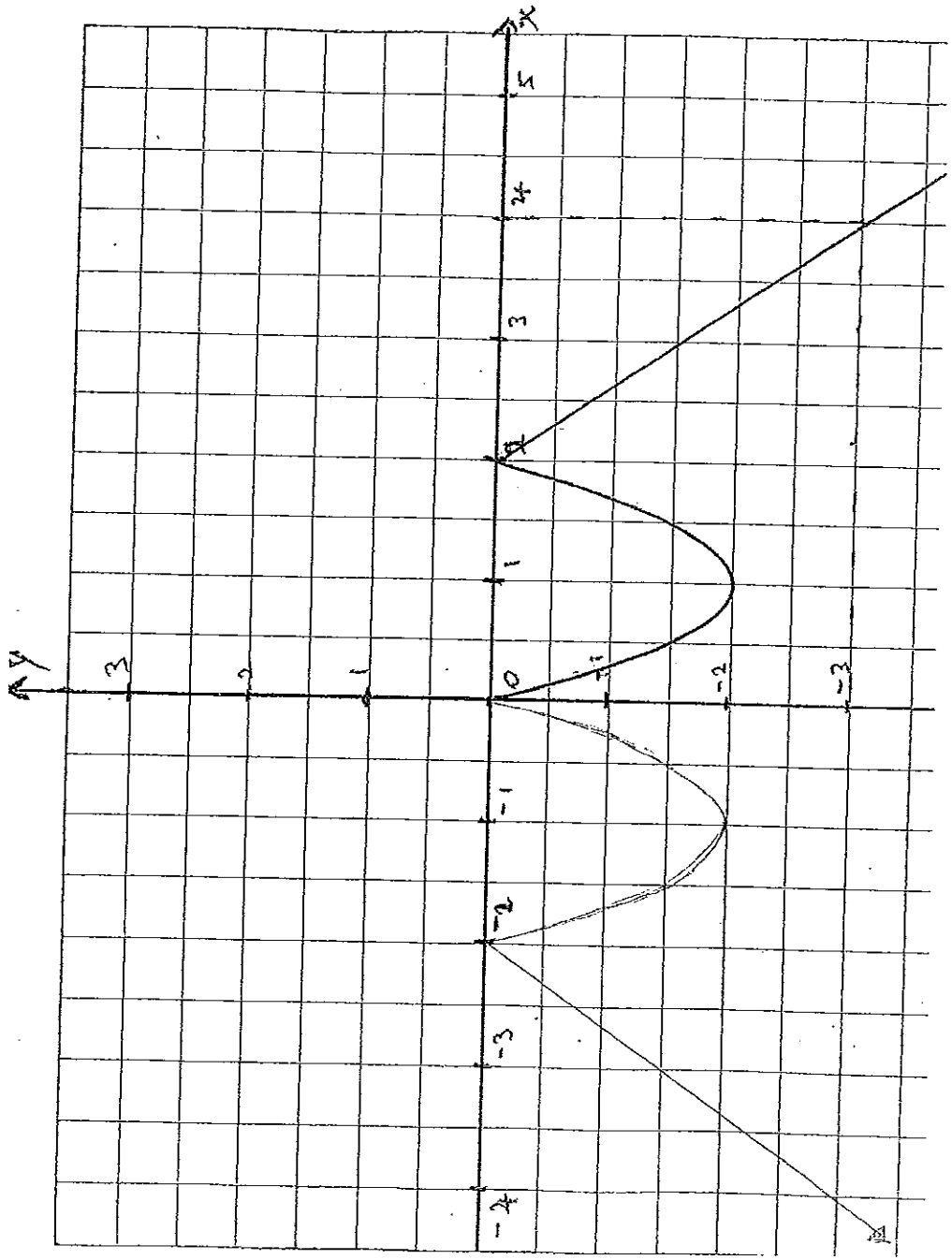
$-3 < x < 2$

QUESTION 3



$-2 < x < 2$

b) ON SHEET



QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

b) $\frac{17}{x} = \cos 41°$

$x = \frac{17}{\cos 41°}$

$x = 22.525221...$

$x = 22.5$ (1 dec. place)

c) $\sin\theta = \frac{1}{1.7}$

$\theta = 144°$ (to nearest degree)

d) Let BC intersect DE at P

$\angle BPE = 31° + 15°$ (exterior angle of $\triangle PEC$)

$\angle BPE = 46°$

As $\angle ABP = \angle BPE$, then AB // DE (alternate angles equal)

OR $\angle EDC + 31° = \angle 82°$ (exterior angle of $\triangle ECD$)

$\angle EDC = 51°$

$\angle BAC + 46° = 97°$ (exterior angle of $\triangle BAC$)

$\angle BAC = 51°$

As $\angle EDC = \angle BAC$ then AB // DC (corresponding angles equal)

Question 6

a) $\sin\theta = \frac{1.7}{2.9}$

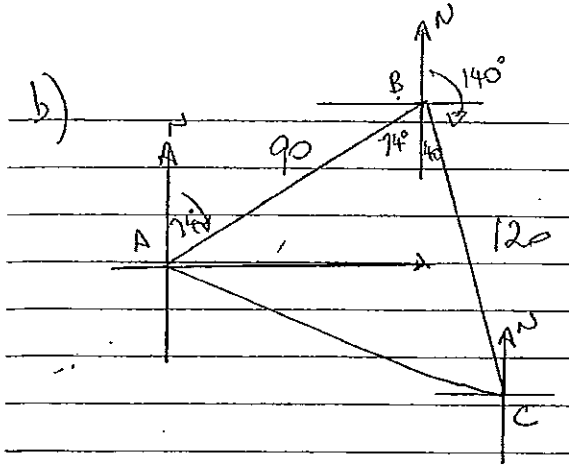
$\theta = 35° 53'$ (to nearest minute)

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

b)



$AC^2 = 90^2 + 120^2 - 2(90)(120)\cos 114°$

$= 8100 + 14400 - (21600)\cos 114°$

$= 31\,285.511...$

$AC = 176.87711...$

AC is 177 km

$AC = 177$ km

ii) $\frac{\sin C}{90} = \frac{\sin 114°}{AC}$

$\sin C = \frac{90 \sin 114°}{AC}$

$\sin C = 0.464837...$

$C = 27.699...$

Bearing of A from C $= 360° - (40° + 28°)$T

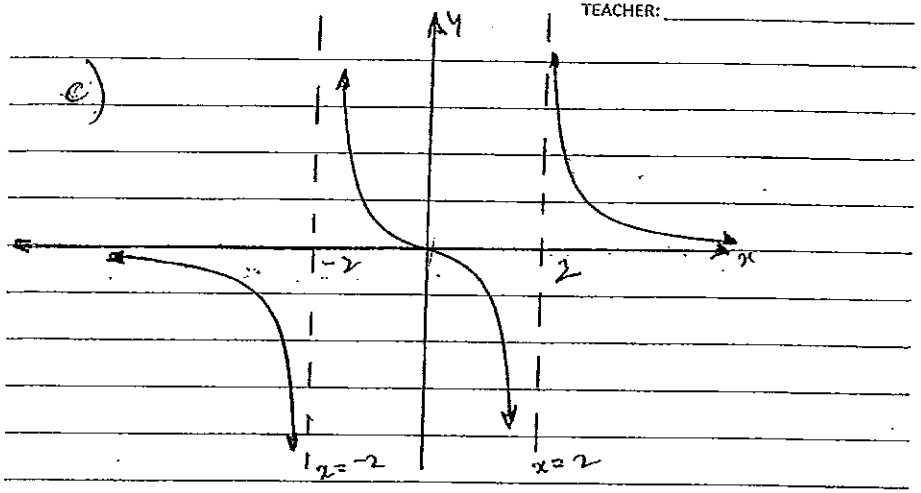
$= 292°$T

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

c)



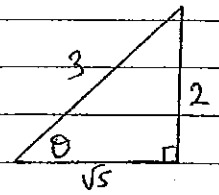
Question 4

a) $\sin(-225°) = \sin 135°$

$= \sin 45°$

$= \frac{1}{\sqrt{2}}$ ∴ C

b)



(i) $\cos\theta = \frac{\sqrt{5}}{3}$

(ii) $\cot\theta = \frac{\sqrt{5}}{2}$

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

c) $\angle ACB = 70°$ (AB = AC (given); base angles of isosceles ΔABC)

$\angle BAC = 40°$ (angle sum of ΔABC)

$\angle ACD = 40°$ (alternate angles; AB // DC)

$\angle ADC = 40°$ (AD = AC (given); base angles of ΔADC)

∴ $\theta = 100°$ (angle sum of ΔADC)

d) $\sin(x - 280°) = \frac{1}{2}$ as $\sin 30° = \frac{1}{2}$

where $0° \leq x \leq 360°$

$-280° \leq x - 280° \leq 80°$

So $x - 280° = 30°$ or $-210°$

$x = 310°$ or $70°$

Question 5

a) A

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

Question 7

a) $\frac{\cos(90-\theta)}{\sin\theta} = \frac{\sin\theta}{\sin\theta}$

$= 1$

b) $\cos\theta = \frac{8^2+7^2-9^2}{2\times8\times7}$

$= \frac{64+49-81}{112}$

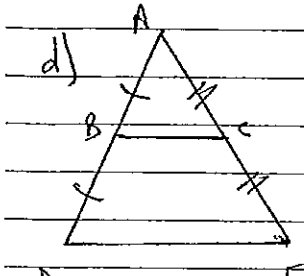
$= \frac{2}{7}$

$\theta = 73°$ (to nearest degree)

c) $\sin\theta = 0$ $\sin\theta$ = y co-ordinate

$\theta = 0°, 180°, 360°, 540°, 720°$

d)



$\frac{AB}{AD} = \frac{1}{2}$

$\frac{AC}{CE} = \frac{1}{2}$

$\angle A$ is common

$\therefore \triangle ABC \,|||\, \triangle ADE$ (two pairs of sides in proportion; included angles equal)

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

$\therefore \frac{BC}{DE} = \frac{1}{2}$ (ratio of sides in similar triangles)

$\frac{1}{2}DE = BC$

Question 8

a) $(1-\tan x)^2 + (1+\tan x)^2 = 2\sec^2 x$

$LHS = (1-\tan x)^2 + (1+\tan x)^2$

$= 1 - 2\tan x + \tan^2 x + 1 + 2\tan x + \tan^2 x$

$= 2 + 2\tan^2 x$

$= 2(1 + \tan^2 x)$

$= 2\sec^2 x$ as $1+\tan^2 x = \sec^2 x$

$= RHS$.

b) i) Opposite angles of a parallelogram 1

ii) $PS = QR$ (opposite sides of a parallelogram) 1/2

$AP = QR$ (given) 1/2

$\therefore PS = AP$ (both equal to QR)

iii) $PS = PA = BR = QR$ (by (ii) and given)

In $\triangle APS$ $PS = AP$ (given) $\therefore \triangle APS$ is isosceles

$\therefore \angle PSA = \angle PAS$ (base angles of isosceles triangle)

Similarly in $\triangle BQR$ $\angle RQB = \angle RBQ$

$\therefore \angle PSA = \angle BQR$

QUESTION NUMBER: _____ NAME: _____

CLASS: _____

TEACHER: _____

$\therefore \triangle PAS \equiv RBQ$ (AAS)

(iv) $\angle APB = \angle PAS$ (alternate angles ; PQ // SR)

$\angle QBR = \angle PAS$ (corresponding angles in congruent triangles)

AP // BR (corresponding angles [$\angle APB$, $\angle QBR$] are equal),

and PB // AR (as PQ // SR)

$\therefore$ PBRQ is a parallelogram as opposite sides parallel [PQ // SR from original parallelogram ; AP // BR (above)]

OR $PQ = PB + BQ$

AND $SR = AR + AS$ and $PQ = SR$ (sides of a parallelogram)

$BQ = AS$ (corresponding sides in congruent triangles)

So $PB = AR$

$\therefore$ PBRQ is a parallelogram as it has one pair of opposite sides parallel and equal