

Sydney Technical High School

Trial HSC Certificate Mathematics 2012

Name	 	 	
7			
Teacher			

General Instructions

- ▶ Reading time 5 minutes
- Working time 3 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11–16

Total marks - 100

Section I Pages 2-5 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–13 90 marks
Attempt Questions 11–16
Allow about 2 hours 45 minutes for this section

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a \neq 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}) \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x$, x > 0

Rationalize the denominator of

(A)
$$\frac{9-\sqrt{2}}{7}$$

(B)
$$\frac{9-\sqrt{2}}{79}$$

(C)
$$\frac{9+\sqrt{2}}{11}$$

(D)
$$\frac{9+\sqrt{2}}{83}$$

The solution of the equation 3(p-2) = 5p + 2 is

(A)
$$p = -4$$

(B)
$$p = -2$$

(C)
$$p = -1$$
 (D) $p = 1$

D)
$$p = 1$$

ABC is an equilateral triangle. CDEF is s square.

BCG, ACF, and GFH are straight lines.

$$\angle CGF = 45^{\circ}$$

The size of $\angle EFH$ is

(A) 15°

30°

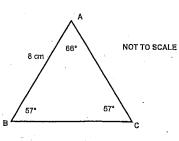
45°

The area of $\triangle ABC$ in square centimetres is closest to.

13.0 (A)

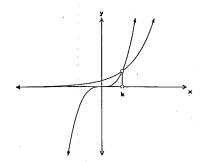
26.8 (B)

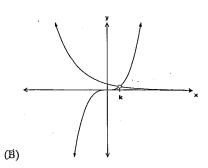
(D) 58.5



E

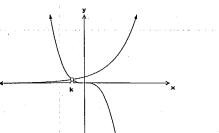
Which of the following could the value of k, be a solution of $2^x - x^3 = 0$

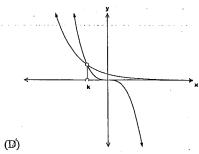






(C)





Given $y = x(x-5)^2$, find the turnings points and determine their nature.

- (A) Minimum at $=\frac{5}{3}$, maximum at x=5
- (B) Minimum at x = -5, maximum at $x = \frac{3}{5}$
- (C) Minimum at $x = \frac{5}{3}$
- (D) Minimum at $=\frac{3}{5}$, maximum at x=-5

Evaluate $\int_{1}^{4} x^5 + 2x^3 - 6x^2 - 10 \, dx$

 $-11\frac{1}{3}$

Which term of the geometric sequence is 5, 15, 45 ... is 885 735?

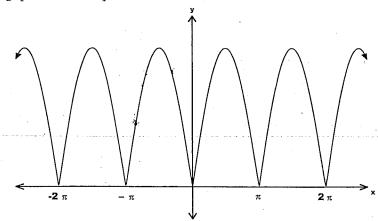
(A) 10^{th}

(B) 11th

(C) 12th

(D) 13th

9. The graph below could represent



(A) $y = \sin |x|$

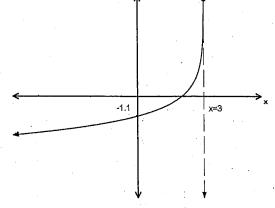
(B) $y = 1 - |\cos x|$

 $(C) |y| = |\sin x|$

(D) $y = |\sin x|$

10. The equation of the graph shown could be

- $(\cancel{A}) \quad y = log_e(3-x)$
- (B) $y = -\log_e(3-x) .$
- $(C) y = e^{(x-2)} .$
- (D) $y = -\log_e(3x 1)$



Section II

Total Marks (90)

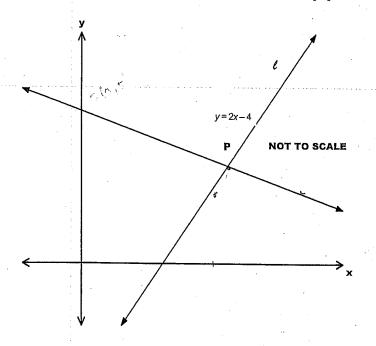
Attempt Questions 11 - 16

Allow about 2 hours 45 minutes for this section.

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page.

Question 11 (15 Marks)

The sketch below shows the line PQ and the line (l) y = 2x - 4, which is perpendicular to PQ



- (i) Show that the point R(3,2) lies on the line L
- (ii) Q is the point (0,5). Find the midpoint of QR.
- (iii) Find the equation of the line PO.
- (iv) Find the gradient of QR.
- (v) Find the distance QR in simplest surd form.
- (vi) Find the distance PQ.

2

1

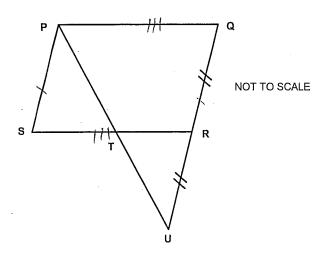
2

1

. . .

•

- The nth term of an arithmetic series is given by $T_n = 3n + 4$.
 - What is the 12th term of the series?
 - What is the sum of the first 20 terms of this series?
- In the diagram, PQRS is a parallelogram. QR is produced to U so that QR=RU. Copy this diagram into your answer booklet



- (i) Giving clear reasons, show that the triangles PST and URT are congruent.
- Hence, or otherwise, show that T is the midpoint of SR.

(15 Marks)

3

Find the values of q if $3qx^2 - 5x + 3q = 0$ is negative definite. Leave your answer in exact form.

b. Prove that
$$\frac{\sin \theta}{\sin \theta} + \frac{\sin \theta}{\sin \theta} = 2 \cos \theta$$

Differentiate:

ii)
$$x^2 \ln x$$

Find:

(i)
$$\int \frac{\cos x}{1 + \sin x} dx$$

$$(ii) \qquad \int_0^{\frac{2\pi}{3}} \sin\frac{x}{2} \ dx$$

e. Given that
$$\frac{d^2y}{dx^2} = \frac{2}{x^2} + 2e^{2x}$$
 and that when $x = 1$, $\frac{dy}{dx} = e^2$ and $y = \frac{e^2}{2}$, find an expression for y in terms of x with no other unknown values.

End of Question 12

Question 13 (START A NEW PAGE)

(15 Marks)

Question 14 (START A NEW PAGE)

(15 Marks)

a. The graphs of $y = 5x - x^2$ and $y = x^2 - 3x$ intersect at the origin and at point A.

(i) Show that the co ordinates of A are (4,4)

(ii) Draw a neat sketch of the two graphs on the same number plane. (use a ruler)

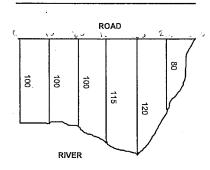
(iii) Find the area enclosed by the two graphs.

4

2

1

A developer wishes to know the area of land bounded by a straight road, a river and a fence at right angles to the road. A developer hires a surveyor who at 40 metre intervals along the road, measures the distance between the road and the river. Shown below is the surveyor's sketch of this area. All measurements are in metres.

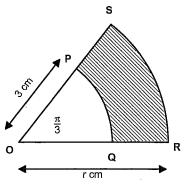


Using Simpson's rule, find the area of the land

A(-1,3) and B(3,1) are two points on the plane. Find the locus of P(x,y) such that PA is perpendicular to PB.

3

In the diagram below PQ and RS are arcs of concentric circles with centre O. $\angle POQ = \frac{\pi}{3}$ radians and OP = 3cm.



(i) Find the area of the sector OPQ

1

(ii) If OR is r cm, find the area of the sector OSR in terms of r.

2

(iii) If the shaded area is $\frac{27\pi}{6}$ cm², find the length of PS

2

b. Consider the curve given by $y = 2 + 3x - x^3$

(i) Find $\frac{dy}{dx}$

1

ii) Locate the stationary points and determine their nature

3

(iii) For what values of x is the curve concave up?

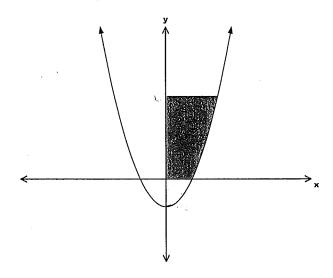
1

(iv) Sketch the curve, for $-2 \le x \le 2$

2.

End of Question 13

The diagram shows the region bounded by the curve $y = 2x^2 - 2$ the line y = 6 and the x and y axis.



Find the volume of the solid of revolution when the region is rotated about the y-axis.

End of Question 14

Question 15	(START A	NEW PAGE)
-------------	----------	-----------

(15 Marks)

3

3

2

a. Solve the following:

 $log_2 x + log_2 (x + 7) = 3$ for x > 0

Find the equation of the normal to the curve $y = x \sin x$ at the point where $x = \frac{\pi}{2}$

c. Consider the function $g(x) = \frac{2}{x^2 - 1}$

) Show that g(x) is an even function

ii) State the domain of y = g(x)

d. Maria is saving for a cruise, She opens an 'Incentive Saver Account' which pays interest at the rate 0.4% per month compounded monthly at the end of each month. Maria decides to deposit \$400 into an account on the first of each month. She makes her first deposit on 1st January 2010 and her last on 1st July 2012. She withdraws the entire amount, plus interest immediately after her final payment on 31st July 2012.

(i) How much did Maria deposit into her 'Incentive Saver Account'?

(ii) How much did Maria withdraw from her account on 31st July 2012?

(iii) Maria's holiday is cancelled due to illness. She then decides to deposit the amount saved for her holiday, into a different account which offers interest at a rate of 5% p.a. compounded quarterly for 2 years. How much will Maria receive at the end of the investment period.

End of Question 15

Question 16 (START A NEW PAGE)

(15 Marks)

- Find the volume generated when the curve $y = \sqrt{\cot x}$ is rotated about the x-axis between $x = \frac{\pi}{3}$ and $x = \frac{\pi}{4}$. Leave your answer in exact form.
- 3

- b. For the parabola with equation $x^2 = -8y$.
 - (i) Find the coordinates of the focus (S) of the parabola.

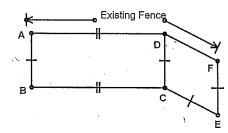
1

(ii) Find the equation of the directrix of the parabola.

1

(iii) Show that the point A(-8, -8) lies on the parabola.

- 1
- (iv) Find the equation of the focal chord of the parabola which passes through A.
- Find the equation of the tangent to the parabola at A.
- A farmer needs to construct two holding paddocks, one rectangular and the other a rhombus for horses and cattle respectively. The diagram below shows the aerial view from directly above the paddocks and how she uses an existing long fence as part of the boundary.



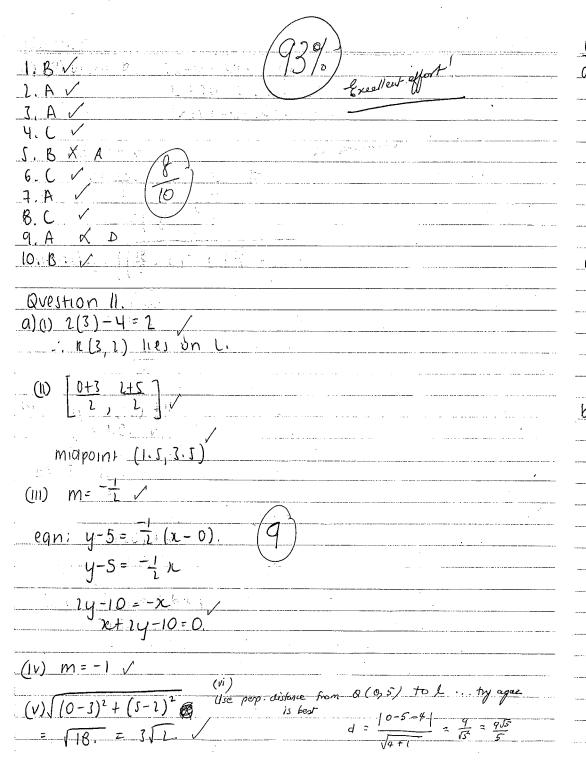
The famer only has 700 m of fencing at her disposal. We also know that $\angle CDF = 30^{\circ}$ By letting AB = x, prove:

(i) A, the area of the paddocks is given by $A = 700x - \frac{7x^2}{2}$

2

(ii) Hence, find the maximum area.

3



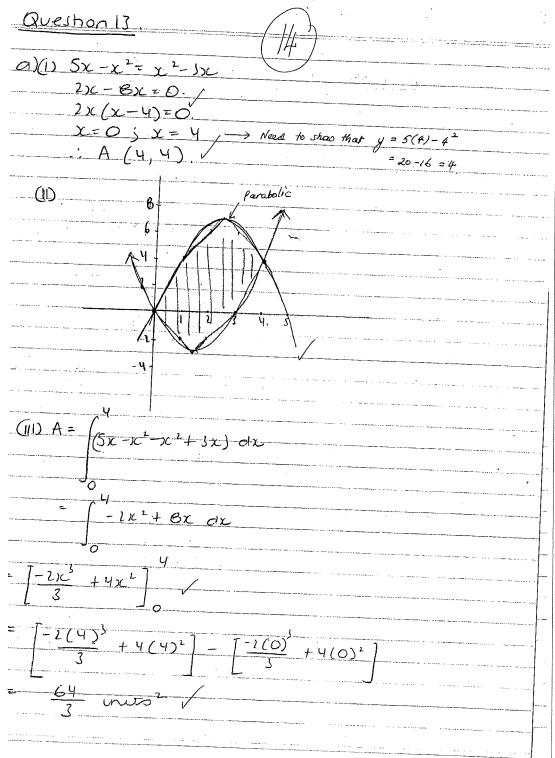
```
avestion 11.
a) (VI) point P: 2x+y-4=0 0
 \rho\left(\frac{16}{5},\frac{16}{5}\right)
distance \log \left( \left( \frac{16}{5} - 0 \right)^2 + \left( \frac{16}{5} - 5 \right)^2 \right)
 3) 100 110 K albart 12
b) (1) T12 = 3 (11) + 42 = = = 40 = 51/2 = 41/3 = 42/2 = 2
 (1) T<sub>1</sub>=3(1)+4.
    T10=3(10)+4
  S10 = 10 [7+64]
```

Question 11.
$C)(1)$ $\Delta P J T = \Delta U K T$
1. PJ=RU (opposite sides of parallelogram are equal)
2. IPTS= LKTU (vertically opposite)
3. LSPT= LTUR (alternate ongles equal on parallel lines Ps) (Qu)
· AAS
(11) PQ=JR (opposite sides of parallelogram are equal)
ST = SR (corresponding sides of congnient mangues
are equality
-: TIS midpointy (3)
Question 12
a) Negative definite: DTO and aTO.
$\Delta = b^2 - 4aC$
$= (-5)^2 - 4(39)(39)$.
$= 25 - 36q^2 < 0.$
(5+6q)(5-6q) (O.
6. 6.
-5 5 9 5
6 . 6
·
·

Question 11.	Question 12
C)(i) APIT = DUKT	
1. PJ=RU (opposite sides of parallelogram are equal)	b) sno + sno = 2 cose co
2. LPTS=LKTU (vertically opposite)	1-6010 1+6010
3. LSPT= LTUR (alternate ongles equal on parallel ines rs) au)	
· AAS	LHS: SIND (1+cost) +SIND (1-cost)
(I) PO - 1 P / DISOLUD (GLOU DA GORDINA DA GOLDINA)	1-60520
(11) PQ = JR (opposite sides of parallelogram are equal)	= (notradio) + to a
ST= SR (corresponding sides of congnient mangues are equal)	= ZINO+INOCOIO + INO - VINO COIO
: Tis midpointy (3)	
	= 25n0
	510 ² 8
	2
Question 12	$=\frac{2}{S\Omega\Theta}$
a) Negative definite: DTO and aTO.	= 200180
$\Delta = b^2 - 4aC$	
$= (-5)^2 - 4(39)(39)$.	<u> </u>
$= 25 - 369^2 \text{ CO}$.	c) (1) tan 1x
(5+69)(5-69) TO.	$\frac{d}{dx} = 2 \sec^2 2x$
	dix /
6. 6.	(1) $\times^2 \ln x$
	U= x2 U1= 1x V= Inx V1= 1c
$\frac{-5}{6}$ $\frac{9}{6}$ $\frac{5}{6}$	$d = 2x \log x + x$
	dx 2

Question 12 d)(1) $\int \frac{\cos x}{1 + \sin x} dx$ + \int + COJIC Use your standard check

1 + SINIC integrals to check = + In (1+sinx) + c $\left[-2\cos\frac{x}{2}\right]^{3}$ $= \left[-2\cos\frac{2\pi}{3} \right] - \left[-2\cos 3\cos\frac{2\pi}{3} \right]$ $= \left[-2 \times 0.5\right] - \left[-2 \times 1\right]$ 1) y"= 2x-2+2e s, but at xol; yo $\int 2x^{-2} + 2e^{2x}$ $y' = -2x' + e^{2x} + c'$ but a+x=1; y1=e2 $-2(1)^{-1}+e^{2}+c=e^{2}$ 41=-1×1+e2x+2 $\int -2x^{-1} + e^{2x} + 2$ y= 21/1x+1e2x+1x+c



10								
b) <u>X</u>	0	40	80	120	160	200	1240.	
<u> </u>	100	100	100	112	120	80. I	0. v	/

$$A = \frac{40}{3} \left[\frac{100 + 100 + 4(100)}{3} \right] + \frac{40}{3} \left[\frac{100 + 120 + 4(115)}{3} \right] + \frac{40}{3} \left[\frac{120 + 0 + 4(80)}{3} \right]$$

$$=\frac{1}{3}8000 + \frac{27200}{3} / + \frac{17600}{3}$$

$$m, \times m, =-1$$
 (perpendicular).

$$\frac{y-3}{x+1} \times \frac{y-1}{x-3} = -1$$

$$(y-3)(y-1) = -(x+1)(x-3)$$

$$y^2 - 4y + 3 = -(x^2 - 2x - 3)$$

$$x^2 - 2x + y^2 - 4y = 0$$

$$(x-1)^2 + (y-2)^2 = 5$$

Question 14.

$$a)(1) A = \frac{1}{2} v^{2} \theta$$

$$= \frac{1}{2} (3)^{2} \left(\frac{\pi}{3} \right)$$

$$= \frac{3\pi}{2} \text{ units}^{2} /$$

(11)
$$A = \frac{1}{4} \times V^2 \times \frac{\pi}{3}$$

$$\frac{= V^2 \pi \quad \text{onts}^2/}{6}$$

$$\frac{\text{(II)} \quad \frac{Y^2 \pi}{6} \quad \frac{3\pi}{2} = \frac{27\pi}{6}$$

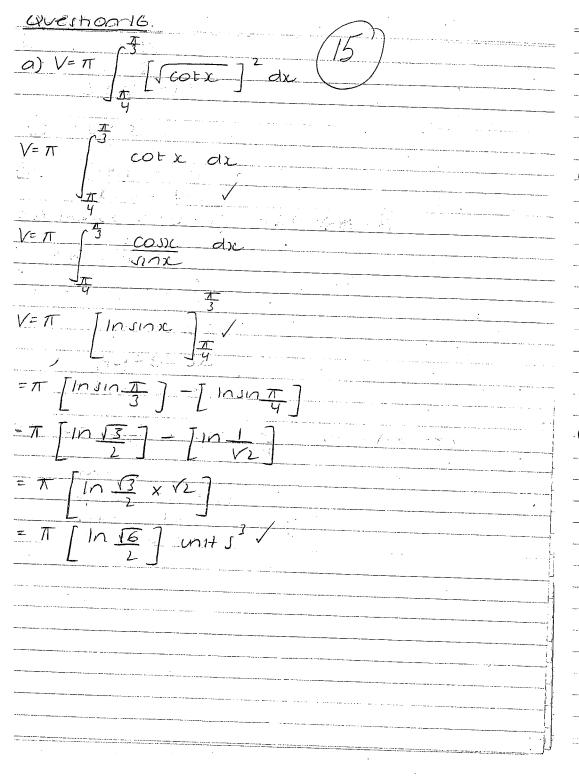
$$\frac{2^{2}\pi - 9\pi - 27\pi}{6}$$

Question 14. (11) Statpts when dy = 0. 3(1+x)(1-x)=0. x=1 ; x=-1when x=1; $\frac{d^{2}y}{dx^{2}}$ (0 (max) uner x=-1; dzy >0 (mn) -- Stat pts (1, 4) and (-1,0) (111) Concave up unes dry >0. (1,4) (-1,0)

• • • • • • • • • • • • • • • • • • • •	:
avestion 14.	
c) y= 2x2-2	
2x2=y+2	
x2= 4+2/	
2	
$x = \sqrt{y+2}$	
V 22	
V-75 6 5 - 2	2
$V = \pi \int_{-\infty}^{\infty} \int \frac{y+2}{2} dy$	
Jo	
76	
V= 1 6 y+2 dy	
<u> </u>	
(- [1/2]]	
$\sqrt{2\pi}$ $\left[\frac{y^2+y}{4}\right]$	
V=T [(6)2 1 C 7 7 1:(0)2	7
$V = \pi \left[\frac{(6)^2}{4} + 6 \right] = \left[\frac{(0)^2}{4} + 0 \right]$	
V= T IS	
=15 T units3	
graduation and the North Company	

Question 15 a) $\log_2 x + \log_2 (x+7) =$ $log_{1} \times (x+2)=3$ $\chi(x+7)=2^3$ X24716-3=0. x2+7x-8=0 (x+8)(x-1)=0 x=-7 + 181 27-8 or 1 Since x >0 but 16 >0. 1. x = -7+VGI & 0.4011. b) y=xsinx pant (# #) Y'= SINX + XCOSX m+ongent = 1 mnormal = -1 egn: $y - \frac{T}{2} = -(x - \frac{T}{2})$ 4- T = -x + T 24-11=-2x+1 $2)c + 2y - 2\pi = 0$. $x+y-\pi=0$ c)(1) $g(x) = \frac{2}{x^2-1}$ $g(-)c) = \frac{c}{(-x)^2 - 1}$ $=\frac{2}{\chi^2-1}$ q(x)=q(-x) .. even, function (11) Domain: X2-1+0. x +1 ; x + -1

```
<u>Question 15</u>
d) (1) 400 x 31
    = $12 400
  (1) A,=400(1.004)
    A,=400(1.004)31 + 400 (1.004) 29
    Az=400(1.004)1 + ... + 400(1.004)
Total: 400(1.004) + ... + 460(1.004)
400 [1.004(1.004]) 7
         0.004
 = $13 226,29 (20p)
(11) Amount saved = $13 226.29
 13226.29(1.0125)
 = $14 608.25 /
```



```
Question 16.
b) (1) 401=B
   (-8)^2 = -8x - 8/
     KHS= LHJ
(v) through (0,-2) and (-B,-B)
 egn: y+2= 3/(x-0)
 y+2 = 3 >c
   4478=32
 3x-44-8=0.
(v) y = -\frac{x^2}{8} y' = -\frac{x}{4} point (-8, -8)
 m=2
y+8=2(x+8)

y+8=2x+16./

2x-y+8=0.
```

```
Question 16.
c) (1) 4x ty = 700.
 A= xy + 2 ( 1 x2 sm30)
 = \chi (100 - 4\chi) + \frac{2\chi^2}{4}
    = 700x-4x2 + x2
               Tx2 / as rego
 (1) dA = 700-716
      Stat pts Ox =0.
   700 - 1 \times = 0
         JC=100,
when x=100; dre <0 (max stat pt)
  " max area when x=100m
  maxarea: A = 700 (100) - 7 (100)2
           = 35 000 m<sup>2</sup>
```