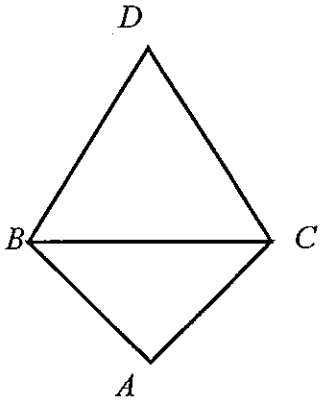
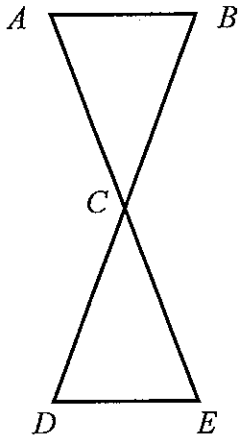
Outcome 4 – Geometry and Similarity (25 Marks)

1. **5**

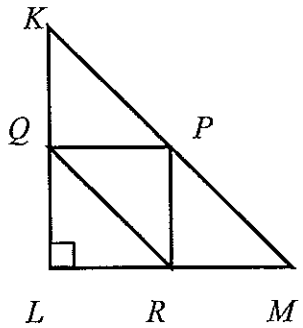
(a) ΔABC is a right-angled isosceles triangle. ΔDBC is equilateral. Find the size of $\angle ABD$.



(b) $AC = BC = DC = EC$. Prove that AB and DE are parallel.



2. ΔKLM is right-angled at L. Q, P and R are the midpoints of the sides of ΔKLM. $KL = 15$ cm, $LM = 10$ cm. Find the: **4**

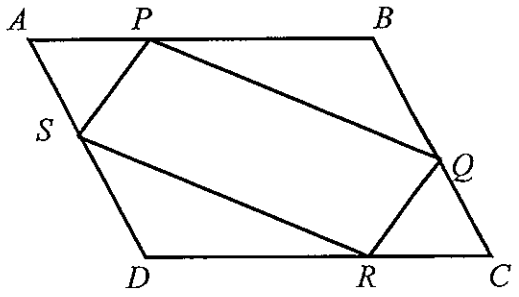


(i) area of ΔKLM

(ii) area of ΔPQR

(iii) ratio of area ΔLRQ to area trapezium $KQRM$. Show all reasons.

3. $ABCD$ is a parallelogram. $AP = AS = CQ = CR$. By using congruent triangles, or otherwise: **5**

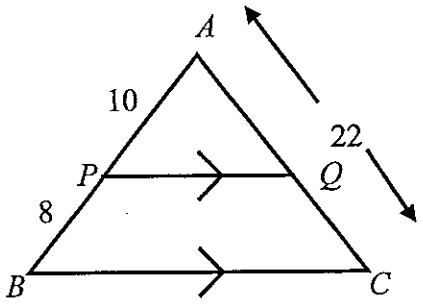


(i) Prove $QR = PS$ and $PQ = SR$.

(ii) What shape is $PQRS$? Justify your answer.

4. (i) Prove that $\triangle APQ$ and $\triangle ABC$ are similar. **4**

(ii) Hence find the length of PQ.



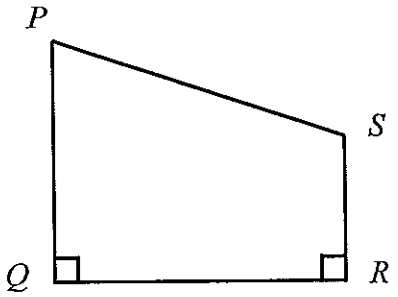
5. Two similar cones have surface areas in the ratio 256:81. What is the ratio of the corresponding: **2**

(i) slant heights (ii) volumes.

6. In the quadrilateral $PQRS$, $PQ \perp QR$ and $SR \perp QR$ **5**

(i) Prove that $PR^2 - QS^2 = PQ^2 - RS^2$

(ii) Hence, prove that $PS^2 - QR^2 = (PQ - RS)^2$



END OF EXAM.

Outcome ④

① (a) $\angle ABC = \frac{1}{2}(180^\circ - 90^\circ)$ (equal base $\angle$ of right isos $\triangle$)
$= 45^\circ$

$\angle DBC = 60^\circ$ ($\triangle DBC$ equilateral)

$\therefore \angle ABD = 105^\circ$

(b) $AC = EC$ (Given)
$BC = DC$ (Given)

$\angle ACB = \angle DCE$ (vert. opp. $\angle$s)

$\therefore \triangle ABC \equiv \triangle DCE$ (SAS)

$\therefore \angle BAC = \angle DEC$ (base $\angle$s of congruent isosceles $\triangle$s)

$\therefore AB \parallel DE$ (alternat $\angle$s equal)

② (a) $75\ cm^2$

(b) $\frac{75}{4}\ cm^2$

(c) $\triangle LRQ \equiv \triangle PRQ \equiv \triangle QPK \equiv \triangle RMP$

Area Trapezium = Area $\triangle PRQ$ + Area $\triangle QPK$ + Area $\triangle RMP$
$= 3 \times$ Area $\triangle LRQ$

$\therefore$ Ratio $= 1:3$

③ (i) $\angle PAS = \angle QCR$ (opp. $\angle$s of $\parallel$gram)
$AP = QC$ (Given)
$AS = RC$ (Given)
$\therefore \triangle APS \equiv \triangle QCR$ (SAS)
$\therefore QR = PS$

$AB = DC$ (Opp sides of $\parallel$gram)

$\therefore BP = AB - AP$
$= DC - RC$ ($AP = RC$ → given)
$= DR$

Similarly $BQ = DS$

$\angle SDR = \angle QBP$ (" " $\parallel$gram)

$\therefore \triangle SDR \equiv \triangle QBP$ (SAS)

(ii) Parallelogram (Opposite sides are equal)

④ (i) $\angle APQ = \angle ABC$ (corresp. $\angle$s in $\parallel$ lines)

$\angle AQP = \angle ACB$ (")

$\therefore \triangle APQ \parallel\parallel \triangle ABC$

(ii) $\frac{AQ}{22} = \frac{10}{18}$ (sides of sim $\triangle$s)

$AQ = 12.2$

⑤ (a) $16:9$

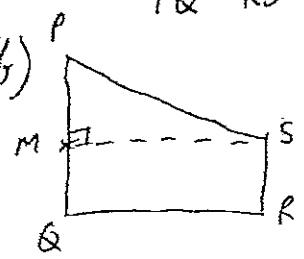
(b) $16^3 : 9^3$

$= 4096 : 729$

⑥ (a) $PR^2 = PQ^2 + QR^2$

$PR^2 - QS^2 = PQ^2 + QR^2 - QS^2$
$= PQ^2 - (QS^2 - QR^2)$
$= PQ^2 - RS^2$

(b)



$PS^2 = QR^2 + PM^2$

$PS^2 - QR^2 = PM^2$
$= (PQ - QM)^2$
$= (PQ - RS)^2$