

Scales and similarity

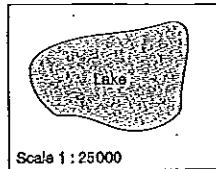
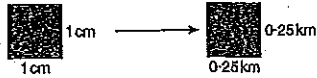
Scales and areas on maps

On the map, the lake has area 4 cm^2 . What is its real area in km^2 ?

1 cm on the map is 25 000 cm on the ground.

$25\,000\text{ cm} = 250\text{ m} = 0.25\text{ km}$.

So 1 cm^2 on the map is $0.25 \times 0.25\text{ km}^2$ on the map = 0.0625 km^2 .



Therefore 4 cm^2 on the map is really $4 \times 0.0625\text{ km}^2 = 0.25\text{ km}^2$.

Similar objects

Example

These glasses are similar.

The area of the base of the small glass is 30 cm^2 and it holds 400 cm^3 .

Find h and the base area and capacity of the large glass.

The glasses are similar, so the large glass is an enlargement of the smaller. Looking at the tops, we can see that the scale factor is $\frac{12}{8} = 1.5$.

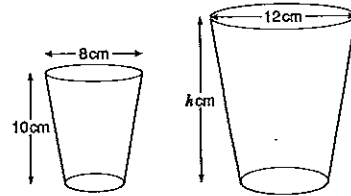
So the length $h\text{ cm} = 1.5 \times 10\text{ cm} = 15\text{ cm}$.

If the length scale factor is 1.5, then the area scale factor is 1.5^2 .

So the base area of the large glass is $30 \times 1.5^2\text{ cm}^2 = 67.5\text{ cm}^2$.

If the length scale factor is 1.5, then the volume scale factor is 1.5^3 .

So the volume of the large glass is $400 \times 1.5^3\text{ cm}^3 = 1350\text{ cm}^3$.



Enlargement \rightarrow page 58
Ratio and proportion
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Remember:

Area scale factor is the square of the length scale factor.

Volume scale factor is the cube of the length scale factor.

- The scale on an Ordnance Survey map is 1:50000.
 - On the map, the distance from Longdon to Upton is 12 cm. What is the actual distance in kilometres?
 - Upton Meadows have an area of 20 cm^2 on the map. What is the real area of the Meadows in square kilometres?
- The Popigai Crater in Siberia is a large circular crater made by the impact of an asteroid or comet. The diameter of the crater is 100 km. A map of Siberia is drawn to a scale of 1 to 8000000. What is the diameter of the crater on the map in millimetres?

MEG/ULEAC (SMP)

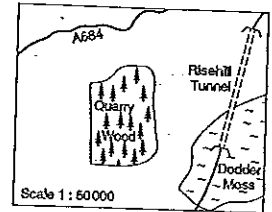
- On a clear day, if you stand at Nefyn in North Wales (sea level) and look towards Snowdon, another mountain, Garnedd-goch, blocks the view. From the map, the distance from Nefyn to Garnedd-goch is 21 km, and from Garnedd-goch to Snowdon is 11 km.

The height of Snowdon is given as 1085 m.

What is the lowest the height of Garnedd-goch could be?

MEG/ULEAC (SMP)

- A map of part of the Yorkshire Dales is drawn to a scale of 1:50000.
 - On the map the length of Risehill Tunnel is 2.3 cm. Calculate the actual length of the tunnel in kilometres.
 - Quarry Wood covers an area of 2 cm^2 on the map. Calculate the actual area of Quarry Wood in hectares. (1 hectare = $10\,000\text{ m}^2$)



- In Xian, China, you can buy solid scale models of the famous Terracotta Warriors. A model 16 cm tall weighs 270 grams and has an armour plate of area 9 cm^2 .
 - Calculate the armour plate area on a similar model of height 24 cm.
 - Calculate the weight of the same 24 cm model.

MEG

MEG (SMP)

- A £1 coin has diameter 22 mm. In 1989 a £2 coin was issued to commemorate the tercentenary of the Bill of Rights. Its diameter is 28 mm. Its mass is double that of the £1 coin and it is made of the same metal.



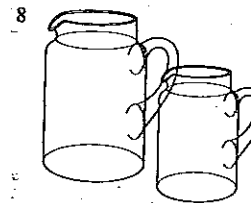
Are they mathematically similar solids (apart from the designs)?

Give reasons and show your working.

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MEG/ULEAC (SMP)

- Sudsy shampoo comes in plastic bottles. Two different sizes are available, and the two bottles are similar. The height of the large size is 1.3 times the height of the small size. The small size costs £1.40 and the large size costs £2.40. Which size gives you more for your money? Explain clearly how you worked it out.



At dinner in a restaurant in France, we had on our table a 50 cl jug for wine and a 25 cl jug for milk. The jugs were mathematically similar in shape. The smaller jug was 15 cm high.

What was the height of the larger jug?

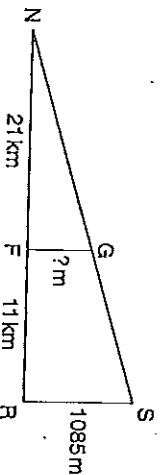
MEG/ULEAC (SMP)

Answers and hints \rightarrow page 130

Scales and similarity (page 60)

- 1 (a) 1 cm on the map is really 50 000 cm
 $= 500 \text{ m} = 0.5 \text{ km}$ on the ground.
 So 12 cm is really 6 km.
- (b) 1 cm stands for 0.5 km, so
 1 cm^2 stands for $0.5 \times 0.5 \text{ km}^2 = 0.25 \text{ km}^2$.
 So 20 cm^2 stands for $20 \times 0.25 \text{ km}^2 = 5 \text{ km}^2$.
- 2 1 mm on the map $= 8000000 \text{ mm}$ on the ground
 $= 800000 \text{ cm} = 8000 \text{ m} = 8 \text{ km}$.
 So 100 km on the ground $= 100 \div 8 \text{ mm}$
 $= 12.5 \text{ mm}$ on the map.
 Alternatively convert 100 km to mm and then
 divide by 8000000.

- 3 First you need a sketch, supposing that Neŷn (N) and the tops of Snowdon (S) and Gamedd-goch (G) are in a straight line.



SRN is an enlargement of GFN, with scale factor $\frac{32}{21} = 1.523 \dots$
 So $GF = SR \div 1.523 \dots = 712.03 \dots \text{ m}$.
 So Gamedd-goch must be at least 720 m high.
 Note that we have rounded up here (because Gamedd-goch must be higher than GF), but it does not really matter much! An answer of 710 m would be quite acceptable.

- 4 (a) 1 cm on the map is 50000 cm on the ground
 $= 500 \text{ m} = 0.5 \text{ km}$.
 So 2.3 cm on the map $= 2.3 \times 0.5 \text{ km}$ on the ground $= 1.15 \text{ km}$
 $= 1.2 \text{ km}$ (to 2 s.f.)
- (b) 1 cm represents 50000 cm $= 500 \text{ m}$, so
 1 cm^2 represents $500 \times 500 \text{ m}^2 = 250000 \text{ m}^2$
 $= 250000 \div 10000 \text{ hectares} = 25 \text{ hectares}$.
 So 2 cm^2 represents $2 \times 25 \text{ hectares}$
 $= 50 \text{ hectares}$.

- 5 (a) The length scale factor from the small to the large model is $\frac{24}{16} = 1.5$.
 So the area scale factor is 1.5^2 .
 So the armour area $= 9 \times 1.5^2 \text{ cm}^2$
 $= 20.25 \text{ cm}^2$
 $= 20 \text{ cm}^2$ (to 2 s.f.)
- (b) Length scale factor $= 1.5$, so volume scale factor $= 1.5^3 = 3.375$.
 So weight (which is proportional to volume)
 $= 270 \times 3.375 \text{ grams}$
 $= 911.25 \text{ grams}$
 $= 910 \text{ grams}$ (to 2 s.f.).

- 6 If the coins are similar, then the length scale factor $= \frac{28}{22} = 1.2727 \dots$
 So the volume factor will be $(1.2727 \dots)^3 = 2.061 \dots$
 In actual fact we are told the weight (volume) factor is 2. The difference between 2 and 2.061 ... is probably within the tolerances we are given in the question, so they probably are similar.

- 7 The volume factor is $1.3^3 = 2.197$.
 If you could buy them, 2.197 of the small bottles would cost $2.197 \times \text{\pounds}1.40 = \text{\pounds}3.0758 = \text{\pounds}3.08$ (to nearest penny).
 This would cost more than the $\text{\pounds}2.40$ cost of the large size, so the large size gives you more for your money.

- 8 The volume scale factor is $\frac{50}{25} = 2$.
 So the length scale factor $= \sqrt[3]{2} = 1.2599 \dots$
 So the large jug is $15 \times 1.2599 \dots \text{ cm}$ high
 $= 18.898 \dots \text{ cm} = 19 \text{ cm}$ (to 2 s.f.).

More help or practice

Similarity \rightarrow Book Y1 pages 81 to 85
 Effects of enlargement on length, area and volume
 \rightarrow Book Y4 pages 9 to 19