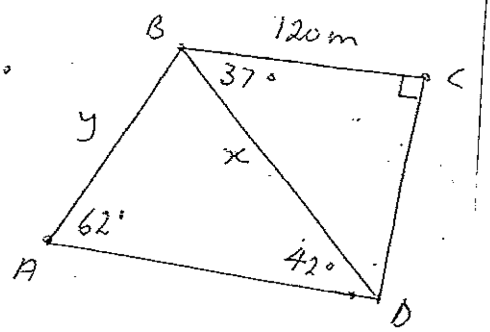
SYDNEY GIRLS H.S - 2007

Class Test Year 10 Name

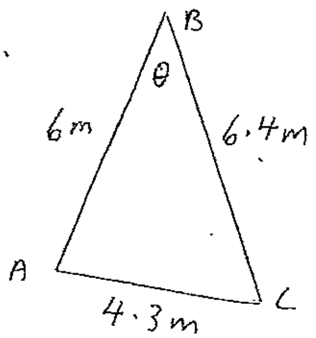
1.



a) Find x (to 2 d.p.)

b) Find y (to 2 d.p.)

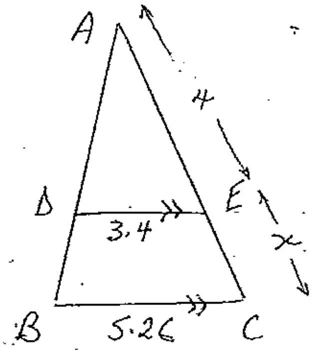
2.



a) Find $\angle ABC$ (to nearest minute)

b) Find area of $\triangle ABC$ to 2 d.p.

3. Prove $\triangle ABC ||| \triangle ADE$ and find value of x (to 2 d.p.) $DE \parallel BC$.

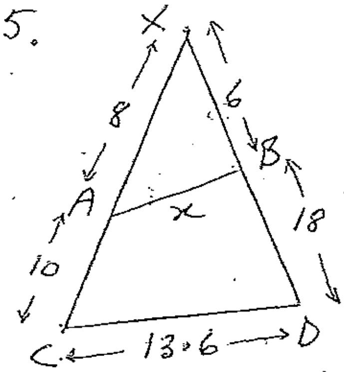


4. Sharon sailed her boat 26 km on a bearing of 042° T from Sydney Harbour to Mimi Island. She then sailed 37 km on a bearing of 126° T and arrived at Minh-Anh Reef.

a) Draw a diagram to show journey.

b) Find the distance and bearing Minh-Anh reef is from Sydney Harbour.

5.



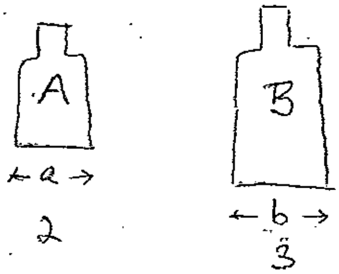
Prove $\Delta CXD \;|||\; \Delta BXA$ and find x

6. Two similar solid have their widths is a ratio of 2:3. What is the ratio of their

a) Surface Areas

b) Volumes

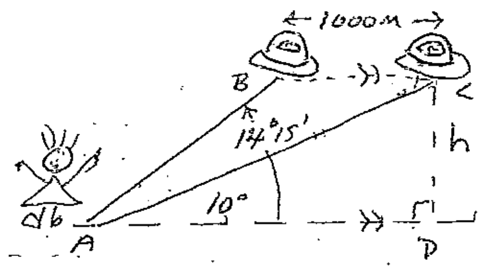
7. The solids below a similar with the ratio of their widths being $a:b$



a) Find the volume of B if the volume of A is $1000\,cm^3$ and $a:b = 2:3$

b) Find the surface area of B if the surface area of A is $1000\,cm^2$ and their volumes are in a ratio of $1:10$ (to 2 d.p.)

8. Two UFO's are at the same altitude and 1000 metres apart. Olivia is at ground level and observes them to have angles of elevation of $14°15'$ and $10°$. Find the height of the UFO.



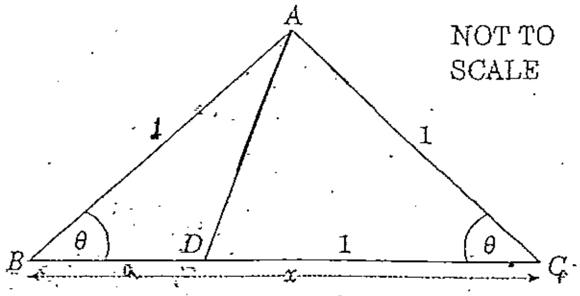
9. Solve for $0 \leq x \leq 360°$

a) $3\sin x = 2$

b) $\cos 2x = -0.8 \quad 0 \leq x \leq 720°$

c) $2\tan^2 x - 8 = 0$

10.



NOT TO SCALE

In the diagram, ABC is an isosceles triangle where $\angle BAC = 108°$ and $AB = AC = 1$.

A 1

The point D is chosen on BC such that $CD = 1$. Let $BC = x$, and let $\angle ABC = \theta$, and note that $\theta = 36°$

(i) Show that $\angle ADC = 2\theta$ and hence show that triangles DBA and ABC are similar.

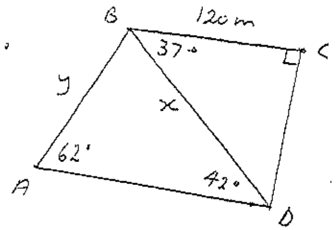
(ii) From part (i) deduce that
$$x^2 - x - 1 = 0.$$

(iii) By using the cosine rule, deduce that
$$\cos\theta = \frac{1+\sqrt{5}}{4}.$$

Class Test Year 10

Name Solutions

1.



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a) Find x (to 2 d.p.)

$\cos 37° = \frac{120}{x}$ $\therefore x = \frac{120}{\cos 37°}$

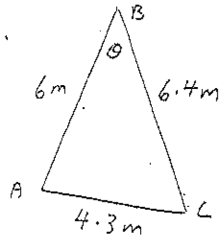
$x \doteq 150.26$ m ✓✓

b) Find y (to 2 d.p.)

$\frac{y}{\sin 42°} = \frac{x}{\sin 62°}$ ✓✓

$y = \frac{\sin 42° x}{\sin 62°} \doteq 113.87$ (to 2 d.p.)

2.



17
17
7
13
43

a) Find ∠ABC (to nearest minute)

$\cos\theta = \frac{6^2 + 6.4^2 - 4.3^2}{2(6)(6.4)} \doteq 0.76$

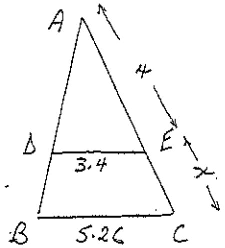
$\therefore \theta = 40°25'$ ✓✓

b) Find area of △ABC to 2 d.p.

$A = \frac{1}{2}(6)(6.4)\sin\theta$

$A = 12.45\,m^2$ (to 2 d.p.) ✓✓

3. Prove △ABC ||| △ADE and find value of x (to 2 d.p.).

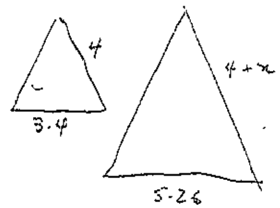


DE // BC

∠ABC = ∠ADE (corresponding ∠'s DE // BC) ✓

∠A is common ✓

∴ △ABC ||| △ADE (equiangular)



$\therefore \frac{4+x}{4} = \frac{5.26}{3.4}$ ✓

$4 + x = 4 \times \frac{5.26}{3.4}$

$x = \frac{4 \times 5.26}{3.4} - 4$ ✓

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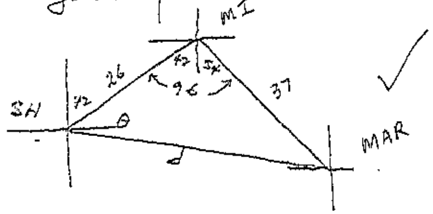
4. Sharon sailed her boat 26 km on a bearing of 042°T from Sydney Harbour to Mimi Island. She then sailed 37 km on a bearing of 126°T and arrived at Minh-Anh Reef.

a) Draw a diagram to show journey.



✓

b) Find the distance and bearing Minh-Anh reef is from Sydney Harbour.

$d^2 = 26^2 + 37^2 - 2(26)(37)\cos 96$

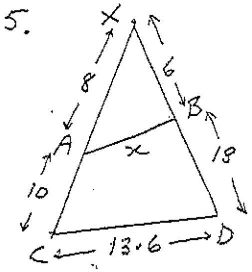
$d \doteq 47.39$ km (to 2 d.p.) ✓✓

$\frac{37}{\sin\theta} = \frac{d}{\sin 96}$ $\therefore \sin\theta = \frac{37 \times \sin 96}{d}$

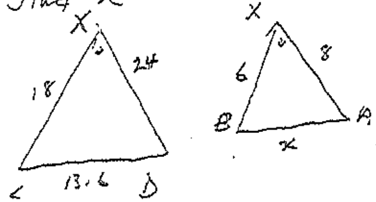
$\angle\theta = 50°56'$ ✓

∴ bearing $= 50°56' + 42° = 092°56'$T ✓

5.



Prove △CXD ||| △BXA and find x



$\frac{CX}{XB} = \frac{18}{6} = 3$ $\frac{XD}{XA} = \frac{24}{8} = 3$ ✓

∠CXD = ∠BXA (given) ✓

∴ △CXD ||| △BXA (two sides in same ratio and equal included angle)

$\frac{x}{13.6} = \frac{1}{3}$, $x = \frac{13.6}{3} = 4.5\dot{3}$ ✓✓

6. Two similar solid have their widths is a ratio of 2 : 3. What is the ratio of their

a) Surface Areas

4 : 9 ✓

b) Volumes

8 : 27 ✓

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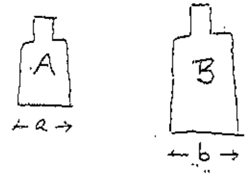
7. The solids below a similar with the ratio of their widths being $a : b$



a) Find the volume of B if the volume of A is $1000 cm^3$ and $a : b = 2 : 3$

$\frac{B}{1000} = \frac{27}{8}$ $\therefore B = \frac{1000 \times 27}{8}$

$B = 3375$

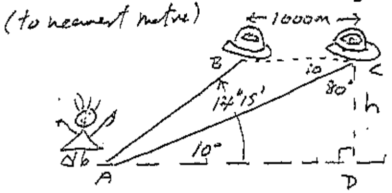
b) Find the surface area of B if the surface area of A is $1000 cm^2$ and their volumes are in a ratio of $1 : 10$. (to 2 d.p.)

Ratio of SA $= (\sqrt[3]{1})^2 : (\sqrt[3]{10})^2$
$= 1 : 10^{2/3}$

$\frac{B}{1000} = \frac{10^{2/3}}{1}$

$\therefore B = 1000 \times 10^{2/3} = 4641.59$ (to 2 d.p.)

8. Two UFO's are at the same altitude and 1000 metres apart. Olivia is at ground level and observes them to have angles of elevation of $14°15'$ and $10°$. Find the height of the UFO (to nearest metre)



$\angle CAB = 4°15'$ $\angle ACB = 10°$

$\angle ACD = 80°$

$\therefore \angle ABC = 180 - (4°15' + 10°) = 165°45'$

$\frac{AC}{\sin 165°45'} = \frac{1000}{\sin 4°15'}$

$AC = \frac{1000 \sin 165°45'}{\sin 4°15'} = 3321.52$

$\sin 10° = \frac{h}{AC}$ $\therefore h = AC \sin 10°$

$\therefore h = 576.78$

or $h = 577$ metres (to nearest metre)

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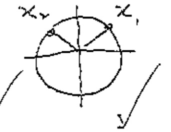
9. Solve for $0 \leq x \leq 360°$

a) $3 \sin x = 2$



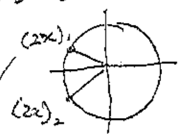
$\sin x = 2/3$

$x_1 = 41°49'$

$x_2 = 180° - 41°49' = 138°11'$

b) $\cos 2x = -0.8$

$(2x)_1 = \cos^{-1}(-0.8) = 143°8'$



$(2x)_2 = 360 - 143°8'$
$= 216°52'$

$(2x)_3 = 143°8' + 360 = 503°8'$

$(2x)_4 = 576°52'$

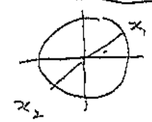
$\therefore x = 71°34', 108°26', 251°34', 288°26'$

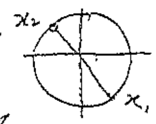
c) $2\tan^2 x - 8 = 0$

$\tan^2 x = 4$

$\therefore \tan x = 2$ or $\tan x = -2$

$x_1 = 63°26'$ $x_1 = -63°26'$





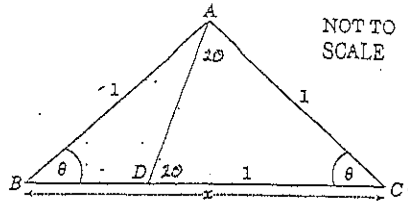
$x_2 = 180° + 63°26'$

$\therefore x_2 = 243°26'$

$x_1 = 360 - 63°26'$
$= 296°34'$

$x_2 = 180 - 63°26'$
$= 116°34'$

10.



In the diagram, ABC is an isosceles triangle where $\angle BAC = 108°$ and $AB = AC = 1$.

The point D is chosen on BC such that $CD = 1$.

Let $BC = x$, and let $\angle ABC = \theta$, and note that $\theta = 36°$

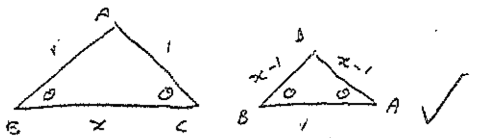
(i) Show that $\angle ADC = 2\theta$ and hence show that triangles DBA and ABC are similar.

(ii) From part (i) deduce that $x^2 - x - 1 = 0$.

(iii) By using the cosine rule, deduce that $\cos\theta = \frac{1+\sqrt{5}}{4}$.

i) $\angle ADC = \frac{180 - 36}{2} = 72°$ (a/

$\theta = 36$ $2\theta = 72° = \angle ADC$



from ext. angle triangle $\angle ADC = \angle BDA + \angle BAD$ .. $\angle BAD = \theta$ $\therefore \triangle DBC ||| \triangle ABC$ (equiangular)

ii) As $\triangle ABC ||| \triangle ABD$

$\frac{1}{x} = \frac{x-1}{1}$ $\therefore 1 = x^2 - x$ or

$x^2 - x - 1 = 0$

iii) $\cos\theta = \frac{1 + x^2 - 1^2}{2x} = \frac{x}{2}$

Solving $x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{1+4}}{2}$

$\therefore x = \frac{1+\sqrt{5}}{2}$ as $\cos\theta = \frac{x}{2}$

$\cos\theta = \frac{1+\sqrt{5}}{...}$

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