

SYDNEY BOYS' HIGH SCHOOL

MOORE PARK, SURRY HILLS



PROGRESS TEST - 5 March 2008

MATHEMATICS

Time allowed — One Period
Examiner: A.M. Gainford

DIRECTIONS TO CANDIDATES

- ALL questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Start each question on a new page.
- If required, additional paper may be obtained from the Examination Supervisor upon request.

Question 1. (15 Marks)

(a) Find $0.\overline{17}$ as a simplified common fraction.

(b) Factorise $2a^2 - a - 3$. $2a^2 - 3a + 2a - 3$

(c) Factorise $a^3 + 27$.

(d) Simplify $\frac{4x+6}{4x^2-9}$.

(e) Solve simultaneously $5x - 4y = 17$
 $3x + 2y = 8$

(f) Solve these inequalities:

(i) $x^2 + 4x - 5 < 0$.

(ii) $\frac{x-2}{6-x} \geq 1$.

(g) Solve the following system of equations:

$$x - 2y = 3; 4y - 3z = 4; x + 3z = 2$$

Question 2. (15 Marks)

(a) (i) Find the points of intersection of the line $y = 3 - x$ and the circle $x^2 + y^2 = 9$.

(ii) Hence sketch the region where $y \geq 3 - x$ and $x^2 + y^2 < 9$ hold simultaneously.

(b) Find the perpendicular distance between the parallel lines $2x - 5y + 8 = 0$ and $2x - 5y - 1 = 0$.

(c) Solve for x : $|2x - 11| = 3x - 4$.

(d) Find the exact value of $\sin 135^\circ + \tan 480^\circ$.
Express your answer as a single fraction with rational denominator.

(e) Simplify $(\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2$.

(f) Find the centre and radius of the circle: $x^2 + y^2 - 6x + 8y = 0$.

(g) Find the equation of the line through the intersection of the lines $2x - 5y - 3 = 0$ and $3x + 4y - 9 = 0$, and the point $P(-1, -3)$.

Question 3. (15 Marks)

(a) Find θ to the nearest minute if $0^\circ \leq \theta \leq 90^\circ$ and $\cos \theta = 0.147$.

(b) Sketch the graph of $y = \frac{x-1}{x+2}$ showing all essential features.

(c) Given that $f(x) = \frac{4x^2}{\sqrt{9-x^2}}$:

(i) Find $f(2)$.

(ii) Show that $f(x)$ is an even function.

(d) Sketch the graphs of the following, showing their principal features:

(i) $y = 1 - x^2$

(ii) $y = |x-1| - 2$

(e) State the natural domain and range of the function $f(x) = \frac{x}{\sqrt{4-x^2}}$.

(f) Prove the identity

$$\frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta} = 2 \sec^2 \theta$$

(g) Consider the function $y = \frac{1}{|x-1|}$.

(i) What is the natural domain of the function?

(ii) Write down the equations of the two branches of the function, and sketch its graph.

This is the end of the paper.