SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2003

YEAR 11 YEARLY EXAMINATION

Mathematics

General Instructions
- Reading time — 5 minutes
- Working time — 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Start each Section in a new booklet

Total marks — 130
- Attempt all questions
- Questions are not of equal value, the mark value is shown beside each part.
- Hand up your paper in five parts:
  Section A, Questions 1 & 2;
  Section B, Questions 3 & 4;
  Section C, Questions 5 & 6;
  Section D, Questions 7 & 8;
  Section E, Questions 9 & 10

Examiner: R.Boros

Total marks - 130
Attempt Questions 1-10

Answer each Section in a SEPARATE writing booklet. Extra writing booklets are available upon request.

Section A Use a SEPARATE writing booklet. **Marks**

Question 1 (16 marks)

(a) Express each of the following as a rational number.

(i) $49^{-\frac{1}{2}} \times 27^{\frac{2}{3}}$ **1**

(ii) The quotient of $\sqrt{7}$ and $\sqrt{63}$ **1**

(iii) $\log_2 8$ **1**

(iv) $\dfrac{\sqrt{32} - \sqrt{8}}{3\sqrt{2}}$ **2**

(b) Find, correct to 2 decimal places, $\dfrac{(3{\cdot}24)^2}{5{\cdot}73 - 2{\cdot}84}$. **1**

(c) Solve for x, $\dfrac{2x}{x-5} = \dfrac{3}{5}$. **2**

(d) Factorise fully the expression $x^3 - x^2 - x + 1$. **2**

(e) Find the centre and radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$. **2**

(f) Solve the inequality $x^2 - 4x < 0$. **2**

(g) Given $v^2 = u^2 - 2aS$, $v = 2{\cdot}5$, $u = 2{\cdot}3$, and $a = 7$, find S correct to 3 significant figures. **2**

Section A continued **Marks**

Question 2 (14 marks)

(a) Given the points $A(-5, 3)$, $B(1, -5)$, and $C(2, 2)$,

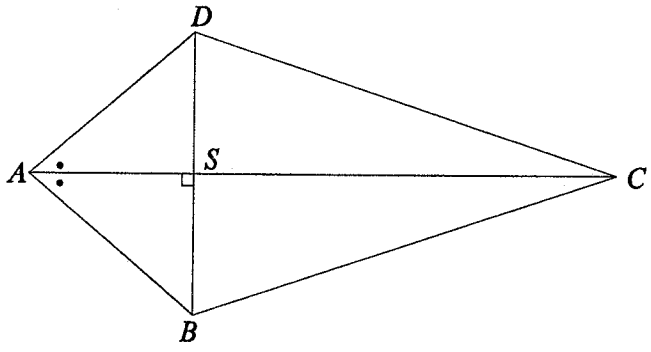
(i) Find the length of AB. **1**

(ii) Find the equation of the line AB written in general form. **2**

(iii) Find the perpendicular distance of C from the line AB. **2**

(iv) Hence or otherwise, calculate the area of the triangle ABC. **1**

(b)



In the above diagram, not to scale, $ABCD$ is a quadrilateral. The diagonals AC and DB intersect at right angles at point S. $D\hat{A}S = B\hat{A}S$.

(i) Prove that ΔASB is congruent to ΔASD. **2**

(ii) Hence prove that $DA = BA$. **1**

(c) Given $f(x) = x^2 + 3x + 2$,

(i) Evaluate $f(-3)$. **1**

(ii) Find a simple expression for $f(a+2)$. **2**

(d) Solve for x, $|2x - 1| \leq 5$. **2**

Section B Use a SEPARATE writing booklet. **Marks**

Question 3 (13 marks)

(a) Doug observes a clifftop A from his yacht at position P. He then sails 500 m towards the cliff to position Q. The angle of elevation to the clifftop from P is 5° and from Q is 8°.

(i) Draw a diagram to illustrate the above information and use the Sine Rule to calculate AQ correct to the nearest metre. **2**

(ii) Hence or otherwise find the distance QB correct to the nearest 10 metres. **2**

(b) Simplify the expression $\dfrac{\tan\theta}{\cot\theta} - \dfrac{\sec^2\theta}{1}$. **2**

(c) (i) Find θ given that $\sin\theta = \frac{1}{2}$ where $0^\circ \leq \theta \leq 180^\circ$. **1**

(ii) Hence find the exact values of $\tan\theta$ and $\sec\theta$. **2**

(d) Simplify $\sin\theta\cos(90^\circ - \theta) + \cos\theta\sin(90^\circ - \theta)$. **2**

(e) If $\sin\theta = \frac{8}{17}$ and θ is an acute angle, find the exact values of $\cos\theta$ and $\tan\theta$. **2**

Section B continued **Marks**

Question 4 (15 marks)

(a) Sketch the area defined by the inequality $y \leq x^2$. **2**

(b) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$. Find the values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $(\alpha + 1)(\beta + 1)$ **1**

(c) State the domain and range of each of these functions.

(i) $x^2 + 3 = y$ **2**

(ii) $y = 3^x$ **2**

(d) Solve for x the equation $9^x - 9(3)^x = 0$. **3**

(e) The roots of the quadratic equation $px^2 - x + q = 0$ are -1 and 3. Find p and q. **3**

Section C Use a SEPARATE writing booklet. **Marks**

Question 5 (15 marks)

(a) A parabola has the equation $y = x^2 - 12x + 20$. Find

(i) where it cuts the x and y axes, **2**

(ii) its axis of symmetry and vertex, **2**

(iii) the focus, by first expressing it in the form $(x-h)^2 = 4a(y-k)$, **2**

(iv) the equation of the directrix. **1**

(b) For each of the quadratics below, evaluate the discriminant and state the relevance of this with regard to the roots of the equation.

(i) $x^2 + 3x + \frac{9}{4} = y$ **2**

(ii) $3x^2 - 2x = y + 5$ **2**

(c) Find the values of M for which the equation $4x^2 - Mx + 9 = 0$ has

(i) exactly one real root, **2**

(ii) real roots. **2**

Section C continued **Marks**

Question 6 (14 marks)

(a) Solve the following equations simultaneously, $4x - y = 3$ and $10x + 3y = 2$. **2**

(b) Find x given $2\log_9\sqrt{3} + \log_9 81 = x$. **3**

(c) Find x correct to 3 decimal places given that $7^x = 15$. **2**

(d) If Ron invests \$500 at 12·5 % p.a. compound interest, how long would it take the investment to grow to a sum of \$1000. (Answer in years, correct to 2 decimal places.) **3**

(e) The r^{th} term of a series is $3 \times 2^{(r-4)}$. Determine which of the numbers 96, ¾, 256 belong to the series. **2**

(f) Evaluate $\sum_{n=-1}^{7} (2n + 3)$. **2**

Section D Use a SEPARATE writing booklet. **Marks**

Question 7 (10 marks)

(a) Let A (0, -2) and B (1, 0) be 2 fixed points and let P (x, y) be a variable point. **3**
Find the locus of P such that the length $(PA)^2$ equals the length $(PB)^2$.

(b) Find the locus of points 2 units away from the line $y = 3$. **2**

(c) Sketch the graph of $y = \sin\theta$ given $-180° < \theta < 90°$. **3**

(d) Find the values of a and b given that **2**

$$(ax - 3)^2 + b = 4x^2 - 12x + 15.$$

Section D continued **Marks**

Question 8 (12 marks)

(a) Differentiate each of the following:

(i) $y = 2x^3 - 8$ **1**

(ii) $y = (2x - 1)^3$ **1**

(iii) $y = \dfrac{2x}{1-3x}$ **2**

(iv) $y = x^2\sqrt{x}$ **2**

(v) $y = \dfrac{7}{2x^3}$ **1**

(b) (i) Find the gradient of the normal to the curve $y = 1 - \frac{1}{2}x^2$ at the point (1, 3). **3**

(ii) Find the point on $y = 1 - \frac{1}{2}x^2$ where the tangent to the curve is parallel to this normal. **2**

Section E Use a SEPARATE writing booklet. **Marks**

Question 9 (11 marks)

(a) Evaluate the following limits:

(i) $\lim_{x \to 5} \left(\frac{x-5}{x^2-25} \right)$ **2**

(ii) $\lim_{x \to \infty} \left(\frac{7-2x-3x^2}{5x^2+3} \right)$ **3**

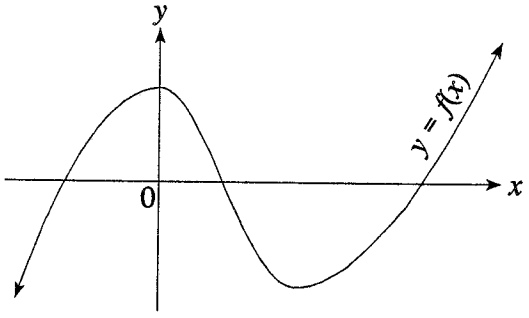
(b) Two cars depart town A at the same time. Car X travels at 60 km/h on a bearing of 345°T, whilst car Y travels at 100 km/h on a bearing of 085°T. How far apart would the cars be after 3 hours? Answer correct to one decimal place. **3**

(c) Differentiate $y = x^2 + x$ from first principles. **3**

Section E continued **Marks**

Question 10 (10 marks)

(a) The diagram below shows $y = f(x)$. **3**



Copy or trace this diagram into your answer booklet and sketch a possible graph for $y = f'(x)$ on the same set of axes.

(b) A farmer wishes to make a rectangular enclosure using a river as one boundary and 400 m of fencing on the other three sides.

(i) Find the maximum possible area of the enclosure. **3**

(ii) What are the dimensions of this enclosure? **1**

(c) Find x correct to 3 decimal places, given that $\log_7 6 - 2\log_7 3 = x$. **3**

END OF THE PAPER

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

SEPTEMBER 2003

YEARLY EXAMINATION

YEAR 11

Mathematics

Sample Solutions

[16 marks]

Question (1)

(a) $49^{-\frac{1}{2}} \times 27^{\frac{2}{3}}$

(i) $= \frac{1}{7} \times (3^3)^{\frac{2}{3}}$

$= \boxed{9/7}$ (1)

(ii) $\frac{\sqrt{7}}{\sqrt{63}} = \boxed{\frac{1}{3}}$ (1)

(iii) $\log_2 8 = 3\log_2 2$

$= \boxed{3}$. (1)

(iv) $\frac{2\sqrt{2}(2-1)}{3\sqrt{2}} = \boxed{\frac{2}{3}}$. (2)

(b). $\frac{(3.24)^2}{5.73 - 2.84} = \boxed{3.63}$ (2d.p). (1)

(c) $\frac{2x}{x-5} = \frac{3}{5}$

$10x = 3x - 15$

$7x = -15$

$\boxed{x = -15/7}$. (2)

(d) $x^2(x-1) - (x-1)$

$= \boxed{(x-1)^2(x+1)}$. (2)

(e) $x^2 - 6x + 9 + y^2 + 4y + 4$ (2)

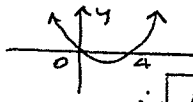
$= 25$

$\therefore (x-3)^2 + (y+2)^2 = 25$

Centre $(3, -2)$	$r = 5$

(f) $x^2 - 4x < 0$

$x(x-4) < 0$



(2)

$\therefore \boxed{0 < x < 4}$.

(g) $(2.5)^2 = (2.3)^2 - 14(s)$. (2)

$14s = 0.96$

$\therefore \boxed{s = -0.0686}$ (3s.f.)

Question (2). [14 marks]

(a) (i) $AB = \sqrt{(-6)^2 + 8^2}$

$= 10$ (1)

(ii) $m = \frac{-8}{6} = -4/3$

$y - 3 = -4/3(x+5)$ (2)

$3y - 9 = -4x - 20$

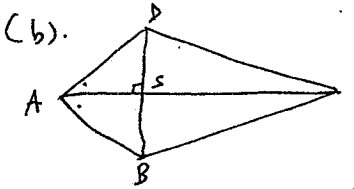
$\therefore \boxed{4x + 3y + 11 = 0}$

(iii) $d = \frac{|8 + 8 + 11|}{\sqrt{25}} = \frac{27}{5} = 5\frac{2}{5}$ (2)

(iv) $\therefore A = \frac{1}{2} \times 10 \times \frac{2\pi}{5} 5$

$= \boxed{25 \text{ sq units}}$. (1)

(b).



(2)

$\angle DAS = \angle BAS$ (given)

$\angle DSA = \angle BSA$ (st. line)

AS is common

$\therefore \triangle ASB \equiv \triangle ASD$ (AAS)

$\therefore DA = BA$

(Corresp. sides of cong. $\triangle$s are equal) (1).

(c) $f(x) = x^2 + 3x + 2$

(i) $f(-3) = 2$ (1)

(ii) $(a+2)^2 + 3(a+2) + 2$

$= \boxed{a^2 + 7a + 12}$ (2)

$(a+3)(a+4)$

(d) $|2x - 1| \leq 5$

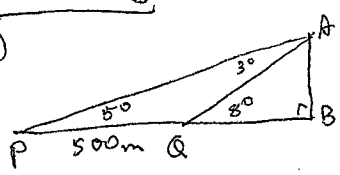
$-5 \leq 2x - 1 \leq 5$

$-4 \leq 2x \leq 6$

$\boxed{-2 \leq x \leq 3}$ (2)

Question 3

(a)



[1]

(i) $\angle PAQ = 3°$ (ext. angle thm)

$$\frac{AQ}{\sin 5°} = \frac{500}{\sin 3°}$$

$$AQ = \frac{500 \sin 5°}{\sin 3°}$$

$$\doteq 833\text{ m}$$

[1]

(ii) $\cos 8° = \frac{QB}{AQ}$

$\therefore QB = AQ \cos 8°$

$\doteq 820$ m (nearest 10m)

[2]

(b) $\frac{\tan\theta}{\cot\theta} - \sec^2\theta$

$= \tan^2\theta - \sec^2\theta$

$= \tan^2\theta - (1 + \tan^2\theta)$

$= -1$

[2]

(c) (i) $\sin\theta = \frac{1}{2}$ $\quad 0° \leq \theta \leq 180°$

$\theta = 30°$ or $150°$

[1]

(ii) $\tan\theta = \frac{1}{\sqrt{3}}$ or $\frac{-1}{\sqrt{3}}$

$\sec\theta = \frac{2}{\sqrt{3}}$ or $-\frac{2}{\sqrt{3}}$

[2]

(d) $\sin\theta \cdot \cos(90-\theta) + \cos\theta \sin(90-\theta)$

$= \sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta$

$= \sin^2\theta + \cos^2\theta$

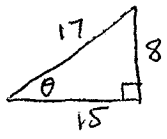
$= 1$

[2]

(e) $\sin\theta = \frac{8}{17}$

$\cos\theta = \frac{15}{17}$

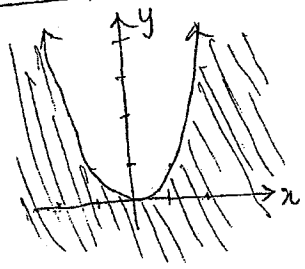
$\tan\theta = \frac{8}{15}$



[2]

Question 4

(a)



[2]

(b) $x^2 - 5x + 2 = 0$

(i) $\alpha + \beta = 5$ [1]

(ii) $\alpha\beta = 2$ [1]

(iii) $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$

$= 2 + 5 + 1$

$= 8$

[1]

(c) (i) $y = x^2 + 3$ $\quad D = \{x : x \in \mathbb{R}\}$

$R = \{y : y \geq 3\}$

(ii) $y = 3^x$ $\quad D = \{x : x \in \mathbb{R}\}$

$R = \{y : y > 0\}$

Question 4 (cont'd)

(d) $9^x - 9(3)^x = 0$

$(3^x)^2 - 9(3^x) = 0$

Let $u = 3^x$

$u^2 - 9u = 0$

$u(u-9) = 0$

$\therefore u = 0$ or $u = 9$

$\therefore 3^x = 0$ or $3^x = 9$

No soln $\quad x = 2$ [3]

(e) $px^2 - x + q = 0$

Roots -1 and 3

$\alpha + \beta = \frac{1}{p} \qquad \alpha\beta = \frac{q}{p}$

$\therefore \frac{1}{p} = 2 \qquad \frac{q}{p} = -3$

$p = \frac{1}{2} \qquad 2q = -3$

$q = \frac{-3}{2}$

[3]

[2] 5. (a) (i) On the x-axis, $y = 0$; so $(x-10)(x-2) = 0$.
On the y-axis, $x = 0$; so $y = 20$.
$\therefore$ The curve meets the x-axis at 2, 10, and the y-axis at 20.

[2] (ii) The midpoint of 2 and 10 is 6, so the axis of symmetry is $x = 6$.
When $x = 6$, $y = -16$, so the vertex is $(6, -16)$.

[2] (iii) $x^2 - 12x + 36 = y - 20 + 36$,
$(x-6)^2 = 4 \times \frac{1}{4} \times (y+16)$.
$\therefore$ The focus is at $(6, -15\frac{3}{4})$.

[1] (iv) The directrix is $y = -16\frac{1}{4}$.

[2] (b) (i) $\Delta = 9 - 9 = 0 \Longrightarrow$ one real root.

[2] (ii) $\Delta = 4 + 60 = 64 \Longrightarrow$ two real, rational roots.

[2] (c) (i) $\Delta = M^2 - 144 = 0$,
$M^2 = 144$.
$\therefore$ One root when $M = \pm 12$.

[2] (ii) $M^2 - 144 \geq 0$,
$M^2 \geq 144$,
$|M| \geq 12$.
$\therefore$ $M \leq -12$, $M \geq 12$.

[2] 6. (a) $4x - y = 3$, $\cdots\cdots(1)$
$10x + 3y = 2$. $\cdots\cdots(2)$
$y = 4x - 3$ from (1), then substitute in (2).
$10x + 3(4x - 3) = 2$,
$10x + 12x - 9 = 2$,
$22x = 11$,
$x = \frac{1}{2}$,
$y = -1$.

[3] (b) $\log_9 3 + \log_9 9^2 = x$,
$\log_9 9^{\frac{1}{2}} + \log_9 9^2 = x$,
$\frac{1}{2}\log_9 9 + 2\log_9 9 = x$,
$\frac{1}{2} + 2 = x$,
i.e., $x = 2\frac{1}{2}$ (or $\frac{5}{2}$).

[2] (c) $\log 7^x = \log 15$,
$x \log 7 = \log 15$,
$x = \dfrac{\log 15}{\log 7}$,
$\approx 1{\cdot}392$.

3 (d) $\$1\,000 = \$500\left(1+\frac{12{\cdot}5}{100}\right)^n$,

$2 = (1{\cdot}125)^n$,

$\log 1{\cdot}125^n = \log 2$,

$n \log 1{\cdot}125 = \log 2$,

$n = \dfrac{\log 2}{\log 1{\cdot}125}$

$n \approx 5{\cdot}88$ years.

2 (e) $96 \div 3 = 32$, $= 2^5$.

$\frac{3}{4} \div 3 = \frac{1}{4}$, $= 2^{-2}$.

$256 \div 3 = 85\frac{1}{3}$, Not a power of 2.

$\therefore$ $96, \frac{3}{4}$ belong to the series.

2 (f) $\displaystyle\sum_{n=-1}^{7}(2n+3) = 1+3+5+7+9+11+13+15+17,$

$= 81.$

QUESTION 7

(a) $A(0,-2)$ $B(1,0)$ $(PA)^2=(PB)^2$

$(x-0)^2+(y+2)^2=(x+1)^2+(y-1)^2$

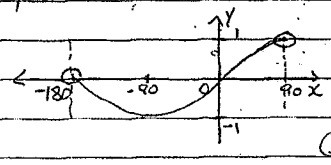
$x^2+y^2+4y+4=x^2-2x+1+y^2$

$4y+2x+3=0$

$2x+4y+3=0$ (3)

(b) $y=5$ or $y=1$ (2)

(c) $y=\sin\theta$ $-180^\circ\leq\theta<90^\circ$



(3)

(d) $(ax-3)^2+b=4x^2-12x+15$

$a^2x^2-6ax+9+b=4x^2-12x+15$

equating coefficients

$a^2=4$ $-6ax=-12x$ $9+b=15$

$a=\pm 2$ $a=2$ $b=6$

$a=2$, $b=6$

or

$(ax-3)^2+b=4x^2-12x+9+6$

$(ax-3)^2+b=(2x-3)^2+6$

$a=2$ $b=6$ (2)

QUESTION 8

(a)(i) $y=2x^3-8$

$y'=6x^2$ (1)

(ii) $y=(2x-3)^3$

$y'=3(2x-3)^2\times 2$

$=6(2x-3)^2$ (1)

(iii) $y=\frac{2x}{1-3x}$

$y'=\frac{v\,du-u\,dv}{v^2}$

$=\frac{(1-3x)\times 2-2x\times -3}{(1-3x)^2}$

$=\frac{2}{(1-3x)^2}$ (2)

(iv) $y=x^2\sqrt{x}$

$=x^{\frac{5}{2}}$

$y'=\frac{5}{2}x^{\frac{3}{2}}$

$=\frac{5}{2}\sqrt{x^3}$ (2)

(v) $y=\frac{7}{2x^3}=\frac{7}{2}x^{-3}$

$y'=-\frac{21}{2}x^{-4}$ (1)

$=-\frac{21}{2x^4}$

(b) $y=1-\frac{1}{2}x^2$

(i) $y'=-x$

when $x=1$ grad of tangent $=-1$

" normal $=1$ (3)

(ii) $y'=-x$

$-x=1$ when $x=-1$

point $(-1,\frac{1}{2})$ (2)

Question 9.

(a) (i) $\lim_{x\to 5} \dfrac{x-5}{x^2-25}$

$= \lim_{x\to 5} \dfrac{x-5}{(x-5)(x+5)}$

$= \lim_{x\to 5} \dfrac{1}{(x+5)}$

$= \dfrac{1}{10}$

(ii) $\lim_{x\to \infty} \dfrac{7-2x-3x^2}{5x^2+3}$

$= \lim_{x\to \infty} \dfrac{\frac{7}{x^2}-\frac{2}{x^2}-\frac{3x^2}{3x^2}}{\frac{5x^2}{5x^2}+\frac{3}{x^2}}$

$= -\dfrac{3}{5}$

(b) In 3 hours car X travels 180km.
In 3 hours car Y travels 300km.

$XY^2 = AY^2 + AX^2 - 2AY.AX.\cos A$

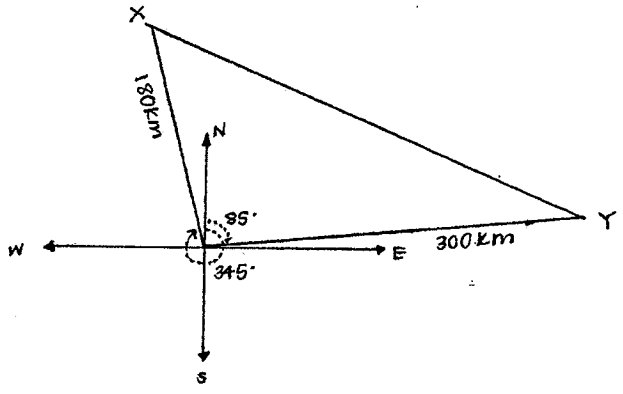
$= 300^2 + 180^2 - 2(300)(180)\cos 100°$

$XY^2 = AY^2 + AX^2 - 2AY.AX.\cos A$

$= 300^2 + 180^2 - 2(300)(180)\cos 100°$

$XY = 375.7$

$\therefore XY = 375.7$ km (1d.p.)



(c) Let $f(x) = x^2 + x$

Let $f(x+h) = (x+h)^2 + (x+h)$

$$f'(x) = \lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to \infty} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h}$$

$$= \lim_{h \to \infty} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$$

$$= \lim_{h \to \infty} \frac{2xh + h^2 + h}{h}$$

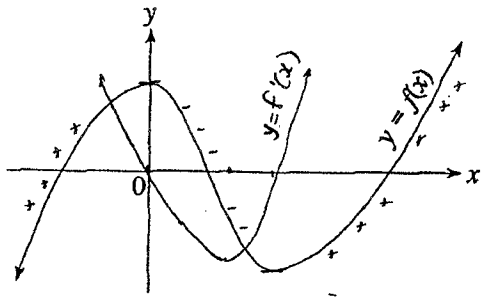
$$= \lim_{h \to \infty} \frac{h(2x + h + 1)}{h}$$

$$= 2x + 1$$

$\therefore f'(x) = 2x + 1$ by first principles.

Question 10.

(a)



(b) (i) Let $P = 2x + y$, where $P = 400$.

$$400 = 2x + y$$
$$\therefore y = 400 - 2x$$

$$\text{Area} = xy$$
$$= x(400 - 2x)$$
$$= 400x - 2x^2$$

Using the axis of symmetry formula, $x = -\dfrac{b}{2a}$, where $a = -2, b = 400$.

$$x = -\frac{b}{2a}$$
$$= -\frac{400}{2(-2)}$$
$$= 100$$

At $x = 100, y = 200$.

Hence the maximum possible area of this enclosure is $A = 20000$ m^2

(ii) Hence the dimensions of this enclosure are 100m by 200m.

(c)

$$\log_7 6 - 2\log_7 3 = x$$
$$\log_7 6 - \log_7 3^2 = x$$
$$\frac{\log_{10} 6}{\log_{10} 7} - \frac{\log_{10} 9}{\log_{10} 7} = x$$
$$0.9207 - 1.1291 = x$$
$$x = -0\cdot 20845$$
$$\therefore x = -0\cdot 208 \text{ (3dp).}$$

or

$$\log_7\left(\frac{6}{3^2}\right) = x$$
$$\log_7\left(\frac{2}{3}\right) = x$$
$$7^x = \frac{2}{3}$$
$$x\log 7 = \log\frac{2}{3}$$
$$\therefore x = -0.208$$